Matrix model description of dilatations in N=4 super Yang-Mills theory

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Outline

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   - AdS/CFT correspondence
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AdS/CFT correspondence

At $N \to \infty$:

$$(\mathcal{N} = 4 \text{ SYM})_{\mathcal{M}_{1,3}} \Leftrightarrow (\text{string theory})_{\mathrm{AdS}_5 \times S^5}$$

Identification of symmetry groups → "AdS/CFT dictionary" – the correspondence between operators of SYM and states of ST, e.g. Dilatations correspond to time shifts

For $N < \infty$ the string interactions are included with rate $\sim N^{-1}$.

$$g_s \sim J^2/N, \qquad J - \text{classical dimension/length}$$

SYM: $N \to \infty$ — invariance of single trace operators. Single trace operators do not mix with multi-trace ones under renormalization. Integrability (See the talk of K.Zarembo)

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“AdS/CFT dictionary”

$AdS_5 \times S^5$ strings	$\mathcal{N} = 4$ SYM
states	Composite operators
AdS symmetry	Conformal symmetry
Spherical symmetry	R-symmetry
Time shift	Dilatation, RG-flow
Hamiltonian, H	Dilatation operator, Mixing matrix, Δ
...	...

SYM dilatation operator

"Alphabet": $\{W_A\} = \{F_{\mu\nu}, \phi, \psi, \nabla F, \nabla\phi, \nabla\psi \ldots\}$

"Language": gauge invariant combinations of letters

"Words": simplest gauge invariants, one-trace composite operators,

$$\mathcal{O}_{A_1 A_2 \ldots A_L} = \operatorname{tr} W_{A_1} W_{A_2} \ldots W_{A_L}$$

"Phrases":

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Operator mixing: as $N \to \infty$ the trace structure becomes invariant

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The SYM dilatation operator can be perturbatively expanded

$$\Delta = \sum_k H_{2k}, \qquad H_{2k} \sim g_{\mathrm{YM}}^{2k}$$

H_{2k} can be written in a compact form in terms of fields and derivatives. For first few k it was obtained by [Staudacher–Beisert]. One-loop "Hamiltonian"

$$H_2 = \sum_j h(j)(P_j)_{AB}^{CD} : [W^A, \check{W}_C][W^B, \check{W}_D] :,$$

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SU(2) sector

Generated by two complex scalars: $\Phi_1 = \phi_1 + \mathrm{i}\phi_2$ and $\Phi_2 = \phi_5 + \mathrm{i}\phi_6$

Spin interpretation (Heisenberg $\mathrm{XXX}_{1/2}$ model+chain interactions):

$$\begin{aligned} \Phi_1 &\leftrightarrow \text{spin } \uparrow \\ \Phi_2 &\leftrightarrow \text{spin } \downarrow \end{aligned}$$

The one-loop dilatation operator is reduced to

$$H_2 = -\frac{g_{\rm YM}^2}{16\pi^2} : \mathrm{tr}[\Phi^a, \Phi^b][\check{\Phi}_a, \check{\Phi}_b] :$$

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$$S(\Psi, \bar{\Psi}, A) =$$
$$\int \mathrm{d}t \left(\mathrm{tr}\, \frac{\mathrm{i}}{2} (\bar{\Psi}_a \nabla_0 \Psi^a - \nabla_0 \bar{\Psi}_a \Psi^a) + \frac{g_{\mathrm{YM}}^2}{16\pi^2} \mathrm{tr} [\Psi^a, \Psi^b][\bar{\Psi}_a, \bar{\Psi}_b] \right)$$

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Finite Temperature

We can consider a thermodynamical system based on our matrix mechanics. In particular, the thermal partition function is

$$Z(\beta) = \mathrm{tr}\, \mathrm{e}^{-\beta\Delta}$$

where H is the Hamiltonian of the system and $\beta = 1/kT$.
SYM: partition function formally is the Fourier/Laplace transform of the anomalous dimension density $\rho(\lambda)$

$$\rho(\lambda) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}\tau\, \mathrm{e}^{\mathrm{i}\tau\lambda} Z(\mathrm{i}\tau)$$

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Conserved Quantities

There is a number of conserved charges in the model. To obvious Energy $E = H_2$ and momentum P there are also additional quadratic charges

- "Total length"

$$L = \mathrm{tr}\, \bar{X}^a X_a \equiv H_0$$

- "Total Spin"

$$\vec{S} = \frac{1}{2} \mathrm{tr}\, \bar{X}^a \vec{\sigma}_a{}^b X_b$$

where $\vec{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$ are Pauli matrices

$$S \leq L/2, \qquad S = |\vec{S}|$$

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Chemical potentials

Consider partial partition function restricted to subspace with L and $\vec{S}$ fixed

$$Z(L,\vec{S};\beta) \equiv \mathrm{e}^{\mathcal{S}(L,\vec{S};\beta)} = \mathrm{tr}_{L,\vec{S}}\, \mathrm{e}^{-\beta\Delta}$$

where $\mathrm{tr}_{L,\vec{S}}$ is restricted to states with total length L and total spin S.

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For large L and Temperature the statistically the probability is uniformly distributed among the states inside of a subspace with fixed L and and $\vec{S}$, i.e. one can take the expansion

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$$\mathcal{F}(\mu,\vec{x};\beta)=\mathcal{F}_0(\mu,\vec{x};\beta)-\beta\langle H_2\rangle_0$$

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Gauged matrix oscillator

After integration over the matrix fields one is left with integral over the gauge field A, which can be reduced to the integral over its $N-1$ eigenvalues

$$Z_0(\mu,\vec{x}) = \frac{2^{-\frac{1}{2}N(N+1)}e^{N^2\mu}}{[\sinh(\mu_+/2)\sinh(\mu_-/2)]^N}\int\prod_n d\theta_n \times \prod_{m>n}\frac{1-\cos\theta_{mn}}{(\cosh\mu_+ - \cos\theta_{mn})(\cosh\mu_- - \cos\theta_{mn})}$$

where $\mu_\pm = \mu \pm x/2$ and $\theta_{mn} = \theta_m - \theta_n$.

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We should find the minimum of the function

$$F(\theta;\mu,\vec{x}) = -N^2\mu + N[\ln\sinh(\mu_+/2) + \ln\sinh(\mu_-/2)] + \frac{1}{2}\sum_{\substack{m,n\\ n\neq m}} \big(-\ln(1-\cos\theta_{mn}) + \ln(\cosh\mu_+ - \cos\theta_{mn}) + \ln(\cosh\mu_- - \cos\theta_{mn})\big)\,,$$

Function $F(\theta;\mu,\vec{x})$ can be expanded in powers '$\mathrm{e}^{-\mu_\pm}$' as

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where $\rho(\theta)$ is the eigenvalue distribution.Since,

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When,

$$\left(1 - 2\mathrm{e}^{-\omega\mu}\cosh\left(\frac{\omega x}{2}\right)\right) = 0,$$

there is an apparent zero mode $\Rightarrow$ singularity in the Free energy?

Singularity = Phase transition

Evaluation by Polya Enumeration Theorem ($x = 0$): Phase transition at $\mu_c = \ln 2$!
Another interesting feature: Contribution $\sim O(N^2)$ and $\sim O(N)$ canceled! The leading contribution is at most finite as $N \longrightarrow \infty$!

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Small chemical potential μ

- When μ is large the repulsion dominates and one expects an eigenvalue-θ distribution which is almost uniform.
- The scale of interaction is $\ell_{int} \sim \sqrt{\mu_+\mu_-}$ and when $\ell_{int} \lesssim 2\pi$ the eigenvalues can condense in a compact region of this size.
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Incompressible liquid approximation

Consider situation when $\sqrt{\mu_+\mu_-} \ll 2\pi$ and assume that the eigenvalues θ_n condensed in some region of size Λ. Approximate the density inside the condensate to be constant.(In fact, the constant mode is only expected to contribute to the thermodynamical functions.)

The zero pressure condition at the center of the condensate gives $\Lambda = 2\sqrt{\mu_+\mu_-}$.

Evaluation of the effective action (Entropy) yields,

$$S_{\text{eff}}(L,S) = 4N^2 \left(\sqrt{\frac{L_+}{L_-}} \ln \left| \frac{1+\sqrt{\frac{L_-}{L_+}}}{1-\sqrt{\frac{L_-}{L_+}}} \right| + \sqrt{\frac{L_-}{L_+}} \ln \left| \frac{1+\sqrt{\frac{L_+}{L_-}}}{1-\sqrt{\frac{L_+}{L_-}}} \right| \right)$$

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Inclusion of H_2

Knowing the eigenvalue distribution we can compute the ev $\langle H_2 \rangle_0$ for each case.

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Conclusions

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- For large L we consider the one-loop contribution to the dilatation operator as a perturbation to the classical one described by the gauged matrix oscillator
- We find signals of phase transition; compatibility with PET computation
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- More loose ends. . .

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