The baryon vertex with magnetic flux

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References: hep-th/0606264

Pre-strings 2007

Workshop on Gravitational Aspects of Strings and Branes

Granada (Spain), 18 - 22 june 2007

Invited speakers: G. Horowitz, J. Maldacena, A. Sen

Outlook

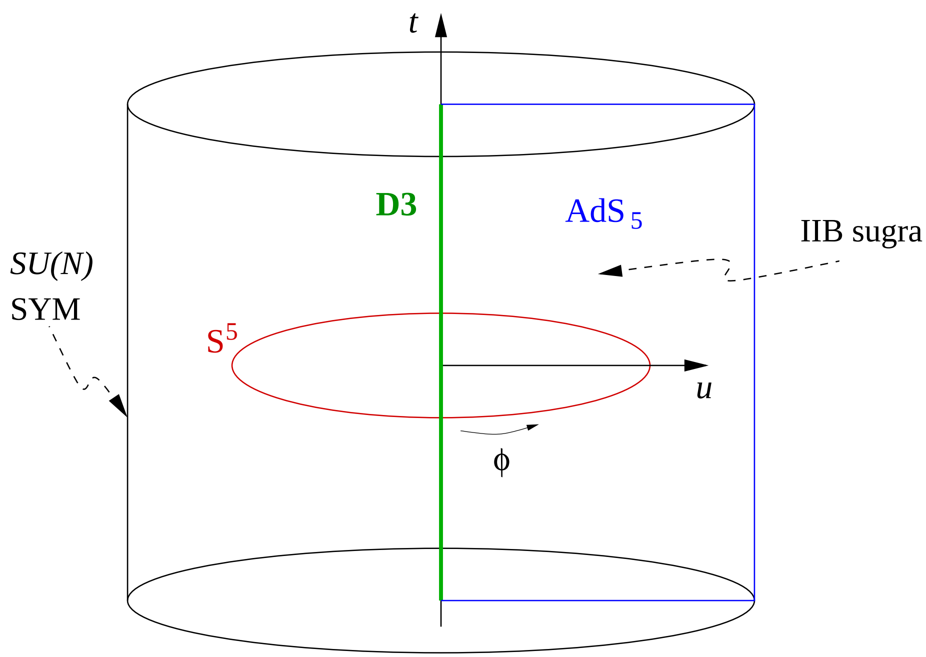
1. The (standard) baryon vertex
2. Adding magnetic flux
3. Bound on the magnetic flux
4. The baryon vertex as a dielectric effect
5. Conclusions

1 The (standard) baryon vertex

Type IIB supergravity (strings) on $AdS_5 \times S^5$
$\sim \ \mathcal{N} = 4$ Super Yang-Mills with gauge group $SU(N)$

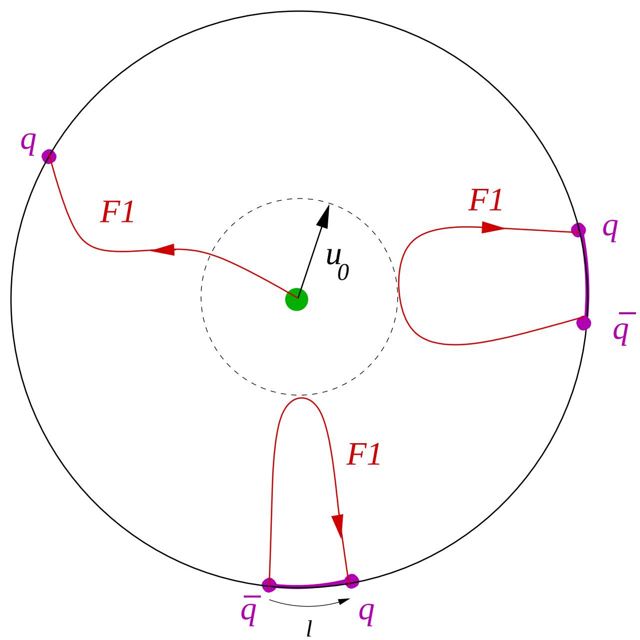
[Maldacena]

$$ds^2 = \frac{u^2}{L^2}\eta_{ab}dx^a dx^b + \frac{L^2}{u^2}du^2 + L^2 d\Omega_5^2, \qquad G_5 = 4L^{-1}\sqrt{|g_{AdS}|} + 4L^4\sqrt{|g_S|}$$



quark of SYM ∼ F1 between horizon and boundary

$q\bar{q}$-pair (meson) in SYM ∼ string "hanging" from boundary



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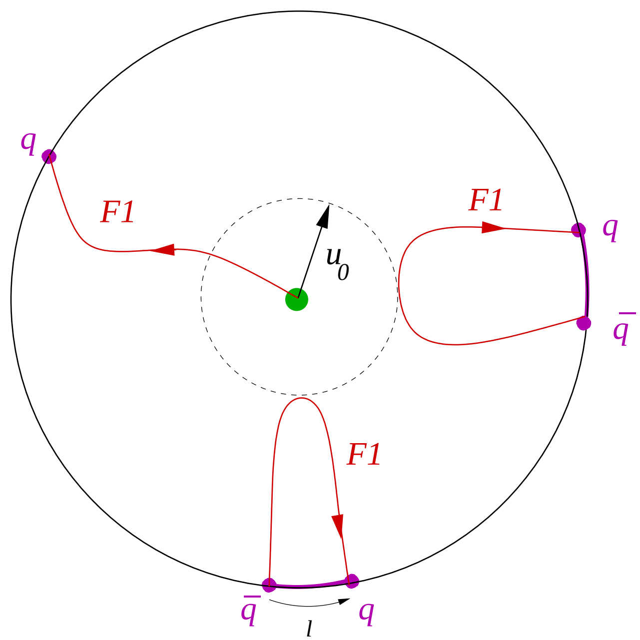
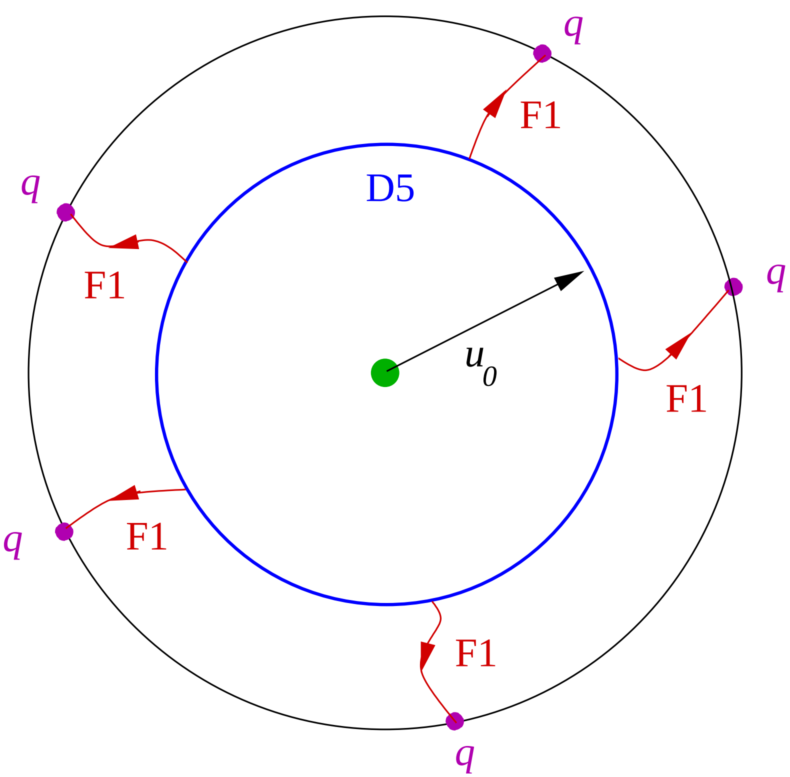


Does there exist a baryon configuration? [Witten]

= colourless antisymmetric bound state of N quarks

NB: No dymanical quarks in $AdS_5 \times S^5$ $\Longrightarrow$ Baryon vertex

Baryon vertex = D5-brane wrapped around S^5 with N strings extending to boundary [Witten]



$$S_{CS} = -T_5 \int_{\mathbb{R}\times S^5} P[C^{(4)}] \wedge F = T_5 \int_{S^5} P[G^{(5)}] \int_{\mathbb{R}} dt A_t = N T_1 \int dt A_t,$$

$\rightarrow$ N units of BI charge induced on D5 worldvolume (= compact)

$\rightarrow$ Add N F1 with same orientation (all q's), between D5 and boundary

$\Longrightarrow$ Baryon vertex

2 The baryon vertex with magnetic flux

Baryon vertex = D5-brane wrapped around S^5

S^5 is $U(1)$ fibre bundle over CP^2

$$d\Omega_5^2 = (d\chi - B)^2 + ds^2_{CP^2},$$

$$B = -\frac{1}{2}\sin^2\varphi_1(d\varphi_4 + \cos\varphi_2 d\varphi_3),$$

$$ds^2_{CP^2} = d\varphi_1^2 + \frac{1}{4}\sin^2\varphi_1\Big(d\varphi_2^2 + \sin^2\varphi_2 d\varphi_3^2 + \cos^2\varphi_1\big(d\varphi_4 + \cos\varphi_2 d\varphi_3\big)^2\Big)$$

Fibre connection B satisfies

$$dB = {}^\star(dB), \qquad\qquad dB \wedge dB \;\sim\; \sqrt{g_{CP^2}} \;\sim\; \sqrt{g_{S^5}}$$

$\rightarrow$ B non-trivial gauge field on CP^2, with non-zero instanton number

Turn on magnetic Born-Infeld flux

$$F = 2n\, dB$$

$$\Rightarrow \int_{CP^2} F \wedge F = 8\pi^2 n^2 .$$

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F is magnetic $\Rightarrow$ no extra terms in Chern-Simons action

New contributions to Born-Infeld action

$$S_{DBI} = -T_5 \int d^6\xi \, \frac{u}{L} \sqrt{\det\Big(g_{\alpha\beta} + F_{\alpha\beta}\Big)}$$

$$= -T_5 \int d^6\xi \, u \, \sqrt{g_{S^5}} \, \Big(L^4 + 2F_{\alpha\beta}F^{\alpha\beta}\Big)$$

$$E = 8\pi^3 T_5 \, u \left(n^2 + \frac{L^4}{8}\right)$$

NB: $\det(g_{\alpha\beta} + F_{\alpha\beta})$ is perfect square $\Longrightarrow$ BPS bound

$F = \sqrt{2n}\, dB$ induces D1 charge in D5 worldvolume

$$S_{D5} \;=\; \frac{1}{2} T_5 \int_{\mathbb{R}\times S^5} P[C^{(2)}] \wedge F \wedge F \;\;=\;\; n^2 T_1 \int_{\mathbb{R}\times S^1} P[C^{(2)}] \;\;=\;\; n^2 \;\, S_{D1}$$

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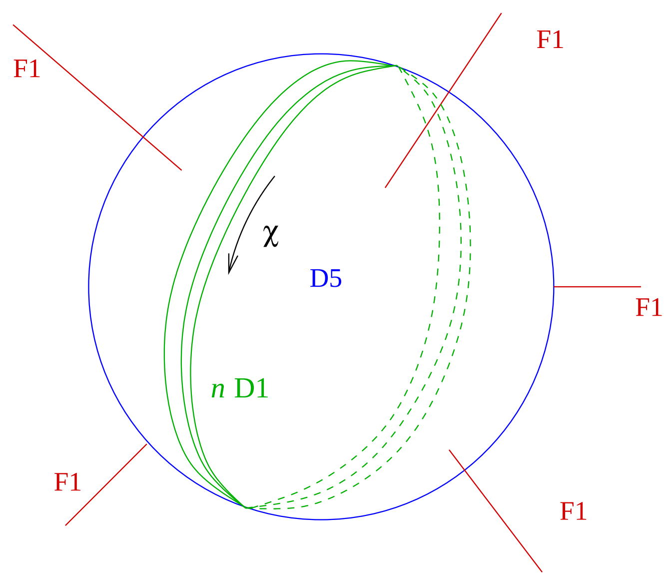
$\longrightarrow n^2$ D1-branes: extended in t- and χ-directions dissolved in D5 worldvolume



NB: n^2 dissolved D1's not to be confused with the N baryon vertex F1s!

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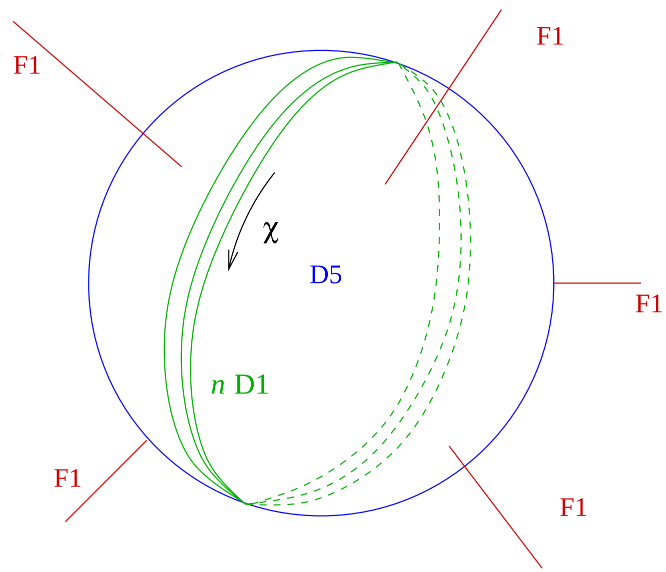

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$\rightarrow$ Alternative, microscopic description in terms of non-Abelian D1's?

Cfr Dielectric effect $\longrightarrow$ See section 4 [Emparan] [Myers]

3 Bound on n

baryon vertex with $n = 0$: stable under perturbations in x^i
stable under perturbations in u

$\rightarrow$ analysis of dynamics due to external F1's

[Brandhuber, Itzhaki, Sonnensshein, Yankielowicz]

What is the influence of $n \neq 0$?

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What is the influence of $n \neq 0$?

$$S \;=\; S_{D5} + N S_{F1} \;=\; S_{D5} + \int dt dx \sqrt{(u')^2 + \frac{u^4}{L^4}}$$

Bulk eqn & boundary eqns combine into:

$$\frac{u^4}{\sqrt{(u')^2 + \frac{u^4}{L^4}}} \;=\; \beta\, u_0^2 L^2 \qquad \text{with} \quad \beta^2 \;=\; 1 - \frac{1}{16}\Big(1 + \frac{8\pi n}{N}\Big)^2$$

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Observation:

u is real $\Longleftrightarrow$ β should be real $\Longleftrightarrow$ $0 \leq \frac{n}{N} \leq \frac{3}{8\pi}$

$\rightarrow$ Upper bound on $\frac{n}{N}$ (relation to string exclusion principle?)

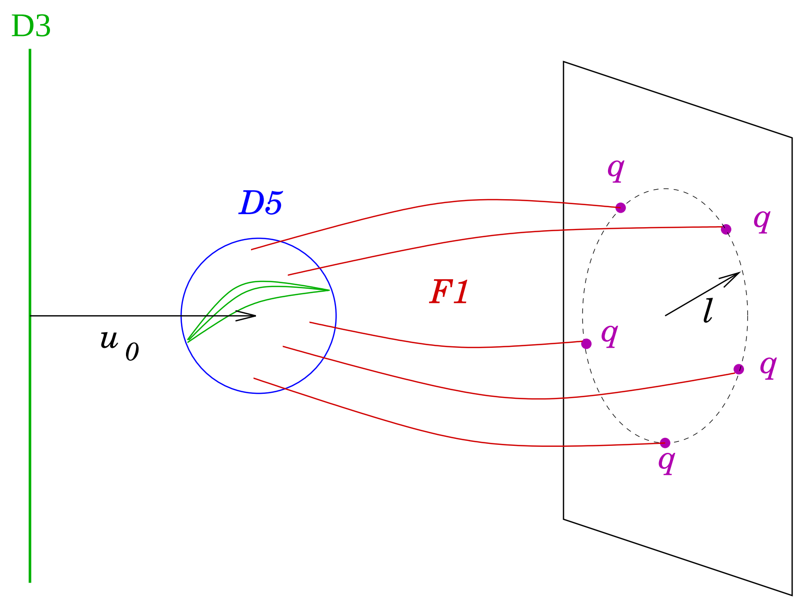
[Maldacena, Strominger]

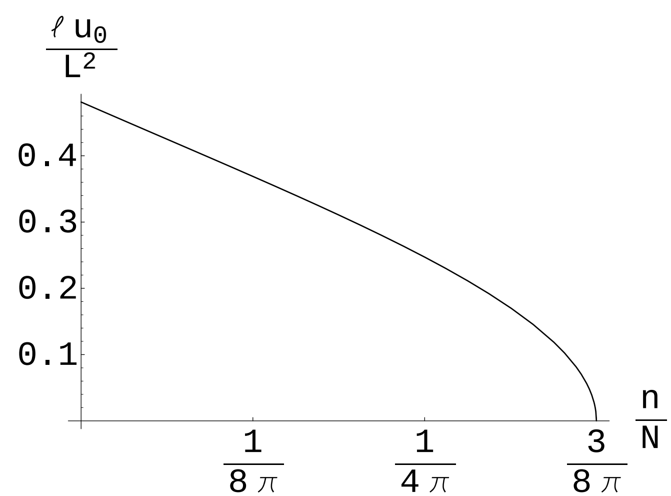
Size ℓ of the baryon vertex (in boundary)

$$\ell = \frac{L^2}{u_0} \int_1^\infty dy \frac{\beta}{y^2 \sqrt{y^4 - \beta^2}}$$

NB: Size of baryon vertex is inversely proporcional to u_0

Size of baryon vertex is function of n/N





Energy E of the baryon vertex

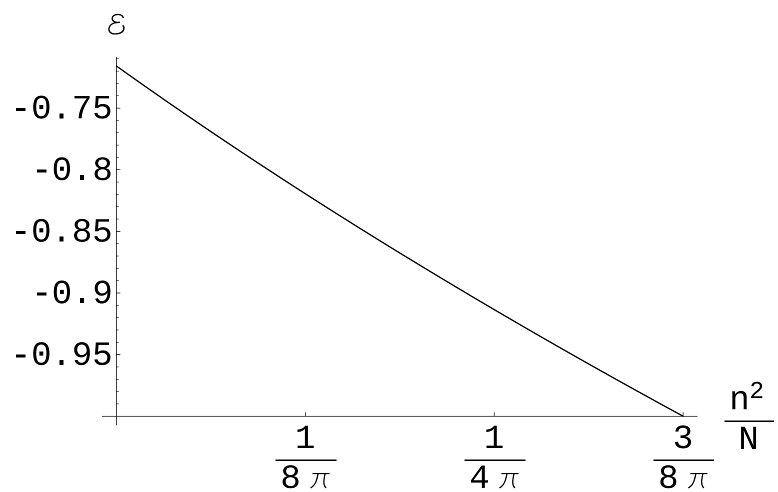
$$E = T_1 u_0 \Big\{ \int_1^\infty dy \Big[\frac{y^2}{\sqrt{y^4 - \beta^2}} - 1 \Big] - 1 \Big\}.$$

Energy E of the baryon vertex is:

proportional to u_0 (conformal invariance)

proportional to $\sqrt{g_{YM} N}$

a function of n/N



4 The microscopical description

N coinciding Dp-branes $\Longrightarrow$ Fuzzy D$(p+q)$-brane [Myers]

n^2 coinciding D1-branes $\Longrightarrow$ Fuzzy baryon vertex?

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n^2 coinciding D1-branes $\Longrightarrow$ Fuzzy baryon vertex?

$$S_{n^2 D1} \;=\; -T_1 \int d^2\xi \; \mathrm{STr}\Big\{ \sqrt{\Big| \det\Big(P[g_{\mu\nu} + g_{\mu i}(Q^{-1} - \delta)^i{}_j g^{jk} g_{k\nu}] \Big) \det Q \Big|} \;\Big\}$$

$$+\, T_1 \int d^2\xi \; \mathrm{STr}\Big\{ P[i(\mathrm{i}_X \mathrm{i}_X) C^{(4)} \;-\; \tfrac{1}{2}(\mathrm{i}_X \mathrm{i}_X)^2 C^{(4)} \wedge \mathcal{F} \Big\}$$

with

$$Q^i{}_j = \delta^i_j + i[X^i, X^k] g_{kj} \qquad \Big((\mathrm{i}_X \mathrm{i}_X) C^{(4)} \Big)_{\mu\nu} \;=\; \tfrac{1}{2}[X^\lambda, X^\rho] C^{(4)}_{\rho\lambda\mu\nu}$$

$$\mathcal{F} = 2\partial\mathcal{A} + i[\mathcal{A}, \mathcal{A}] \qquad (\mathrm{i}_X \mathrm{i}_X)^2 C^{(4)} \;=\; \tfrac{1}{4}[X^\lambda, X^\rho][X^\nu, X^\mu] C^{(4)}_{\mu\nu\rho\lambda}$$

$\rightarrow$ n^2 D1's (wound in fibre) expand into fuzzy CP^2

(Fuzzy S^5 is Abelian $U(1)$ fibre over fuzzy CP^2) [B.J., Lozano, Rodr.-Gómez]

CP^2 is coset manifold $SU(3)/U(2)$
embedded in $\mathbb{R}^8$ via

$$\sum_{i=1}^{8} x^i x^i = 1 \qquad\qquad \sum_{j,k=1}^{8} d^{ijk} x^j x^k = \frac{1}{\sqrt{3}} x^i$$

Fuzzy CP^2 generated by $SU(3)$ generators T^i in (anti-)fundamental repres

$$X^i = \frac{T^i}{\sqrt{(2n-2)/3}} \qquad\qquad [X^i, X^j] = \frac{i f^{ijk}}{\sqrt{(2n-2)/3}} X^k$$

[Alexanian, Balachandran, Immirzi, Ydri]

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Substituting in D1 action:

$$S_{nD1} = -T_1 \int dt d\chi \, u \, \mathrm{STr}\Big\{ \mathbb{1} + \frac{L^4}{4(2n-2)} \mathbb{1} \Big\}$$

$$E_{nD1} = 2\pi u T_1 \left(n + \frac{nL^4}{8(n-1)} \right)$$

$$\left[E_{D5} = 8\pi^2 u T_5 \left(n + \frac{L^4}{8} \right), \qquad T_1 = 4\pi^2 T_5 \right]$$

D5-brane in baryon vertex is expanded D1-branes

Where are F1's that form vertex?

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Where are F1's that form vertex?

→ Chern-Simons coupling:

$$\begin{aligned} S_{CS} &= T_1 \int dt d\chi \, \mathrm{STr}\Big\{ P[(\mathrm{i}_X \mathrm{i}_X) C^{(4)}] \; - \; P[(\mathrm{i}_X \mathrm{i}_X)^2 C^{(4)}] \wedge \mathcal{F} \Big\} \\ &= -\frac{T_1}{4} \int dt d\chi \, \mathrm{STr}\Big\{ [X^i, X^j][X^k, X^l] G^{(5)}_{\chi ijkl} \mathcal{A}_t \Big\} \end{aligned}$$

$$\rightarrow G^{(5)}_{\chi ijkl} = L^4 f^m_{[ij} f^n_{kl]} X^m X^n$$

$$= \frac{L^4 T_1}{2(n-1)} \int dt d\chi \, \mathrm{STr}\Big\{ \mathcal{A}_t \Big\}$$

$$\rightarrow \mathcal{A} = A_t(t) \, \mathbb{1} dt$$

$$= \frac{n}{n-1} N T_1 \int dt \; A_t$$

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⇒ N BI charges as $n \rightarrow \infty$, cancelled by N extrnal F1's

5 Conclusions

- Witten's baryon vertex is D5-brane wrapped around S^5 in $AdS_5 \times S^5$
- $S^5 \xrightarrow{S^1} CP^2$ permits to add magnetic BI flux $F = \sqrt{2n}\, dB$
  $\Rightarrow$ Generalised baryon vertex
- Upperbound on n/N, related to string exclusion principle?
- $F = \sqrt{2n}\, dB$ introduces n D1-branes in D5 worldvolume
- Microscopic description: D1-branes expanding into S^5
  $\Rightarrow$ agreement for $n \gg 1$
  (Witten's baryon vertex not included)

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