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An Alternative for Moduli Stabilisation

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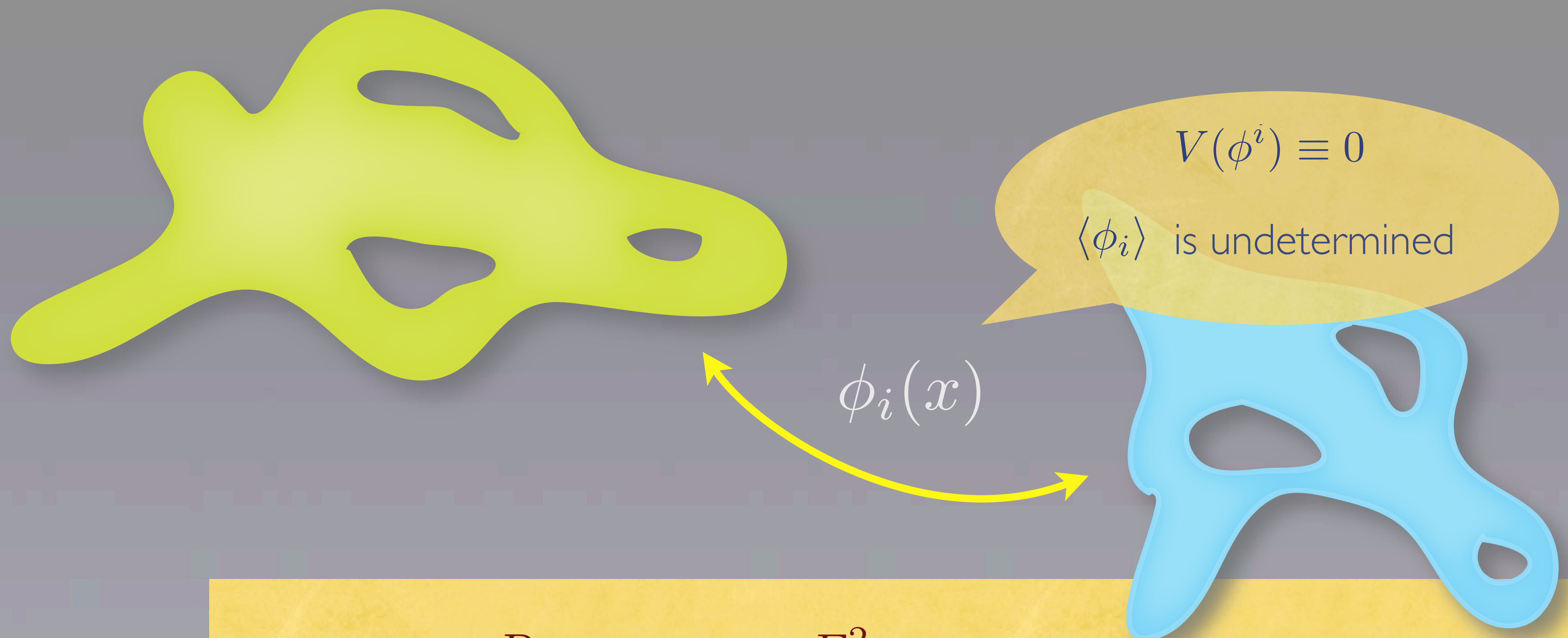
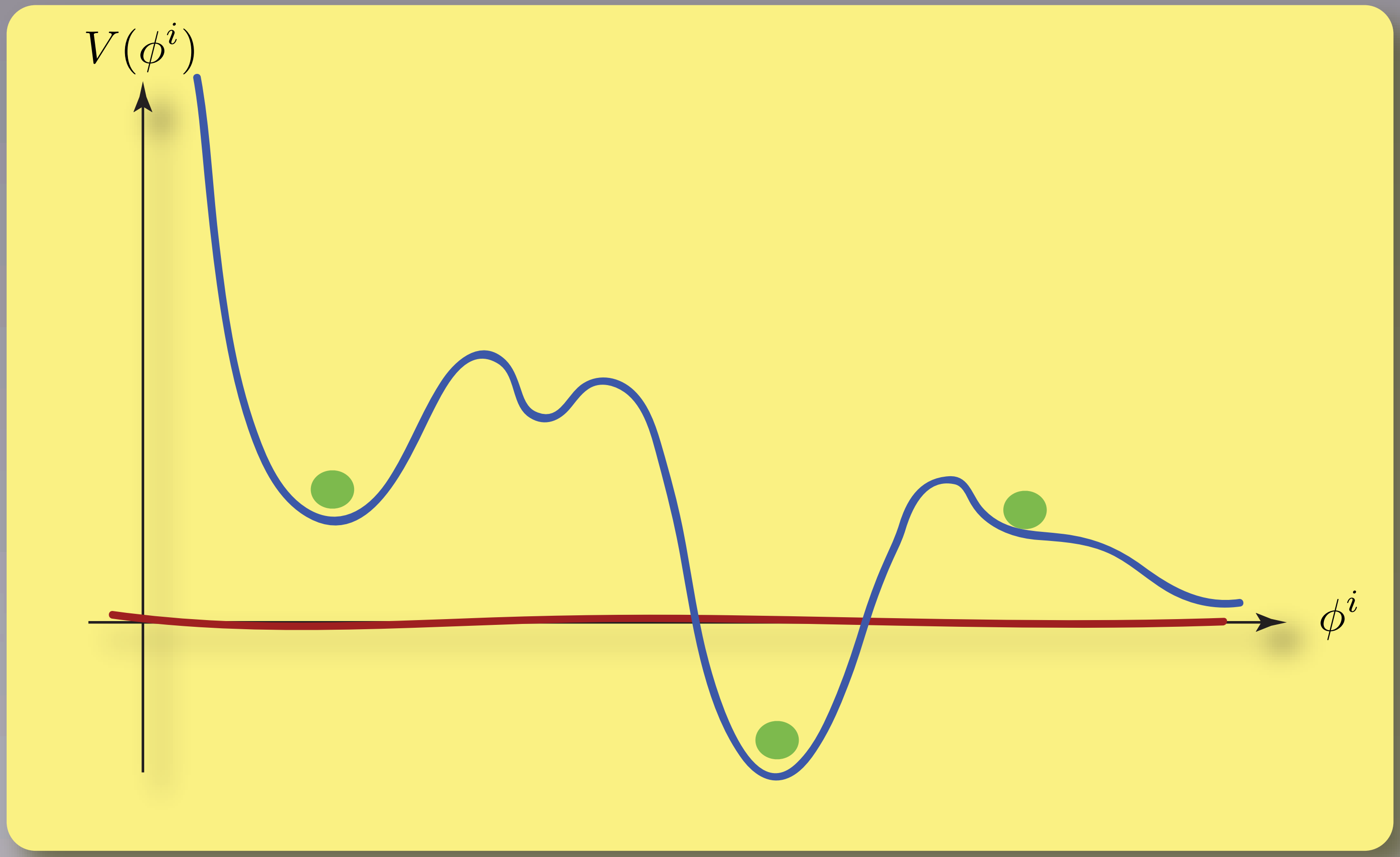
String compactifications are typically characterised by a moduli space of supersymmetric vacua.



$$\mathscr{L} \sim \frac{R}{8\pi(\alpha')^4 g_s^2} + \frac{F^2}{(\alpha')^{(p-3)/2} g_s}$$

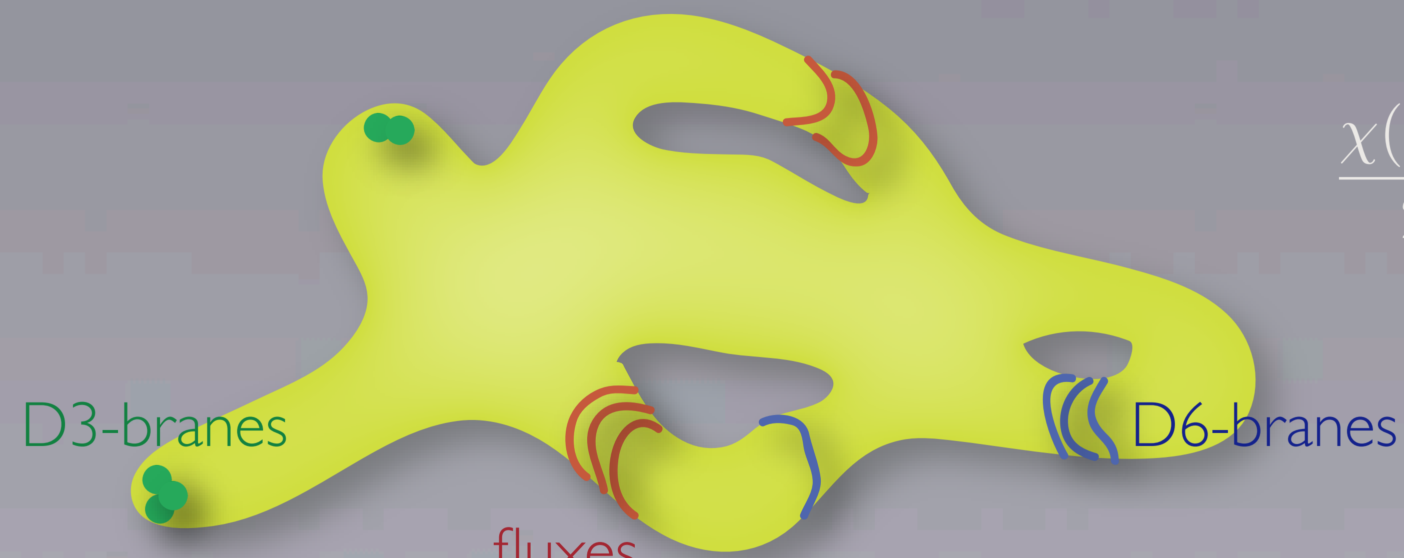
upon compactification $\longrightarrow$ $$\mathscr{L} \sim \frac{R}{16\pi G_{\mathrm{N}}(\phi)} + \frac{F^2}{g_{\mathrm{YM}}(\phi)} + G_{ij}(\phi)\partial\phi^i\,\partial\phi^j$$

It is thus important to device mechanisms to lift the moduli space



In the last few years we have witnessed an increasing activity in the search of ways to stabilise moduli

In general, most of the constructions involve non-trivial fluxes for NS-NS and R-R potentials



$$H_3, F_3 \in H^3(\mathrm{CY}, \mathbb{Z})$$

$$\frac{\chi(\mathrm{CY})}{24} = N_{\mathrm{D3}} + \frac{1}{2\kappa_{10}^2 T_3} \int_{\mathrm{CY}} H_3 \wedge F_3$$

generate the superpotential

[Gukov,Witten]

$$W \sim \int_{\mathrm{CY}} \Omega \wedge (F_3 + \tau H_3)$$

fixes the complex structure moduli

[Giddings,Kachru,Polchinski]

Kähler class moduli can be also stabilised by

Non-perturbative effects

- gaugino condensation
- D-instantons

[KKLT]

quantum corrections

$$\mathscr{M} \neq \mathscr{M}_{\rm complex} \times \mathscr{M}_{\rm Kahler}$$

[Berg,Haack,Körs]

In fact, this analysis relies on a low-energy (gauged) supergravity description

The supergravity approach is actually valid in the large-volume regime where (higher-order) quantum corrections can be neglected

A two-dimensional sigma-model description is actually missing

$B_{IJ} \neq \text{const}$
difficult to solve
(only few known cases)

$$\int_\Sigma \Big(\partial_a X^I \partial^a X^J G_{IJ} + \epsilon^{ab} \partial_a X^I \partial_b X^J B_{IJ}(X)$$

$$+ F_{IJK} \; ? \; \Big)$$

In the NSR string
no RR couplings
(pure spinors ??)

[See Grassi and Billò's talk for recent progress on RR backgrounds]

Other approaches to complex-structure and Kähler-class moduli stabilisation relies on oblique D-brane fluxes and/or coisotropic branes

[Antoniadis, (Kumar), Maillard]
[Bianchi, Trevigne]
[Kumar, Mukhopadhyay, Ray]
[Font, Ibañez, Marchesano]

A CFT description is in principle available
(at least for Abelian fluxes)

However, no vacuum solutions with stabilised moduli are known at present

In what follows, I am going to present
an ***alternative*** *bona-fide string mechanism*
for moduli stabilisation

... it relies on
non-supersymmetric string compactifications,
where quantum corrections to the moduli
space are generated perturbatively

Outline

- Scherk-Schwarz reductions in Field and String Theory
- The one-loop quantum potential. The standard lore
- The Hagedorn phase transition
- A stringy Scherk-Schwarz deformation
- The vacuum energy and moduli stabilisation
- Conclusions and outlooks

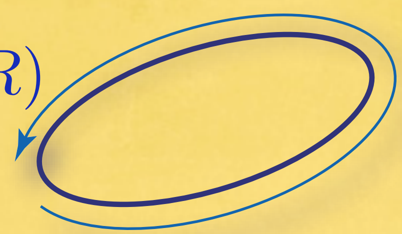
Scherk-Schwarz Supersymmetry Breaking in Field Theory

[Scherk, Schwarz]

In dimensional reductions use internal symmetries to affect the Kaluza-Klein excitations of bosons and fermions

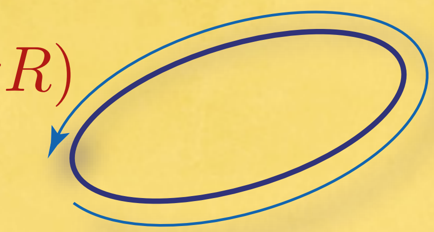
$$\mathscr{M}_{d+1} \to \mathscr{M}_d \times \mathscr{S}^1$$

$+\phi(y+2\pi R)$



$\phi(y)$

$-\psi(y+2\pi R)$



$\psi(y)$

$$\phi(y) = \sum_n \phi_n \, e^{iny/R} \qquad \psi(y) = \sum_n \psi_n \, e^{i(n+\frac{1}{2})y/R}$$

$$\Delta M \sim 1/R \qquad \Lambda \sim -\frac{1}{R^d}$$

In String Theory Scherk-Schwarz deformations can be conveniently described in terms of freely acting orbifolds

[Kiritsis, Kounnas]

$$\mathscr{S}^1 / (-1)^F \, \delta$$

space-time fermion index

$X \to X + \pi R$

$$\Gamma \sim -\frac{1}{2} \int_{\mathscr{F}} \frac{d^2\tau}{\tau_2^{11/2}} \sum_{m,n} \left[\left| \frac{V_8 - S_8}{\eta^8} \right|^2 \Lambda_{m,n} + \left| \frac{V_8 + S_8}{\eta^8} \right|^2 (-1)^m \, \Lambda_{m,n} \right.$$

$$\left. + \left| \frac{O_8 - C_8}{\eta^8} \right|^2 \Lambda_{m,n+\frac{1}{2}} + \left| \frac{O_8 + C_8}{\eta^8} \right|^2 (-1)^m \, \Lambda_{m,n+\frac{1}{2}} \right]$$

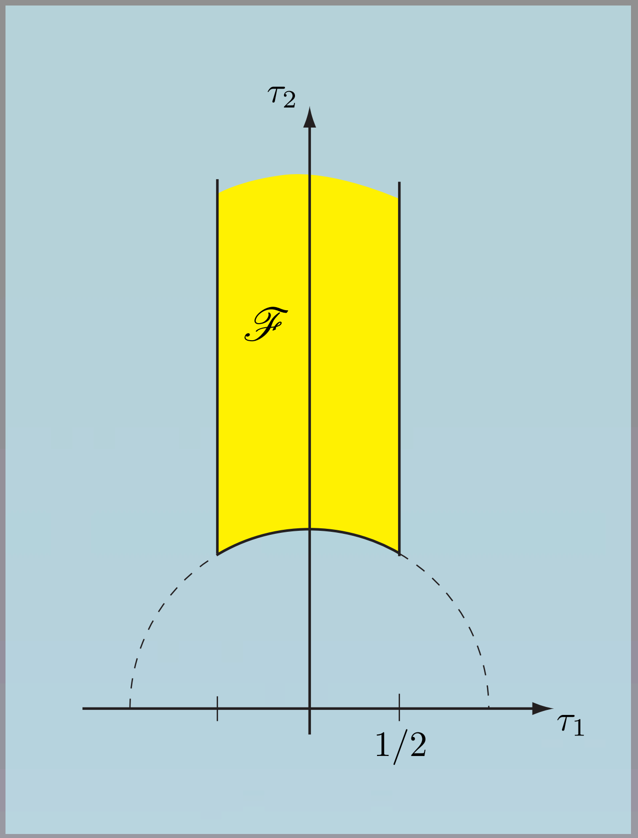
Modular invariance introduces the "tachyonic" twisted sector!

[Rohm; Kounnas, Porrati; Ferrara, Kounnas, Porrati, Zwirner]

The mass of the "would-be" tachyon is $\alpha' m^2 = -1 + \frac{\alpha'}{2} \left(\frac{R}{2\alpha'} \right)^2$

Schematically ...

$$\Gamma \sim \int_{\Lambda}^{\infty} \frac{d\tau_2}{\tau_2^{11/2}} \sum_{\{m^2\}} c(m^2) e^{-4\pi\tau_2 m^2(R)}$$



divergent in the IR because of tachyons

Perturbation theory breaks-down at small radius,
hence we work in the large *R* regime

We must compute

$$\Gamma \sim \int_{\tilde{\mathscr{F}}} \frac{d\tau_2}{\tau_2^{11/2}} \sum_{m,n} \left| \frac{\theta_2^4}{\eta^{12}} \right|^2 (-1)^m \, q^{\frac{\alpha'}{4}\left(\frac{m}{R}+\frac{nR}{\alpha'}\right)^2} \, \bar{q}^{\frac{\alpha'}{4}\left(\frac{m}{R}-\frac{nR}{\alpha'}\right)^2}$$

However, the large-radius behaviour is not clear. To this end it is useful to go to the Lagrangian formalism by Poisson resumming over momenta

$$\Gamma \sim \frac{R}{\sqrt{\alpha'}} \int_{\tilde{\mathscr{F}}} \frac{d^2\tau}{\tau_2^6} \left| \frac{\theta_2^4}{\eta^{12}} \right|^2 \sum_{\ell,n} e^{-\frac{\pi R^2}{4\alpha' \tau_2} |2\ell+1+2n\tau|^2}$$

After unfolding the fundamental domain

$$\mathscr{V} = -\frac{R}{2\alpha'} \int_{-1/2}^{1/2} d\tau_1 \int_0^\infty \frac{d\tau_2}{\tau_2^6} \left| \frac{\theta_2^4}{\eta^{12}} \right|^2 \sum_{p \text{ odd}} e^{-\frac{\pi R^2 p^2}{4\alpha' \tau_2}}$$

$$\left(\frac{\theta_2^4}{\eta^{12}} = \sum_{N=0}^{\infty} d_N \, q^N \right)$$

$$\sim -\frac{a}{R^9} - \frac{b}{R^4} \sum_{p \text{ odd}} \sum_{N=1}^{\infty} d_N^2 \, \frac{N^{5/2}}{p^5} \, K_5 \left(2\pi p \sqrt{N R^2/\alpha'} \right)$$

The large radius behaviour is $\mathscr{V}(R) \sim \dfrac{1}{R^9}$ as expected from field theory

As expected,

$$\mathscr{V} \sim -\frac{a}{R^9} - \frac{b}{R^4} \sum_{p \text{ odd}} \sum_{N=1}^{\infty} d_N^2 \frac{N^{5/2}}{p^5} K_5\left(2\pi p \sqrt{N R^2/\alpha'}\right)$$

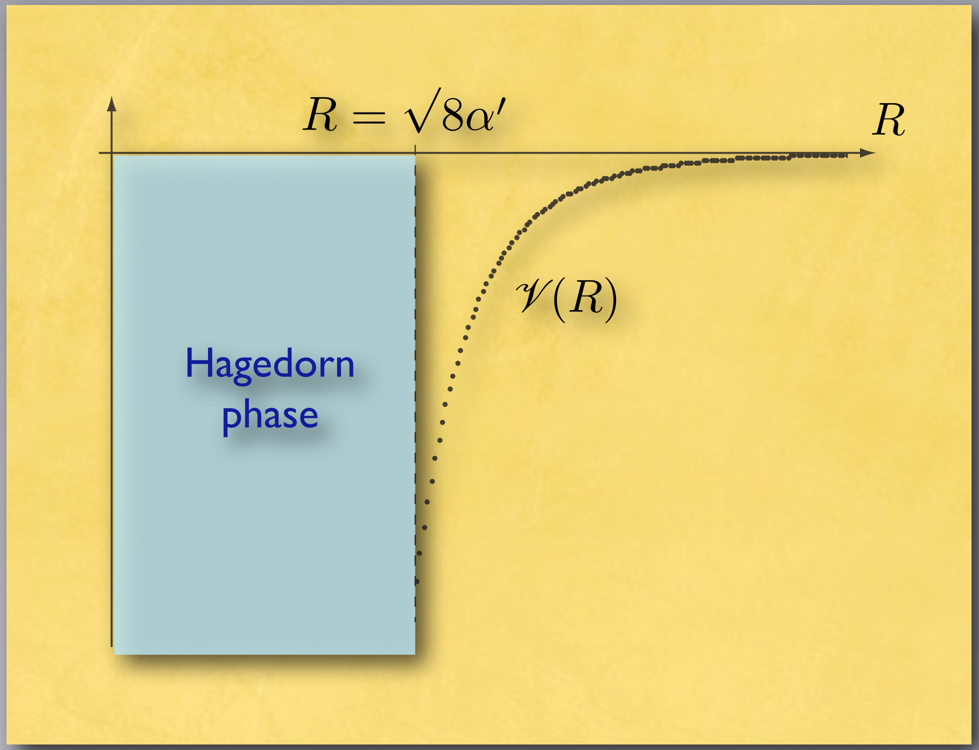
is not finite for any value of *R*

Indeed, from the Hardy-Ramanujan formula $d_N \sim N^{-11/4} e^{\pi\sqrt{8N}}$

and from the asymptotic behaviour of Bessel functions $K_n(x) \sim \frac{e^{-x}}{\sqrt{x}}$

the *N*-series is convergent only for

$$R \geq \sqrt{8\alpha'} =: R_{\text{Hagedorn}}$$



Finite temperature analogue: $R \sim \frac{1}{T}$

- ✓ temperature is the (inverse) radius of compact Euclidean time
- ✓ bosons are periodic while fermions are antiperiodic
- ✓ supersymmetry is broken
- ✓ first-order Hagedorn phase-transition if exponential growth of states

[Hagedorn]

- In heterotic string the new phase is non-critical d=7 strings
- In type II superstrings the new phase is not known

[Antoniadis, Kounnas]
[Atick, Witten]

So far, we have used twists that affect the Kaluza-Klein excitations

String theory, however, can afford more possibilities

$$(-1)^F\,\delta_1$$

$$X_{\rm L} \to X_{\rm L} + \frac{\pi R}{2}$$
$$X_{\rm R} \to X_{\rm R} + \frac{\pi R}{2}$$

$$(-1)^F\,\delta_2$$

$$X_{\rm L} \to X_{\rm L} + \frac{\pi\alpha'}{2R}$$
$$X_{\rm R} \to X_{\rm R} - \frac{\pi\alpha'}{2R}$$

$$(-1)^F\,\delta_3$$

$$X_{\rm L} \to X_{\rm L} + \frac{\pi R}{2} + \frac{\pi\alpha'}{2R}$$
$$X_{\rm R} \to X_{\rm R} + \frac{\pi R}{2} - \frac{\pi\alpha'}{2R}$$

Clearly, δ_1 and δ_2 shifts share the same fate

$$(-1)^F\,\delta_1$$

$$X_{\rm L} \to X_{\rm L} + \frac{\pi R}{2}$$
$$X_{\rm R} \to X_{\rm R} + \frac{\pi R}{2}$$

T-duality

$$(-1)^F\,\delta_2$$

$$X_{\rm L} \to X_{\rm L} + \frac{\pi\alpha'}{2R}$$
$$X_{\rm R} \to X_{\rm R} - \frac{\pi\alpha'}{2R}$$

Hence, well defined in the small radius regime $\mathscr{V} \sim R^9$

while, Hagedorn-like phase transition for $R \geq \sqrt{\dfrac{\alpha'}{8}}$

What about the asymmetric Scherk-Schwarz deformation?

$$(-1)^F \, \delta_3$$

$$X_{\rm L} \rightarrow X_{\rm L} + \frac{\pi R}{2} + \frac{\pi \alpha'}{2R}$$

$$X_{\rm R} \rightarrow X_{\rm R} + \frac{\pi R}{2} - \frac{\pi \alpha'}{2R}$$

T-duality invariant deformation!

Hence, we expect both the small and large radius regimes to be consistent

In the twisted sector the (anti-)holomorphic masses read

$$m_{\mathrm{L}}^2 = -\frac{1}{2\alpha'} + \frac{\alpha'}{4}\sum_{i=1}^{2d}\left(\frac{m_i+\frac{1}{2}}{R_i} + \frac{(n_i+\frac{1}{2})R_i}{\alpha'}\right)^2 + N^{(X)} + N^{(\psi)}$$

$$m_{\mathrm{R}}^2 = -\frac{1}{2\alpha'} + \frac{\alpha'}{4}\sum_{i=1}^{2d}\left(\frac{m_i+\frac{1}{2}}{R_i} - \frac{(n_i+\frac{1}{2})R_i}{\alpha'}\right)^2 + N^{(X)} + N^{(\psi)}$$

Level-matching

$$N^{(X)} + N^{(\psi)} - \tilde{N}^{(X)} - \tilde{N}^{(\psi)} + \left(m+\frac{1}{2}\right)\cdot\left(n+\frac{1}{2}\right) = 0$$

$$N^{(X)} + N^{(\psi)} - \tilde{N}^{(X)} - \tilde{N}^{(\psi)} + n\cdot m + \frac{1}{2}\cdot(n+m) + \frac{2d}{4} = 0$$

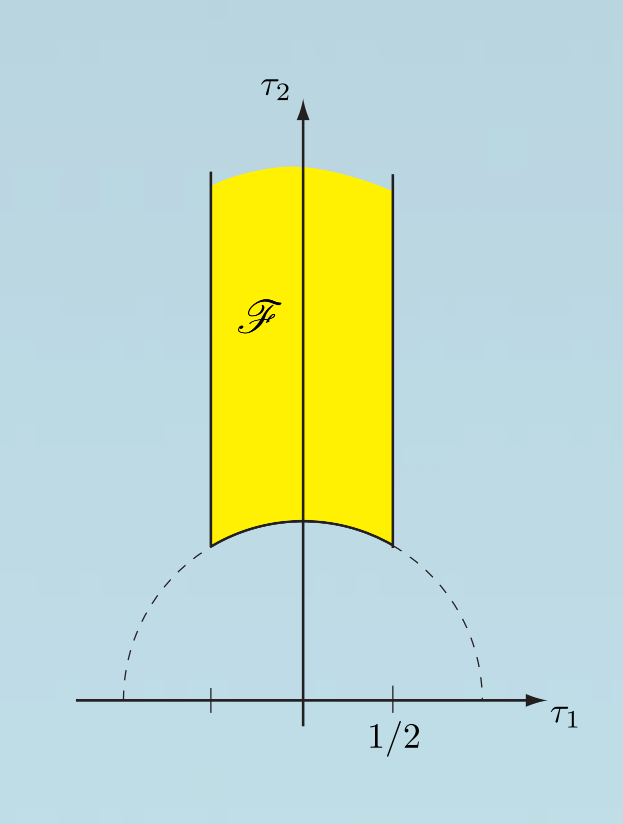
requires an even number of internal coordinates to be affected

In the twisted sector the would-be tachyon has

$$\alpha' m^2_{\text{min}} = -1 + \alpha' d \left[\left(\frac{1}{2R} \right)^2 + \left(\frac{R}{2\alpha'} \right)^2 \right]$$

$$= \frac{d}{4} \left(\frac{\sqrt{\alpha'}}{R} - \frac{R}{\sqrt{\alpha'}} \right)^2 + \frac{d-2}{2} \geq 0 \quad \text{for} \quad d = 2, 3$$

$$\mathscr{V} \sim -\frac{V_{10-2d}}{4(4\pi^2\alpha')^{5-d}} \int_{\mathscr{F}} \frac{d^2\tau}{\tau_2^{6-d}} \sum_{m^2} c(m^2)\, q^{m^2_{\text{L}}}\, \bar{q}^{m^2_{\text{R}}} < \infty$$



No divergence!

The one-loop partition function

$$\mathscr{V} = -\frac{V_{10-2d}}{4(2\pi^2\alpha')^{5-d}} \int_{\mathscr{F}} \frac{d^2\tau}{\tau_2^{6-d}} \sum_{\vec{m},\vec{n}} \left[\left| \frac{V_8 - S_8}{\eta^8} \right|^2 \Lambda_{\vec{m},\vec{n}} + \left| \frac{V_8 + S_8}{\eta^8} \right|^2 (-1)^{(\vec{m}+\vec{n})\cdot\vec{\epsilon}} \Lambda_{\vec{m},\vec{n}} \right.$$

$$\left. + \left| \frac{O_8 - C_8}{\eta^8} \right|^2 \Lambda_{\vec{m}+\frac{1}{2},\vec{n}+\frac{1}{2}} + \left| \frac{O_8 + C_8}{\eta^8} \right|^2 (-1)^{d+(\vec{m}+\vec{n})\cdot\vec{\epsilon}} \Lambda_{\vec{m}+\frac{1}{2},\vec{n}+\frac{1}{2}} \right]$$

$[\vec{\epsilon} = (1,\ldots,1)]$

where

$$\Lambda_{\vec{m}+\vec{a},\vec{n}+\vec{b}}(R) = q^{\frac{\alpha'}{4}\left(\frac{m+a}{R} + \frac{(n+b)R}{\alpha'}\right)^2} \bar{q}^{\frac{\alpha'}{4}\left(\frac{m+a}{R} - \frac{(n+b)R}{\alpha'}\right)^2}$$

is the (modified) Narain lattice for a 2d-dimensional torus

Large-radius behaviour

After Poisson re-summing over momenta

$$\mathscr{V} = -\frac{R^6}{(\alpha')^3}\int_{\tilde{\mathscr{F}}}\frac{d^2\tau}{\tau^6}\left|\frac{\theta_2^4}{\eta^{12}}\right|^2\sum_{n,\ell}e^{i\pi n\cdot\epsilon}\,\exp\left\{-\frac{\pi R^2}{4\alpha'\tau_2}|2\ell+1+2n\tau|^2\right\}$$

To disentangle the integrals we restrict to a single rep. of each SL(2,Z) orbit

degenerate orbit

$$n_i = 0$$

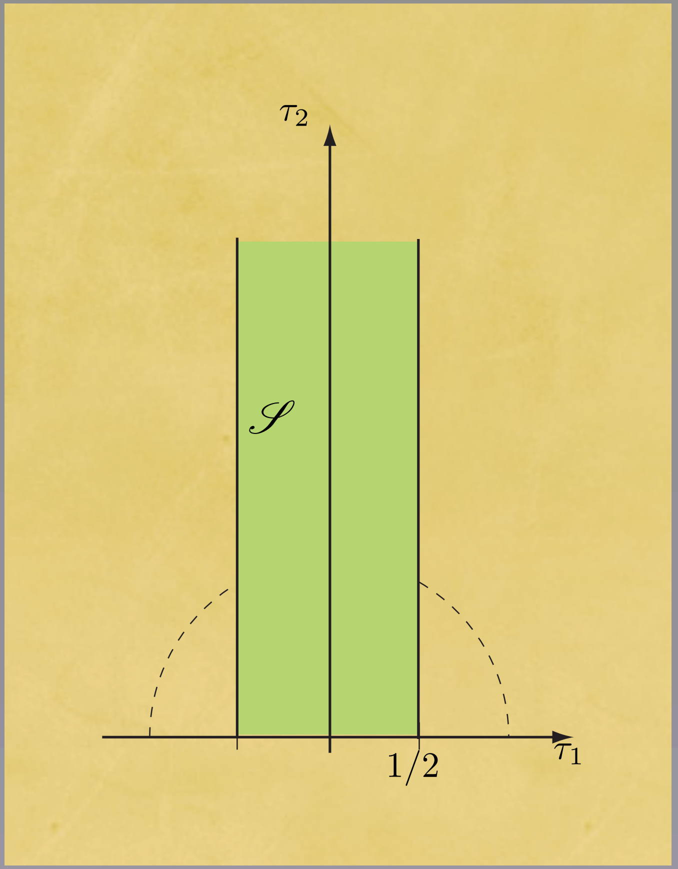
non-degenerate orbit

$$n_i \neq 0$$

The degenerate orbit

$$\mathscr{V} \sim R^6 \int_{-1/2}^{1/2} d\tau_1 \int_0^\infty \frac{d\tau_2}{\tau_2^6} \left| \frac{\theta_2^4}{\eta^{12}} \right|^2 \sum_\ell e^{-\frac{\pi R^2}{4\tau_2 \alpha'} \|2\ell+1\|^2}$$

$$\sim R^6 \int_0^\infty \frac{d\tau_2}{\tau_2^6} \sum_{N=0}^\infty \sum_\ell d_N^2 e^{-4\pi N \tau_2} e^{-\frac{\pi R^2}{4\tau_2 \alpha'} \|2\ell+1\|^2}$$



$$\sim -\frac{a^2}{R^4} \sum_{\{\ell_i\}} \left(\sum_i (2\ell_i + 1)^2 \right)^{-5}$$

massless contribution, dominant for large radius

$$-\frac{b^2}{R^6} \sum_{N=1}^\infty \sum_{\{\ell_i\}} d_N^2 \frac{N^{5/2}}{\left(\sum_i (2\ell_i+1)^2\right)^5} K_5 \left(2\pi \sqrt{N R^2 \sum_i (2\ell_i+1)^2 / \alpha'} \right)$$

massive contribution, exp suppressed for large radius

Small-radius behaviour

After Poisson re-summing over windings

$$\mathscr{V} = -\frac{(\alpha')^3}{R^6}\int_{\tilde{\mathscr{F}}}\frac{d^2\tau}{\tau^6}\left|\frac{\theta_2^4}{\eta^{12}}\right|^2\sum_{m,k} e^{i\pi m\cdot\epsilon}\,\exp\left\{-\frac{\pi\alpha'}{4R^2\tau_2}|2k+1+2m\tau|^2\right\}$$

To disentangle the integrals we restrict to a single rep. of each SL(2,Z) orbit

degenerate orbit

$$m_i = 0$$

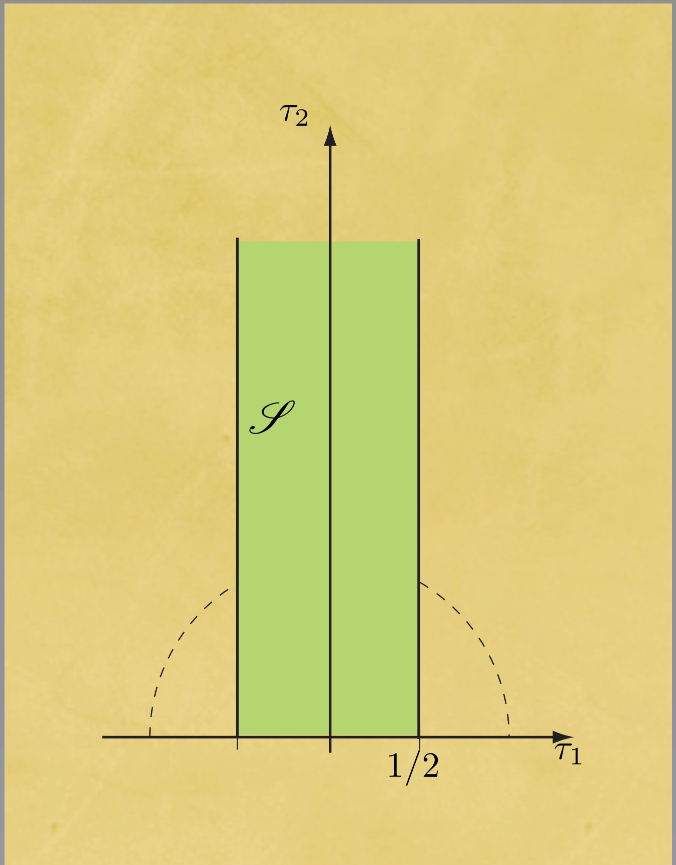
non-degenerate orbit

$$m_i \neq 0$$

The degenerate orbit

$$\mathscr{V} \sim \frac{1}{R^6} \int_{-1/2}^{1/2} d\tau_1 \int_0^\infty \frac{d\tau_2}{\tau_2^6} \left| \frac{\theta_2^4}{\eta^{12}} \right|^2 \sum_k e^{-\frac{\pi\alpha'}{4R^2\tau_2}\|2k+1\|^2}$$

$$\sim \frac{1}{R^6} \int_0^\infty \frac{d\tau_2}{\tau_2^6} \sum_{N=0}^\infty \sum_k d_N^2 e^{-4\pi N \tau_2} e^{-\frac{\pi\alpha'}{4R^2\tau_2}\|2k+1\|^2}$$



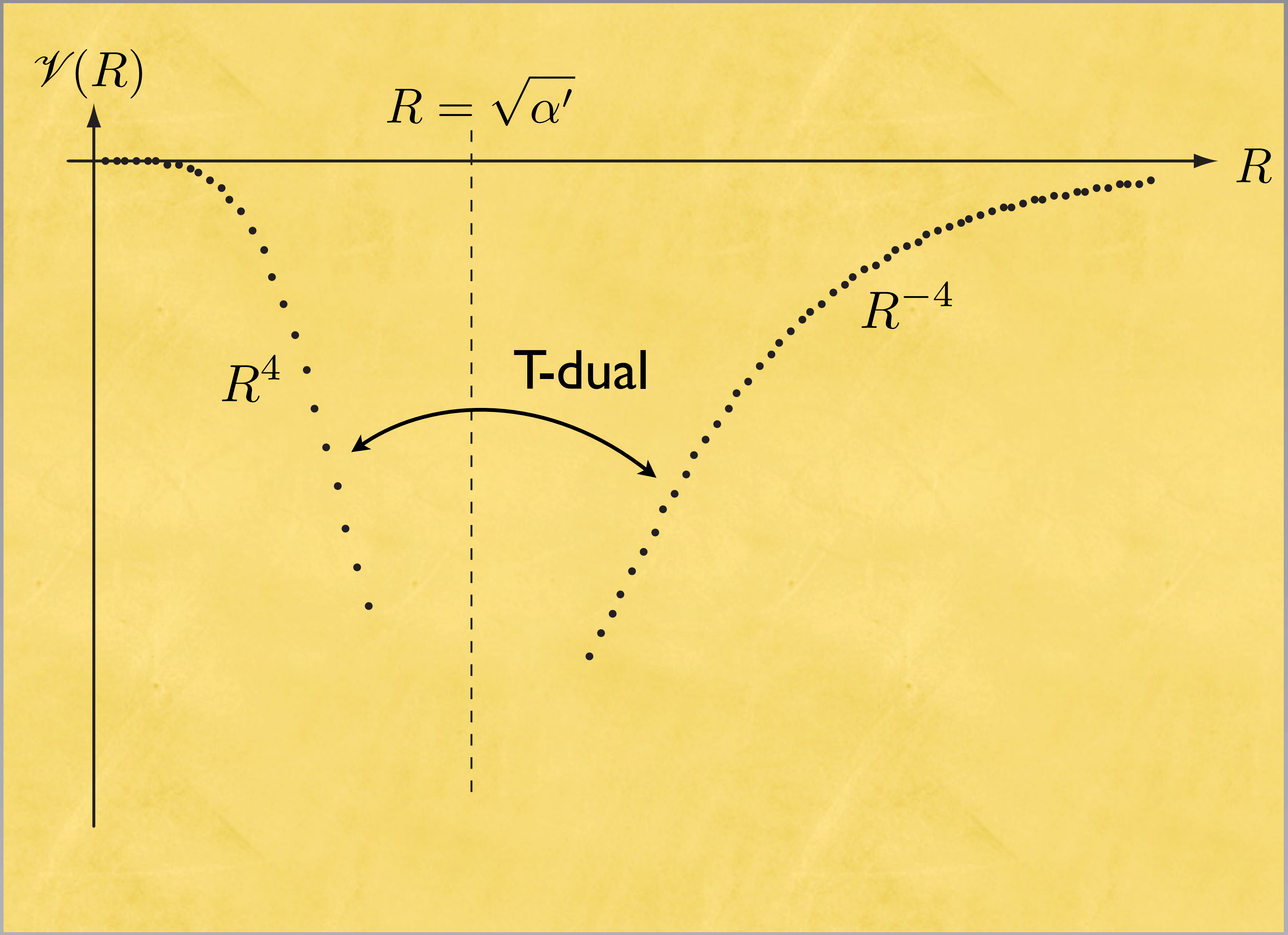
$$\sim -R^4 \sum_{\{k_i\}} \left(\sum_i (2k_i+1)^2 \right)^{-5}$$

massless contribution, dominant for small radius

$$-b^2 R^4 \sum_{N=1}^\infty \sum_{\{k_i\}} d_N^2 \frac{N^{5/2}}{\left(\sum_i (2k_i+1)^2\right)^5} K_5 \left(2\pi \sqrt{N\alpha' \sum_i (2k_i+1)^2/R^2} \right)$$

massive contribution, exp suppressed for small radius

Asymptotically ...



What about intermediate values of R?

$$\frac{\partial \mathscr{V}}{\partial R} = -\frac{V_{10-2d}}{16(4\pi^2\alpha')^{5-d}} \int_{\mathscr{F}} \frac{d^2\tau}{\tau_2^{6-d}} \left| \frac{\theta_2^4}{\eta^{12}} \right|^2$$

$$\times \sum_{m,n} (-1)^{(m+n)\cdot\epsilon}\, q^{\frac{\alpha'}{4}p_L^2}\, \bar{q}^{\frac{\alpha'}{4}p_R^2}\, (-\pi\tau_2) \left(-\frac{\alpha' m^2}{R^2} + n^2 \right) + \dots$$

$= 0$ **at** $R = \sqrt{\alpha'}$

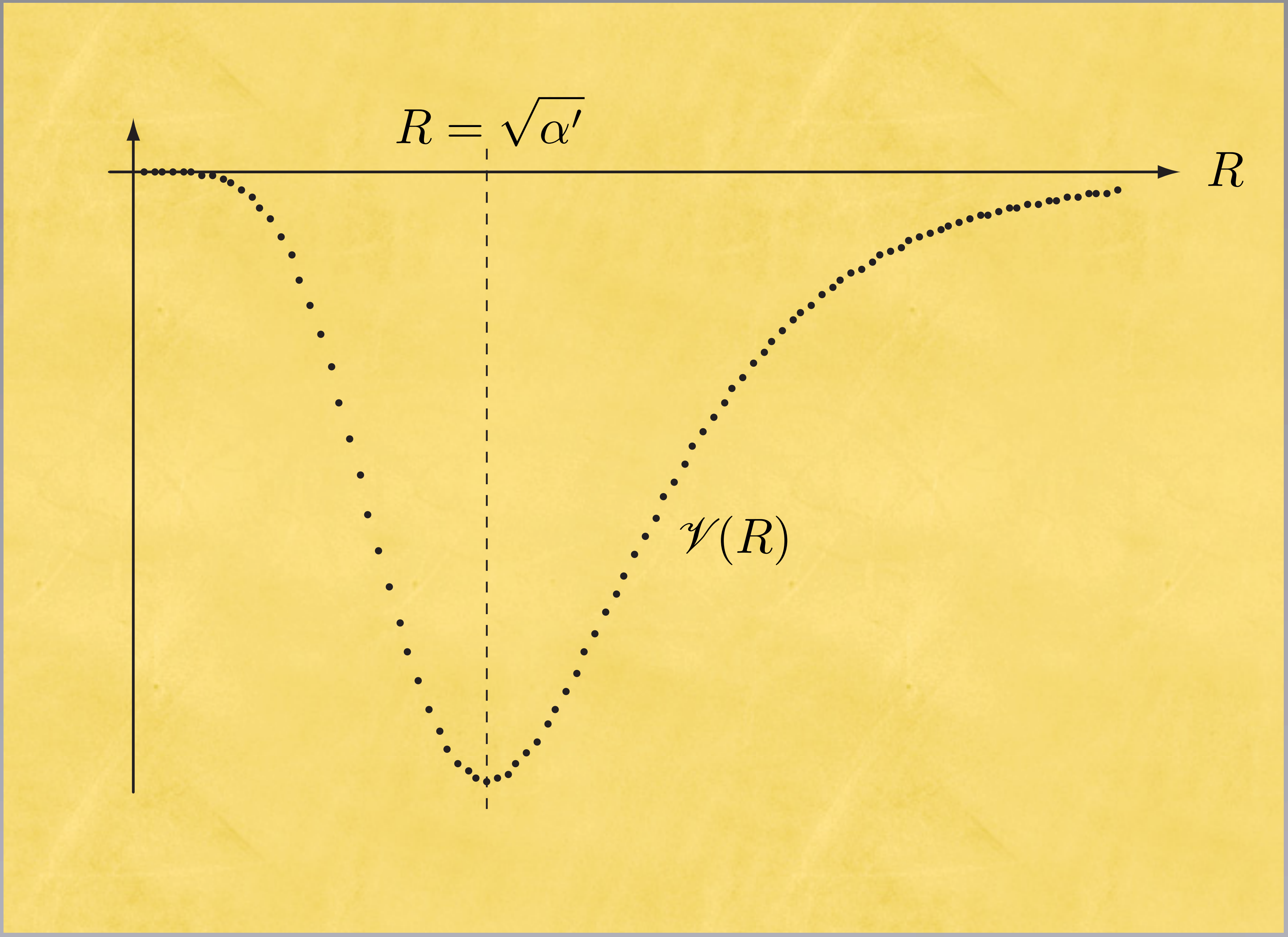
The quantum potential has a minimum at the self-dual radius

However, no analytic result for finite *R*, since

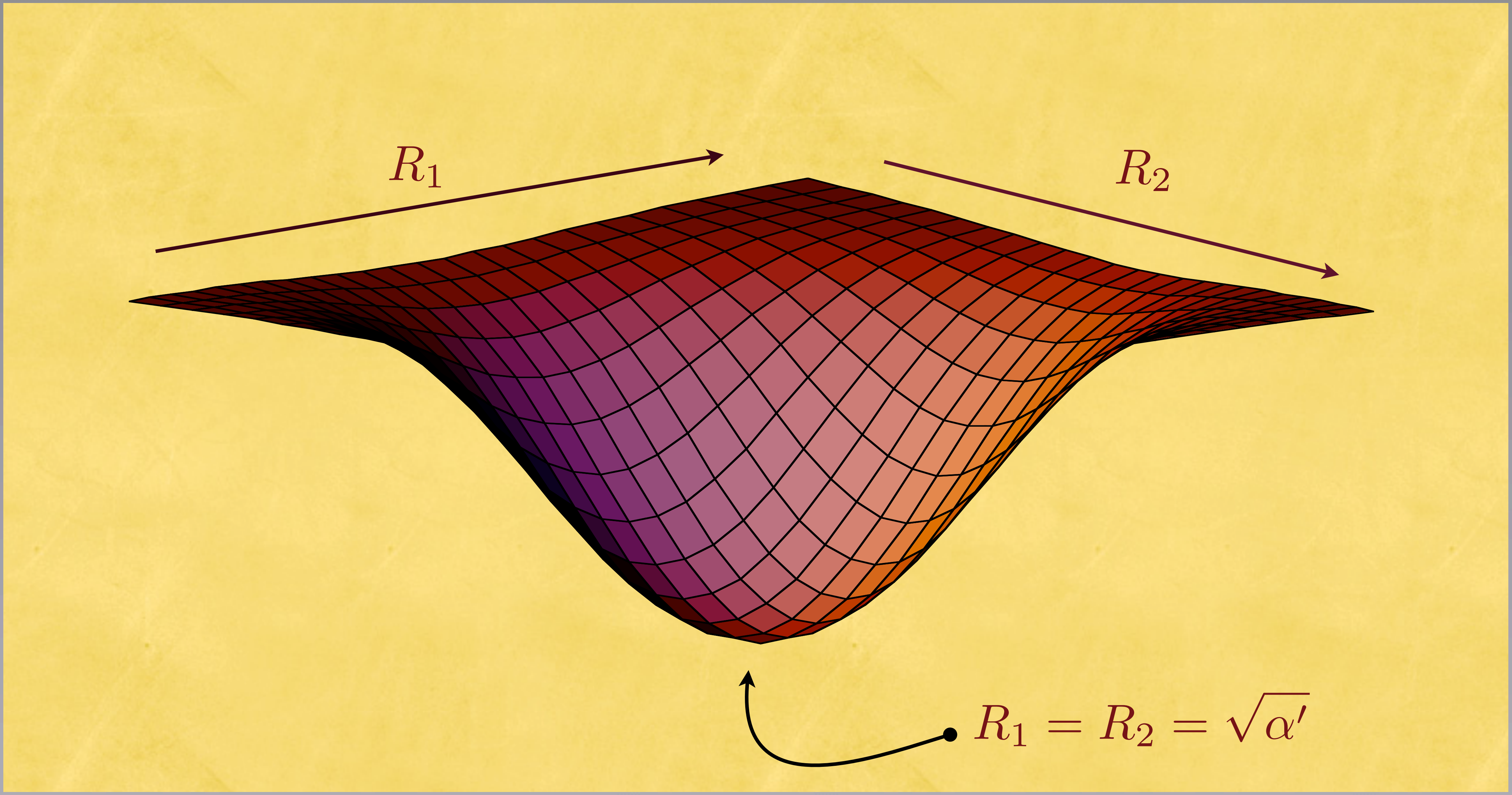
$$\int_0^\infty \frac{d\tau_2}{\tau_2^6} \sum_{N=0}^{\infty} \sum_{\ell} d_N^2 \, e^{-4\pi N \tau_2} \, e^{-\frac{\pi R^2}{4\tau_2 \alpha'} \|2\ell+1\|^2}$$

the *N*-series is not uniformly convergent in the IR, hence we can't exchange summation with integration and write the result in terms of Bessel functions!

Numerically evaluating the integrals ...



A three-dimensional plot ...



A tempting thermodynamics description

$$R \sim T^{-1}$$

large-radius/small-radius duality
high-temperature/low-temperature duality

vacuum energy finite for any value of *R*
free energy finite for any value of *T*

higher-order derivatives logarithmically divergent
second-order phase transition at self-dual temperature

two-dimensional Ising model

What about the stabilisation of the remaining geometric NS-NS moduli?

$$g_{ij} \qquad\qquad B_{ij}$$

Extrema are expected to occur at symmetry enhancement points

However, it seems that at the most symmetric point
the twisted tachyon is actually tachyonic

Is it separated by an energy barrier from other
metastable non-tachyonic local minima?

It is always possible to remove the unstable direction
via orbifold/orientifold projections

For instance ...

Use world-sheet parity to project away B_{ij}

Use $\mathbb{Z}_4 \times \mathbb{Z}_4$ orbifold to cast the metric in a diagonal form

$$\mathbb{Z}_4 = (\tfrac{1}{4}, -\tfrac{1}{4}, 1)$$

$$g_{ij} = \begin{pmatrix} R_1^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & R_1^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & R_2^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_2^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_3^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & R_3^2 \end{pmatrix}$$

Are we sure there aren't additional (divergent!) contributions to the vacuum energy?

Klein-bottle, annulus and Möbius amplitudes are identically vanishing since the states with opposite GSO aren't left-right symmetric

[Sagnotti]

The orbit generated by

$(-1)^F \, \delta_3$  is missing

$g \in \mathbb{Z}_4 \times \mathbb{Z}_4$

Additional amplitudes are identically vanishing (each preserves some supersymmetries)

Conclusions and Outlooks

- I have presented a simple example where a full-fledged string calculation of the quantum potential leads to moduli stabilisation
- This scenario universally applies to IIA/IIB, heterotic strings and orientifolds
- By construction, the quantum potential only depends on the NS-NS moduli
- Is it possible to up-lift the potential *à la* KKLT and obtain de Sitter vacua from String Theory?
- Add D-branes (and O-planes) with MSSM-like spectra.
- Stability versus higher-loop corrections?