Meson decays from string splitting

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Collaboration with L. Martucci, W. Troost (Leuven University) and F. Bigazzi (Paris VI and VII)

Phys. Rev. Lett. 96 (2006); hep-th/0511045 and hep-th/0606059.

Strong coupling gauge theories

The String/Gauge theory correspondence states the equivalence of a string theory (or M-theory) on a non-trivial background and a gauge theory.

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Study certain exclusive decays of high spin mesons into mesons in models of large N_c quenched QCD at strong coupling.

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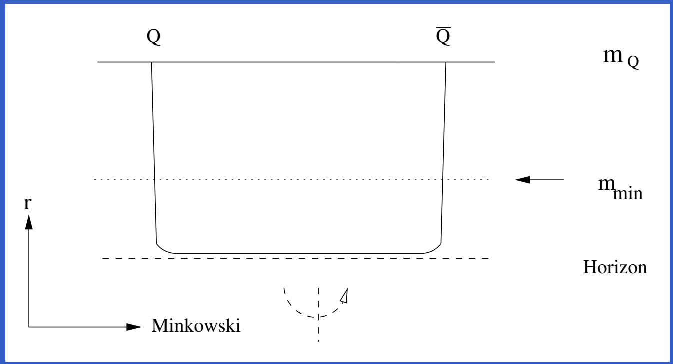
- Lower bound for effective quark mass $\Rightarrow$ Minimum, non-zero value of radial position of flavor branes.
- Large spin $J \Rightarrow$ string almost “straight”:



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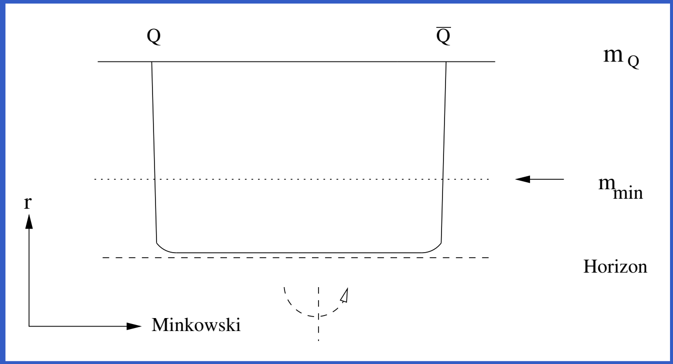
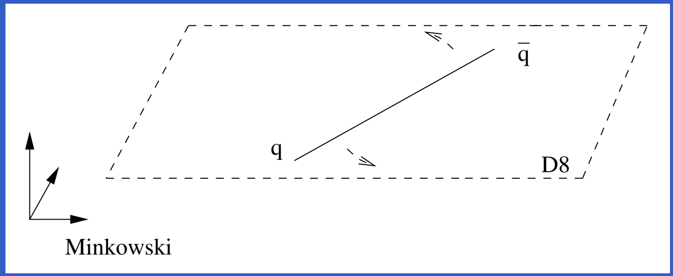


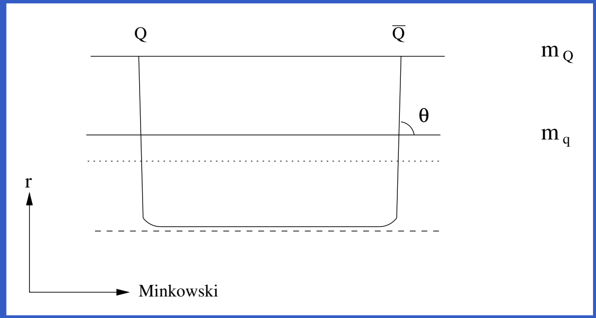
II. [Sakai, Sugimoto 2004]: Add $N_f \ll N_c$ D8/anti-D8 $\Rightarrow$ massless flavors.

- Realization of χSB (massless pions) but no mass parameter.
- String spins on the D8 world-volume:



The process, Model I

Lighter flavor probe brane q: mass $m_q < m_Q$:



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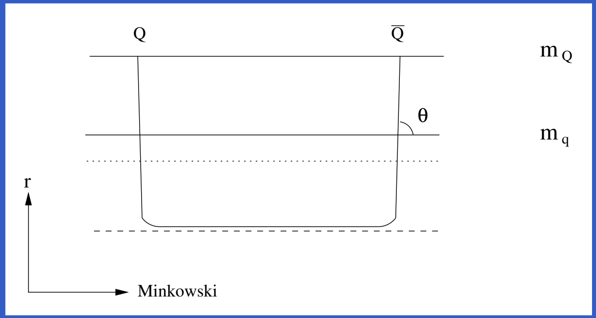

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For the metric in the form:

$$ds^2 = e^{A(r)}(-dt^2 + d\rho^2 + \rho^2 d\eta^2 + dx_3^2) + e^{B(r)} dr^2 + G_{ij} d\phi^i d\phi^j$$

and string configuration: $t = \tau \qquad \eta = \omega\tau \qquad r = \sigma \qquad \rho = \rho(\sigma) \qquad \phi^i = \phi^i_Q$

the angle is: $\cos^2\theta = \frac{(\rho'(r_q))^2}{e^{B(r_q)-A(r_q)} + (\rho'(r_q))^2}.$

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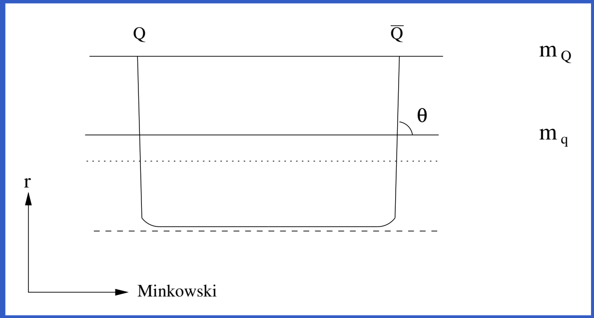

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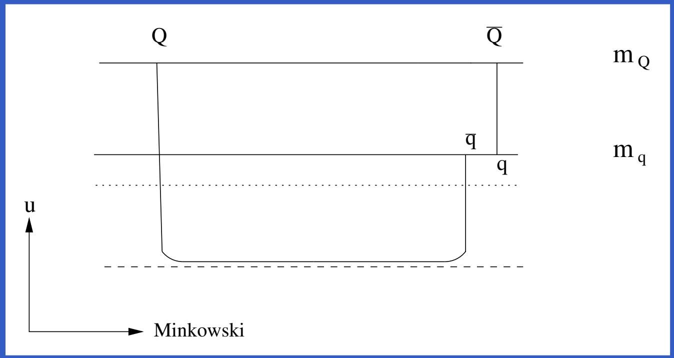
If $\theta \neq \frac{\pi}{2} \Rightarrow$ net transversal force $\Rightarrow$ string can split: $\bar{Q}Q \to \bar{Q}q + \bar{q}Q$.



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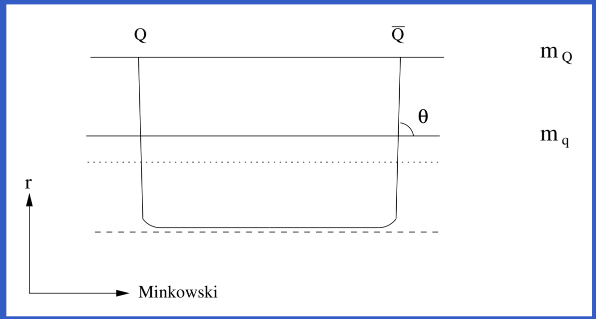

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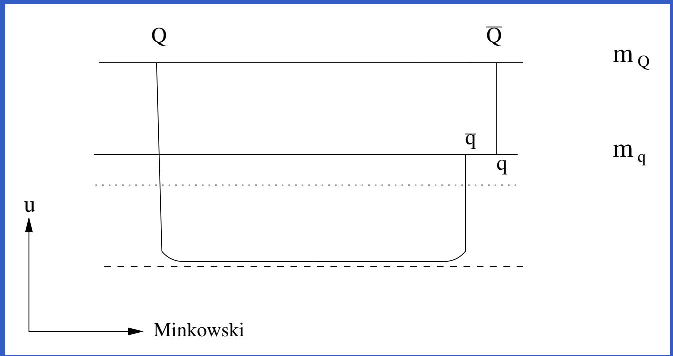

Note: decay highly constrained, no phase space.

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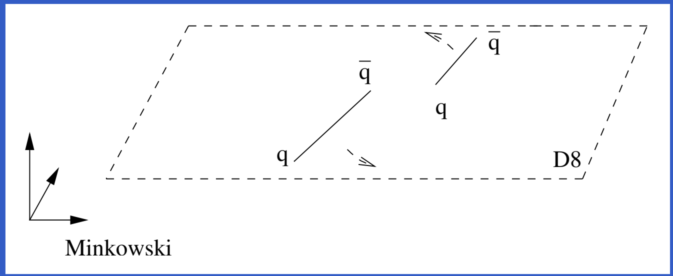

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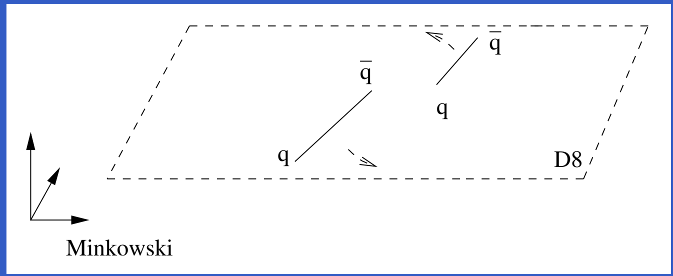

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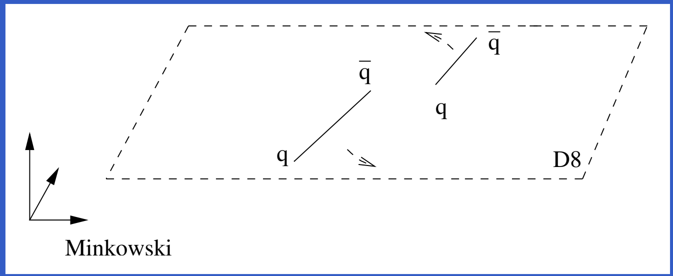

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Space is curved but weakly $\Rightarrow$ compute rate in flat space

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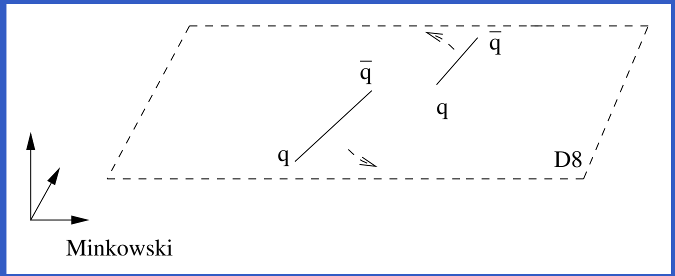

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We computed the rate for splitting of a string intersecting or lying on a Dp-brane.

Using tricks in [Polchinski 1988, Polchinski, Cay 1989, Jackson, Jones, Polchinski 2004].

Decay rate calculations

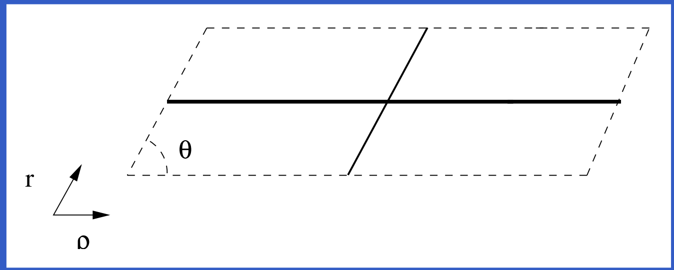
- 1^{st} trick: Compactify space.

  Model I. $R_t \times T^2_\theta \times T^{p-1}_{||} \times T^{8-p}_{\perp}$.

  T^2_θ: $(\rho, r) \simeq (\rho + n_1 l_1 + n_2 l_2 \cos\theta, r + n_2 l_2 \sin\theta),\ n_1, n_2 \in Z.$

  Dp-brane wrapping $T^{p-1}_{||}$ and ρ.

  String wrapping r.



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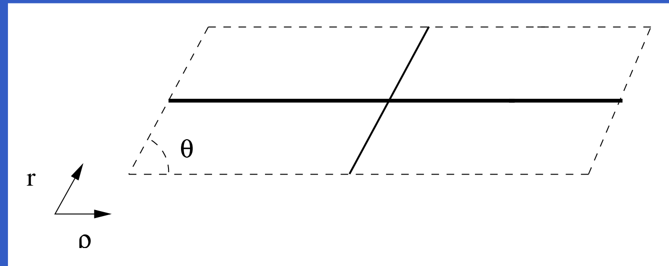

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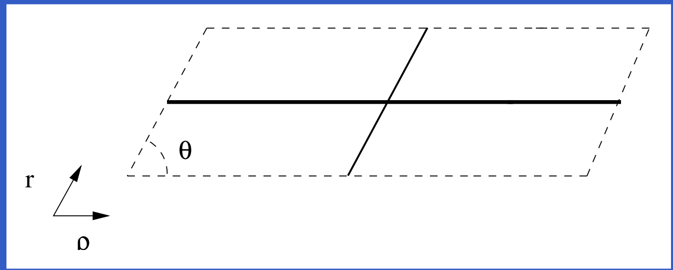

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- 3^{rd} trick: Optical theorem, total decay rate from disk correlator:

$$\mathcal{A} = \langle \mathcal{V}_{(0,0)}(p_L, p_R) \mathcal{V}_{-1,-1}(p'_L, p'_R) \rangle.$$

Decay rate calculations

Vertex operators: $$\mathcal{V}_{(-1,-1)} = \frac{\kappa}{2\pi\sqrt{V}} : e^{-\phi-\tilde{\phi}+ip_L\cdot X+ip_R\cdot\tilde{X}} :$$

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String momenta: Model I: $p = \frac{m}{\sqrt{1-v^2}}(1, 0, 0, \vec{v}, \vec{0})$ $m^2 = (l_2/2\pi\alpha')^2 - 2/\alpha'$

Leading g_s order $\Rightarrow$ D-brane does not recoil $\Rightarrow |\vec{v}| = |\vec{v}'|$.

Model II: $p = m(1, 0, \vec{0}, \vec{0})$ $m^2 = (L/2\pi\alpha')^2 - 2/\alpha'$

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Use: $$\langle X^\mu(z) X^\nu(z')\rangle = -\frac{\alpha'}{2}\eta^{\mu\nu}\log(z-z')$$

$$\langle X^\mu(z) \tilde{X}^\nu(\bar{z}')\rangle = -\frac{\alpha'}{2}G^{\mu\nu}\log(z-\bar{z}')$$

...

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Decay rate calculations

Invariants:

$$\frac{\alpha'}{2} p_L \cdot G \cdot p_R = \frac{\alpha'}{2} p'_L \cdot G \cdot p'_R \equiv -\sigma$$

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Use optical theorem: $\Gamma = \frac{1}{m} \mathrm{Im}\mathcal{M} = \frac{1}{m} \frac{g_s l_2}{32\pi^2} \times \frac{\cos^2\theta}{\sin\theta \alpha'^{3/2}} \times \frac{(2\pi\sqrt{\alpha'})^{8-p}}{V_\perp}$

Decay rate calculations

For long strings obtain:

$$\Gamma_I = \frac{g_s}{16\pi\sqrt{\alpha'}} \cdot \frac{(2\pi\sqrt{\alpha'})^{(8-p)}}{V_\perp} \cdot \frac{\cos^2\theta}{\sin\theta}$$

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- Probability of breaking increases as the string is more parallel to the brane, since the tension creates a bigger transversal force.

Decay rate calculations

For long strings obtain:

$$\Gamma_I = \frac{g_s}{16\pi\sqrt{\alpha'}} \cdot \frac{(2\pi\sqrt{\alpha'})^{(8-p)}}{V_\perp} \cdot \frac{\cos^2\theta}{\sin\theta}$$

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For Model II:

- Rate proportional to L as expected.

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But if impose that in the limit the direction of length $L \sin\theta$ is included among the transverse directions, $V_{\perp(8-p)} = V_{\perp(9-p)}/L\sin\theta$:

$$\Gamma_I \to \frac{g_s}{32\pi^2\alpha'} \cdot \frac{(2\pi\sqrt{\alpha'})^{(9-p)}}{V_{\perp(9-p)}} \cdot L\cos^2\theta$$

is an interpolating rate: $\Gamma_I \to \Gamma_{II} \quad$ for $\quad \theta \to 0$.

Preliminaries

Background:

$$ds^2 = (\frac{u}{R})^{3/2}(dx_\mu dx^\mu + \frac{4R^3}{9u_h} f(u) d\theta_2^2) + (\frac{R}{u})^{3/2} \frac{du^2}{f(u)} + R^{3/2} u^{1/2} d\Omega_4^2$$

$$f(u) = (u^3 - u_h^3)/u^3 \qquad e^\Phi = g_s \left(\frac{u}{R}\right)^{3/4}.$$

String/FT dictionary:

$$u_h = \frac{\lambda m_0 \alpha'}{3}, \qquad g_s = \frac{\lambda}{3\pi N_c m_0 \sqrt{\alpha'}}, \qquad R^3 = \frac{\lambda \alpha'}{3 m_0}, \qquad T = \frac{\lambda m_0^2}{6\pi},$$

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Quark mass in Model II: energy of string stretching from u_Q to u_h:

$$m_Q = \frac{T}{m_0}\int_1^{u_Q/u_h} dz\left[1 - \frac{1}{z^3}\right]^{-\frac{1}{2}}.$$

Meson decay: Model II.

Let us translate Γ_{II}

- First factor: $\frac{g_s}{\alpha'} \longrightarrow \frac{e^{\Phi}}{\alpha'_{eff}} = \frac{\lambda}{N_c} \frac{m_0^2 \lambda^{3/2}}{3^{5/2}\pi}$.

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Rate:

$$\Gamma_{II} = \frac{\lambda}{N_c}\frac{1}{6\sqrt{2}\pi^{3/2}}\frac{1}{\log^{1/2}(1+\frac{8\pi T}{9m_0^2})}\frac{\sqrt{T}}{m_0}M.$$

Rate linear in the mass M of the meson.

$1/N_c$ process, increasing with λ.

In “QCD limit" $T \sim m_0^2 \sim \Lambda_{QCD}$ it is just $\Gamma_{II} \sim \lambda M/N_c$.

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  Profile: $\rho'(u) \simeq \rho'_W(u) + \delta\rho'(u)$ $\delta\rho' \ll 1$

$$\rho' \approx \frac{(Ru_h)^{3/2}}{u_h^3(x^3-1)}\left[1 - \frac{x^3(x-1)}{y(x^3-1)}\right] \qquad x \equiv u_q/u_h, \quad y \equiv u_Q/u_h.$$

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- For $J \gg \lambda$ lower point of the string: $u_0 \sim u_h(1 + e^{-\frac{3m_0 L}{2}})$.
  Since $J \sim L^\sharp \quad \Rightarrow \quad u_0 = u_h$.

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$$\Gamma_I = \frac{\lambda m_0}{16\pi^2 N_c} \frac{\sqrt{x}}{(x^3-1)} \left[1 + \frac{1}{y} \frac{(x-1)(1-2x^3)}{(x^3-1)}\right].$$

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$$\Gamma_I \sim \frac{\lambda}{36\pi^2 N_c} \left(\frac{T}{m_0}\right)^{2} \frac{m_0}{m_q^{2}} \left[1 - \frac{T}{3 m_0 m_Q}\right].$$

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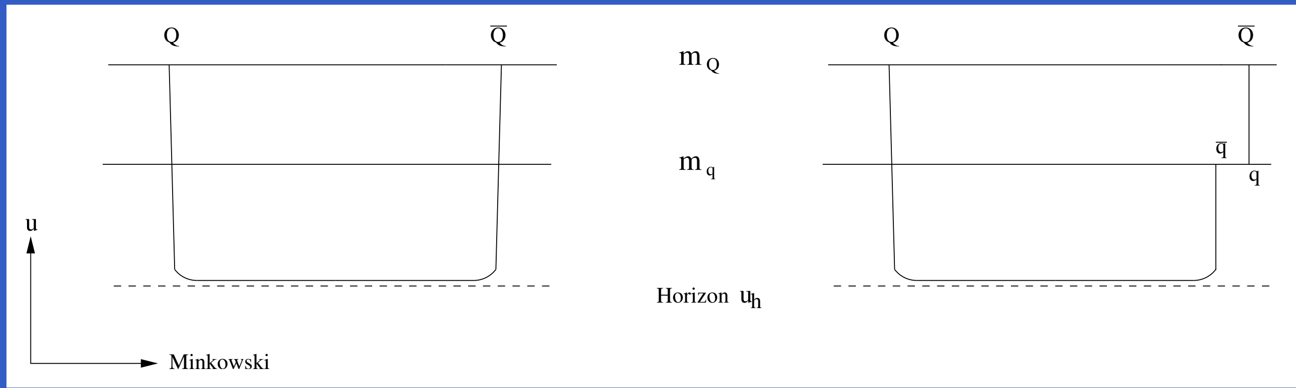
- Same result applies to mesons made up of different heavy quarks.

Physical Picture

- In Model I: flux tube has almost constant energy density apart from small region around the quarks (from shape of the string).



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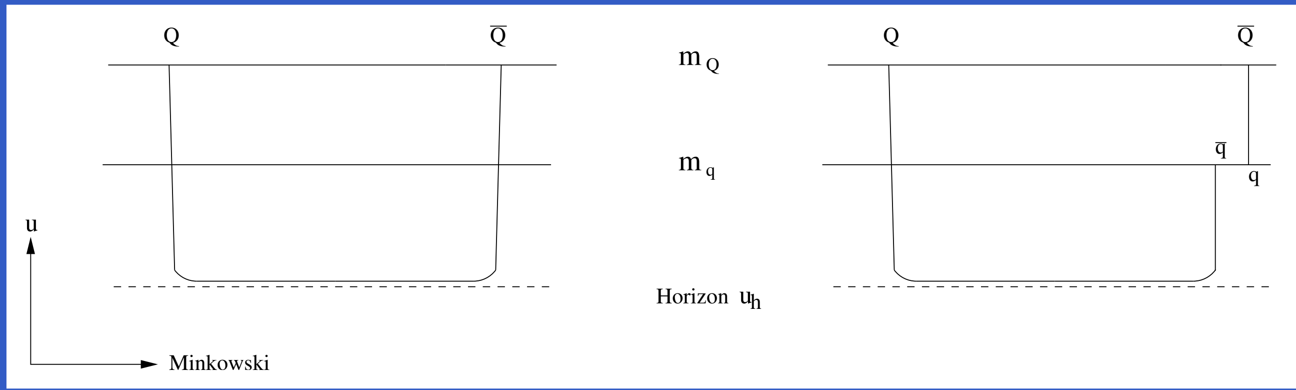

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- In Model II: flux tube has constant energy density everywhere. Decay by pair-production of massless quarks $\Rightarrow$ every piece of the tube has enough energy for the process $\Rightarrow$ rate proportional to the mass (length) of the meson.

Final comments

- Phenomenology.
  For light quarks ($\rightarrow$ Model II) rate maybe linear with M “in average”.
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- Can evaluate along the same lines meson decay rate in $\mathcal{N} = 4 +$ flavors, with nice physical interpretation.
- The high spin glueballs: closed spinning strings at the horizon.
  Semi-classical decay: folded strings...
  Witten model: $\Gamma \sim \frac{\lambda}{N^2} \frac{T^{5/2}}{m_0^4}$ ($\Gamma \sim \frac{\lambda}{N^2} \Lambda_{QCD}$ in “QCD limit”).
  $1/N_c^2$ process that increases with the coupling. No J dependence.
  Large phase space for the decay: can split at any point.