Adding D7-branes to the Polchinski-Strassler gravity background

R. Apreda[1] J. Erdmenger[1] D. Lüst[1,2] C. Sieg[3]

[1] Max-Planck Institut für Physik, München

[2] Ludwig-Maximilians-Universität, München

[3] Università degli Studi di Milano

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Outline

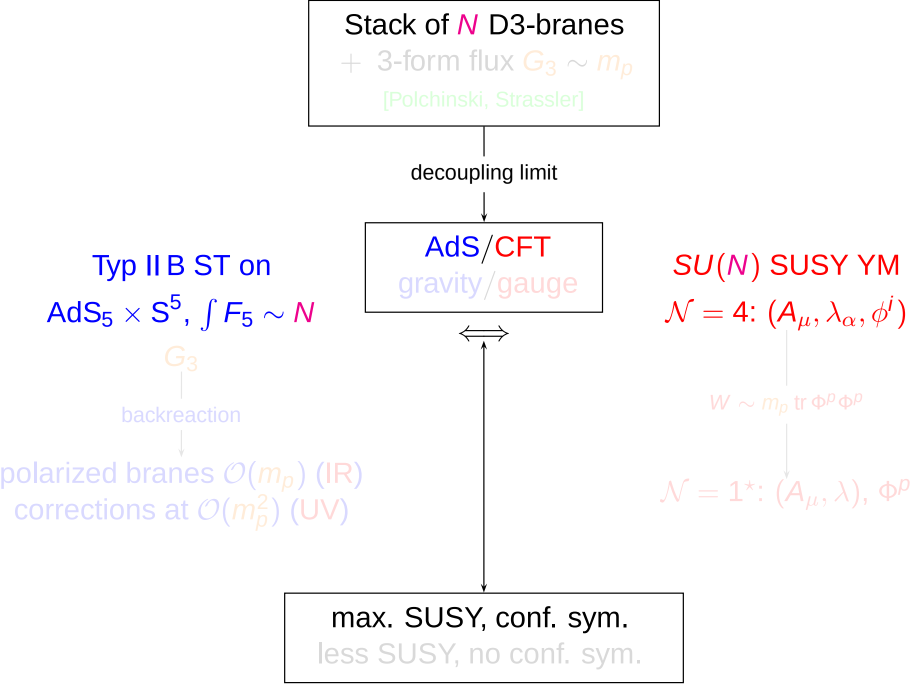
An extension of AdS/CFT: the mass perturbation

- Decoupling limit

Adding D7-branes

- AdS/CFT
- Polchinski-Strassler gravity background
- The symmetric embedding

Conclusions



Stack of N D3-branes

$+$ 3-form flux $G_3 \sim m_p$

[Polchinski, Strassler]

decoupling limit

AdS/CFT

gravity/gauge

Typ II B ST on

$\mathrm{AdS}_5 \times \mathrm{S}^5$, $\int F_5 \sim N$

G_3

backreaction

polarized branes $\mathcal{O}(m_p)$ (IR)

corrections at $\mathcal{O}(m_p^2)$ (UV)

$SU(N)$ SUSY YM

$\mathcal{N} = 4$: $(A_\mu, \lambda_\alpha, \phi^i)$

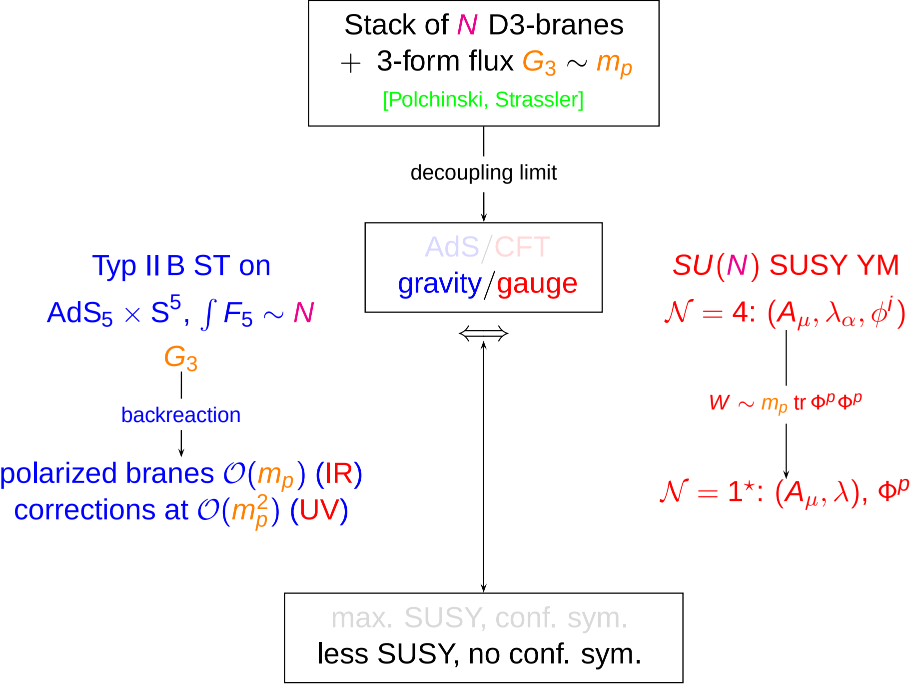
$W \sim m_p \,\mathrm{tr}\, \Phi^p \Phi^p$

$\mathcal{N} = 1^\star$: (A_μ, λ), Φ^p

$\Longleftrightarrow$

max. SUSY, conf. sym.

less SUSY, no conf. sym.



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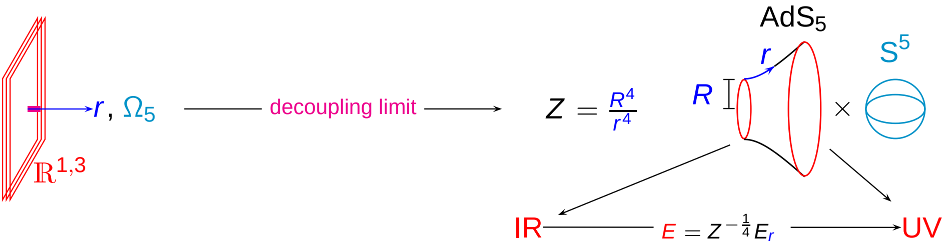
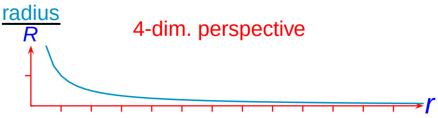
The decoupling limit

metric of D3-branes: $ds^2 = Z^{-\frac{1}{2}} \eta_{\mu\nu}\, dx^\mu\, dx^\nu + Z^{\frac{1}{2}} (dr^2 + r^2 d\Omega_5^2)$



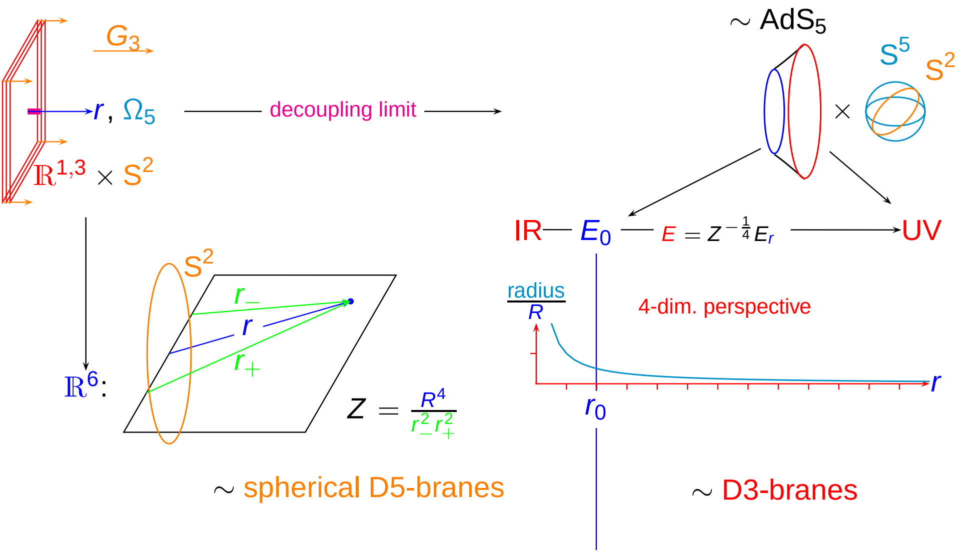
decoupling limit $\longrightarrow$ $Z = \frac{R^4}{r^4}$

IR $\longleftarrow$ $E = Z^{-\frac{1}{4}} E_r$ $\longrightarrow$ UV



4-dim. perspective

The decoupling limit



$E = Z^{-\frac{1}{4}} E_r$

$$Z = \frac{R^4}{r_-^2 r_+^2}$$

4-dim. perspective

$\sim$ spherical D5-branes

$\sim$ D3-branes

Backreaction: higher order corrections

sufficient distance from the shell S^2 (D3-brane description):

$$\mathrm{AdS}_5 \times S^5 : \; g_{MN} \,, \qquad F_5 = dC_4 \,, \qquad \tau = \frac{\theta}{2\pi} + \frac{i}{g_s}$$

- linear approximation:

$$\sim m_p : \qquad G_3 = F_3 - \tau H_3 \xrightarrow[\text{Bianchi id.}]{\text{Hodge }\star} C_6 = \tfrac{2}{3} C_4 \wedge B_2$$

- quadratic approximation:

$$\sim m_p^2 : \qquad \tilde{g}_{MN} \,, \qquad \tilde{C}_4 \,, \qquad \tilde{\tau} \,, \qquad \text{[Freedman,Minahan]}$$

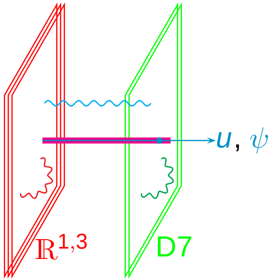
$$C_0 \xrightarrow[\text{Bianchi id.}]{\text{Hodge }\star} C_8 = -\tfrac{1}{6} C_4 \wedge \left(B_2 \wedge B_2 + g_s^2 \tilde{C}_2 \wedge \tilde{C}_2\right)$$

- cubic approximation [Cardoso, Curio, dall'Agata, Lüst]:

$$\begin{tikzcd} & \tilde{\tau} \arrow[d] & \\ \sim m_p^3 : \quad \tilde{G}_3 : & \tilde{G}_{(3,0)} \arrow[r] & \langle \lambda\lambda \rangle \end{tikzcd}$$

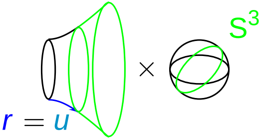
gaugino condensate

Adding D7-branes to AdS/CFT [Karch, Katz]



decoupling limit
rescale u

$\longrightarrow$ $AdS_5 \times S^5 \supset AdS_5 \times S^3$



$N \gg N_f$

$SU(N)$	$SU(N_f)$	string
adj.	–	〰
–	adj.	〰
fund.	fund.	〰

$\longrightarrow$ $\mathcal{N} = 4$ SYM fields

$\longrightarrow$ decouple

$\longrightarrow$ fundamental matter

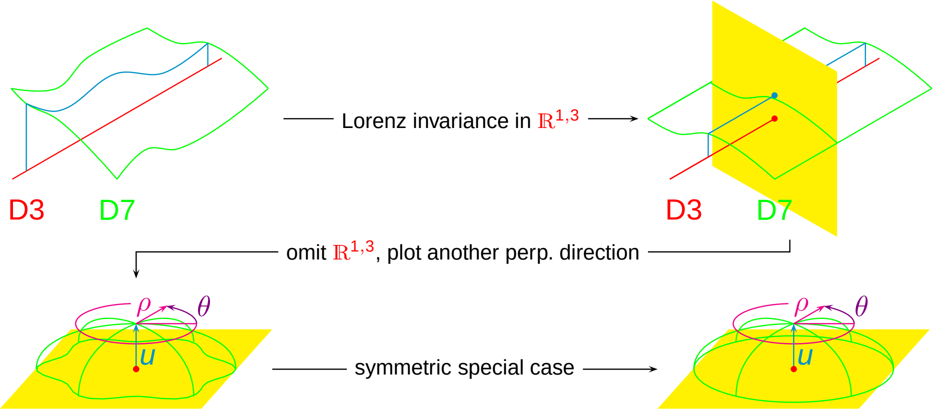
$\mathcal{N} = 2$ SYM

$\mathcal{N} = 4$ SYM fields

$+$

fundamental matter

Embedding of D7-branes



Lorenz invariance in $\mathbb{R}^{1,3}$ →

omit $\mathbb{R}^{1,3}$, plot another perp. direction

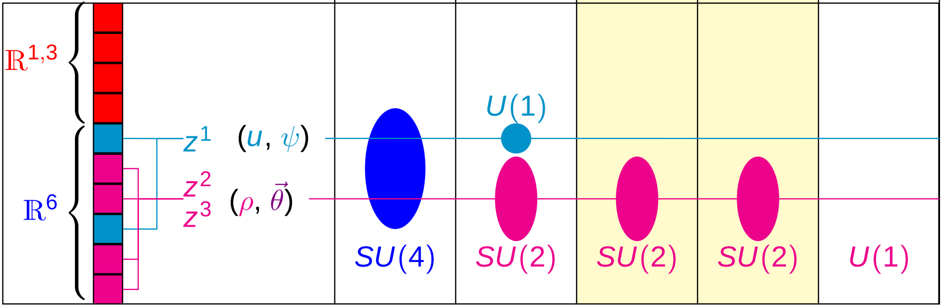
symmetric special case →

$u = u(\rho, \theta)$

$u = u(\rho)$

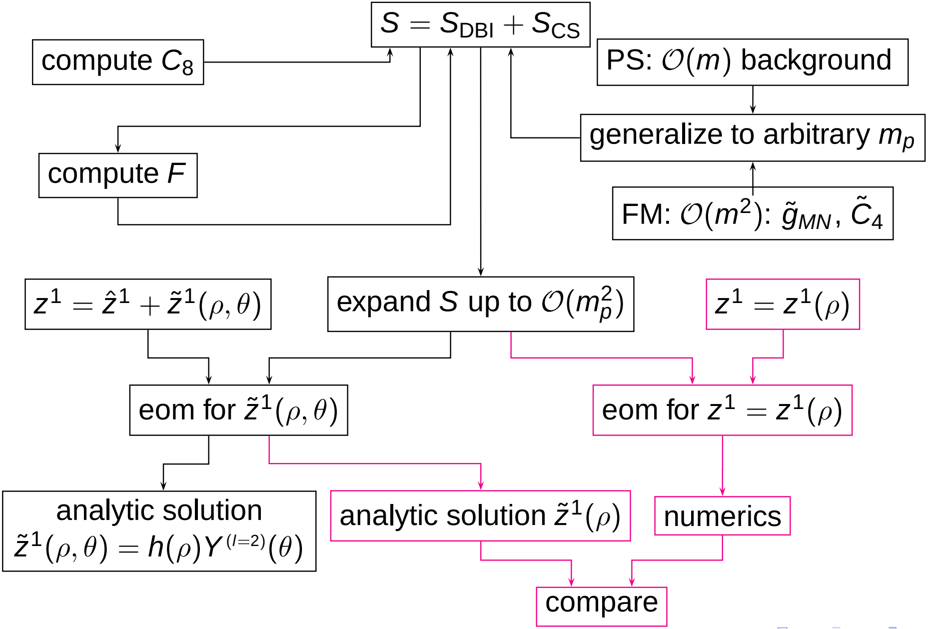
Embedding of D7-branes (max. R-symmetries)

$AdS_5 \times S^5$		$G_3 \neq 0$		
$\mathcal{N} = 4$		$\mathcal{N} = 2$		$\mathcal{N} = 1$
		$m_1 = 0$, $m_2 = m_3$		$m_p \neq 0$
no D7	D7 $\perp z^1$	no D7	D7 $\perp z^1$	(no) D7
$SU(4)$	$U(1)$ $SU(2)$	$SU(2)$	$SU(2)$	$U(1)$



- maximal possible R-symmetries shown
- theories with $SU(2) \times U(1)$ R-symmetry are superconformal:
  $\Rightarrow$ D3-branes must lie 'inside' D7-brane, i.e. $u = 0$
  $\Rightarrow$ $G_3 \neq 0$ must break $U(1)$

Investigation of the (symmetric) embedding



$S = S_{\text{DBI}} + S_{\text{CS}}$

compute C_8

PS: $\mathcal{O}(m)$ background

generalize to arbitrary m_p

compute F

FM: $\mathcal{O}(m^2)$: $\tilde{g}_{MN}$, $\tilde{C}_4$

$z^1 = \hat{z}^1 + \tilde{z}^1(\rho, \theta)$

expand S up to $\mathcal{O}(m_p^2)$

$z^1 = z^1(\rho)$

eom for $\tilde{z}^1(\rho, \theta)$

eom for $z^1 = z^1(\rho)$

analytic solution
$\tilde{z}^1(\rho, \theta) = h(\rho) Y^{(l=2)}(\theta)$

analytic solution $\tilde{z}^1(\rho)$

numerics

compare

The symmetric embedding

- equations of motion for $u(\rho) = \hat{u} + \tilde{u}(\rho)$ (with $\hat{r}^2 = \rho^2 + \hat{u}^2$):

$$\frac{1}{\rho^3}\partial_\rho(\rho^3\partial_\rho\tilde{u}(\rho)) = g(\rho)\,, \qquad g(\rho) = \frac{\hat{u}}{\hat{r}^2}\left(a + \frac{\hat{u}^2}{\hat{r}^2}b\right)\,, \quad a, b = \mathcal{O}(m^2)$$

$\Rightarrow$ corrections to the background enter eom as inhomogenities

- solutions with $u(\infty) = \hat{u}$

fixed: regularity at $\rho = 0$

$$u(\rho) = \hat{u} + \tilde{u}(\rho) = \hat{u} + \frac{\hat{u}}{8\rho^2}\left(b\frac{\hat{u}^2}{\hat{r}^2} + 2(4C - a) - 2a\ln\hat{r}^2\right)$$

unique regular solution: $u(\rho) = \hat{u} - b\frac{\hat{u}}{8\hat{r}^2} - a\frac{\hat{u}}{4\rho^2}\ln\frac{\hat{r}^2}{\hat{u}^2}$

$\rho \ll \hat{u}$

$\rho \gg \hat{u}$

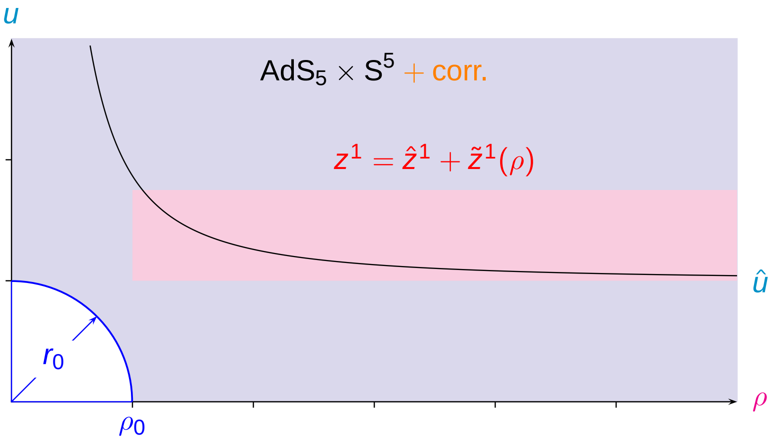
$$\hat{u} - \frac{1}{8\hat{u}}(2a + b)$$

$$\hat{u} - a\frac{\hat{u}}{4\rho^2}\ln\rho^2 - \frac{\hat{u}}{8\rho^2}(b - 2a\ln\hat{u}^2)$$

quark mass m_q

quark condensate? No
(holographic renormalization)

Validities of the approximations



- perturbative treatment of the backreaction at $r > r_0$
- non-regular solutions: expansion valid at $\rho > r_0$
- regular solutions: expansion valid at $\hat{u} \gtrsim m$

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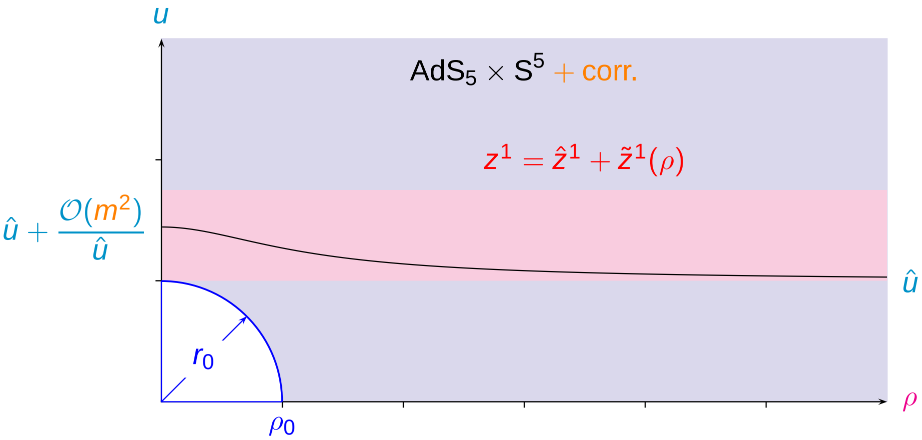

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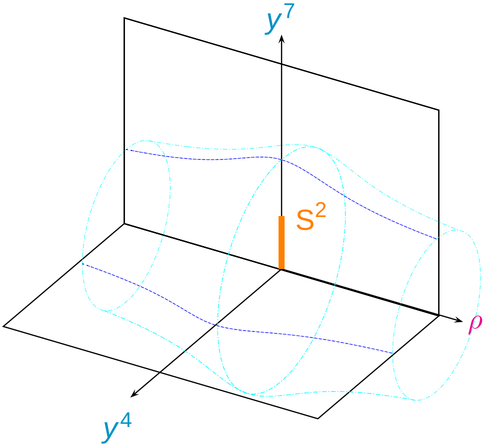
Angular dependence of the embedding

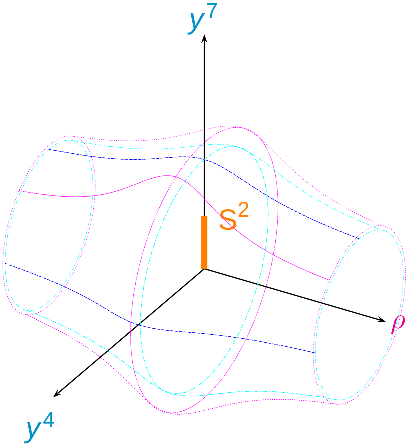
illustration: $\psi_0 = \psi = 0, \frac{\pi}{2}$



for $\psi_0 = 0$ and $\psi_0 = \frac{\pi}{2}$: no ρ-dependent corrections $\tilde{\psi}(\rho)$

Angular dependence of the embedding

illustration: $\psi_0 = \frac{\pi}{4}$



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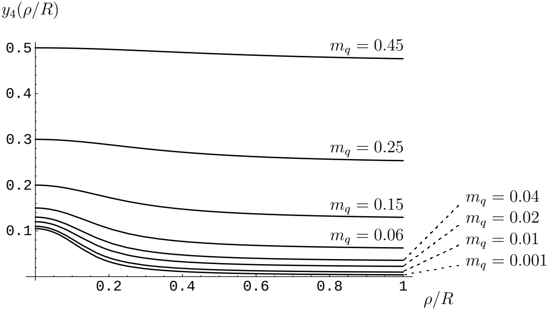
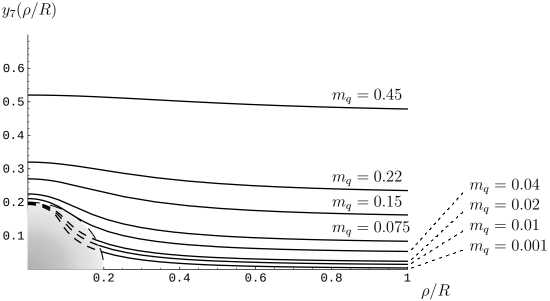
Numerical results

values: $R = 1$, $m = 0.2$



$\psi = \psi_0 = 0$

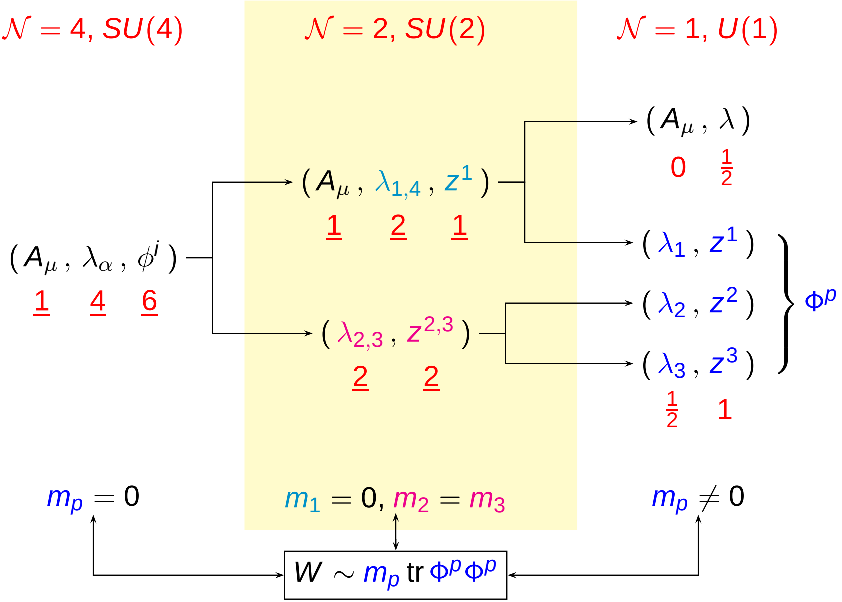


$\psi = \psi_0 = \frac{\pi}{2}$

Conclusions

- Aim: Find gravity/gauge correspondences more close to QCD
- Possibilities:
  - deform AdS/CFT, e. g. by adding flux ↔ masses
  - add D7-branes ↔ fundamental matter
- Combination of these possibilities
  - generalization of all results to generic masses m_p
  - computation of C_8 and F
  - expansion of the action and the equations of motion
  - analytic solutions up to $\mathcal{O}(m^2)$
  - holographic renormalization:
    vanishing action → vanishing quark condensate

Supersymmetries, R-symmetries



$\mathcal{N}=4$, $SU(4)$

$\mathcal{N}=2$, $SU(2)$

$\mathcal{N}=1$, $U(1)$

$(A_\mu\,,\ \lambda_\alpha\,,\ \phi^i)$
$\underline{1}\quad \underline{4}\quad \underline{6}$

$(A_\mu\,,\ \lambda_{1,4}\,,\ z^1)$
$\underline{1}\quad \underline{2}\quad \underline{1}$

$(\lambda_{2,3}\,,\ z^{2,3})$
$\underline{2}\quad \underline{2}$

$(A_\mu\,,\ \lambda)$
$0\quad \frac{1}{2}$

$(\lambda_1\,,\ z^1)$

$(\lambda_2\,,\ z^2)$

$(\lambda_3\,,\ z^3)$
$\frac{1}{2}\quad 1$

Φ^p

$m_p = 0$

$m_1 = 0$, $m_2 = m_3$

$m_p \neq 0$

$$\boxed{W \sim m_p \,\mathrm{tr}\, \Phi^p \Phi^p}$$

Mass perturbation: tensor

gauge theory: diagonal fermion mass term ($p = 1, 2, 3$)

$$m^{\alpha\beta}\lambda_\alpha\lambda_\beta \quad = \quad \underset{\text{fermions (chiral multiplets)}}{\underset{\updownarrow}{m_p\lambda_p\lambda_p}} \quad + \quad \underset{\text{gaugino}}{\underset{\updownarrow}{m_4\lambda\lambda}}$$

supergravity:

- $SO(6)$ symmetry:

$$y^i : \underline{6}\,, \qquad T_3 : (\underline{6}\times\underline{6}\times\underline{6})_{\text{asym.}} = \underline{20}$$

$$\begin{tikzcd} & \underline{20} \arrow[dl, "(\star_6+i)T_3=0"'] \arrow[dr, "(\star_6-i)T_3=0"] & \\ \underline{\overline{10}} & + & \underline{10} \end{tikzcd}$$

z^p : $\underline{\overline{10}}$

$$\boxed{T_3 = m_p\epsilon_{pqr}\mathrm{d}z^p \underset{\underset{T_{(1,2)}}{\updownarrow}}{\wedge} \mathrm{d}\bar{z}^q \wedge \mathrm{d}\bar{z}^r + m_4\epsilon_{pqr}\mathrm{d}z^p \underset{\underset{T_{(3,0)}}{\updownarrow}}{\wedge} \mathrm{d}z^q \wedge \mathrm{d}z^r}$$

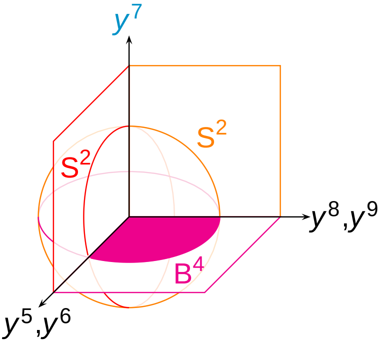
- add radial dependence ($r^2 = y^i y^i = 2z^p\bar{z}^p$):

$$G_3 \sim \mathrm{d}(r^{-4}S_2)\,, \qquad \mathrm{d}S_2 = 3T_3$$

D3-brane polarization revisited

non-commutative coordinates: $[y^i, y^j] \sim \mathrm{Im}\, T_{ijk} y^k$

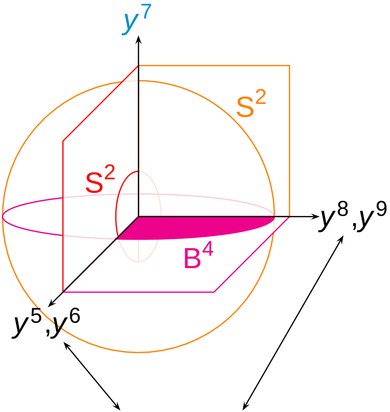
$m_1 = 0,\ m_2 = m_3$



no polarization into y^4

$SO(4) = SU(2) \times SU(2)$

$m_1 = m_2 = m_3$



$U(1) \times U(1)$

$U(1)$ rotating (y^4, y^7) is broken

Holographic renormalization

[Bianchi,de Haro,Freedman,Karch,O'Bannon,Skenderis,Solodukhin]

required to clarify if $u(\rho) \Rightarrow$ quark condensate

QFT		SUGRA
UV divergences	$\longleftrightarrow$	$V_{\text{AdS}} = \infty$
regularization		$V_{\text{AdS}} \to V_{\varepsilon} = V_{\text{AdS}}\big\|_{r \leq \frac{1}{\varepsilon}}$
		$S[\hat{u}] \to S_{\text{sub}}[u_\varepsilon] = \int_{V_\varepsilon} \mathcal{L} + S_{\text{ct}}$
scheme trafo		finite ct $\sim \frac{u_\varepsilon^4}{2} + \frac{m^2 R^4}{3} u_\varepsilon^2$

SUSY preserving scheme

$$S_{\text{sub}} = 0$$

$$\langle \mathcal{O}_u \rangle = \lim_{\varepsilon \to 0} \frac{\delta S_{\text{sub}}}{\delta u_\varepsilon}$$

no quark condensate