Classification of Supersymmetric

**Backgrounds of String Theory** 

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Main results based on:

- Systematics of M-theory spinorial geometry U. Gran, G. Papadopoulos, D.R.; hep-th/0503046,
- Systematics of IIB spinorial geometry U. Gran, J. Gutowski, G. Papadopoulos, D.R.; hep-th/0507143,
- Maximally supersymmetric *G*-backgrounds of IIB supergravity U. Gran, J. Gutowski, G. Papadopoulos, D.R.; hep-th/0604079,
- N = 31 is not IIB
   U. Gran, J. Gutowski, G. Papadopoulos, D.R.; hep-th/0606049.

## Introduction

Supersymmetric solutions of supergravity play an important role in string/M-theory:

- entropy matching: black holes and configurations of M-/D-branes,
- phenomenology: flux compactifications to  $\mathcal{N} = 1$  in four dimensions,
- AdS/CFT:  $AdS_5 \times S^5$ , its Penrose limit and deformations with less supersymmetry,

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Usually via Ansätze of metric and fluxes based on physical intuition. Possible to miss unexpected and exciting backgrounds (e.g. black rings).

Desirable to have classifications!

#### Classifications

What about classifications? e.g.

- D = 4: minimal  $\mathcal{N} = 2$  [a] and coupled to vectors [b]
- D = 5: minimal  $\mathcal{N} = 1$  [c]
- D = 6: minimal  $\mathcal{N} = 1$  [d]

Theories with 8 supersymmetries and solutions with N = 4, 8. Two techniques: Newman-Penrose ('82, '83) & spinor bilinears ('02,...).

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We propose **spinorial geometry** as a new technique for more complicated cases such as maximal supergravity.

We will focus on IIB since that is the most difficult case (coset scalars, more - complex & self-dual - fluxes, chiral fermions), but the story generalises to other supergravities. Supersymmetric backgrounds with fluxes

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With fluxes:  $susy \Rightarrow G$ -structures - no classification.

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Holonomy(D)  $\neq$  gauge group Spin(dim - 1, 1)

Massless/ungauged maximal supergravities: holonomy  $SL(32, \mathbb{R})$  [a] 11D:  $D_m = \partial_M + \frac{1}{4}\Omega_{M,PQ}\Gamma^{PQ} - \frac{1}{288}(\Gamma_M{}^{PQRS}\Gamma_{PQRS} - 8\Gamma_{MPQR}\Gamma^{PQR})$  $\Rightarrow Spin(10, 1) \Rightarrow$  extends this to  $SL(32, \mathbb{R})$  Supersymmetric backgrounds with fluxes

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Inequality is reason for complications with fluxes (e.g. zero curvature  $\Rightarrow$  trivialisable connection  $\Rightarrow$  gauge where Killing spinors are constant). Solving the Killing spinor equations

Which *G*-structures does one find? Different methods to solve the KSE:

Spinor bilinears:

- Uses the  $N^2$  relations  $\nabla \kappa_{ij} \sim F \kappa_{ij}$  where  $\kappa_{ij} = \bar{\epsilon}_i \Gamma^{(p)} \epsilon_j$ ,
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Spinorial geometry:

- Basis in the space of spinors and description in terms of forms,
- Analyses the *N* Killing spinor equations  $D\epsilon_i = 0$  directly,
- Necessary and sufficient conditions.

Basis in space of spinors

Spinor in terms of forms [a]:

space of Dirac spinors  $\epsilon$ of Spin(9,1)dimension 64  $\equiv \begin{array}{c} \text{space of forms } \eta \\ \text{spanned by } e_1, \dots, e_5 \\ \text{(with compl. coeff.)} \\ \text{dimension } 2 \cdot 2^5 \end{array}$ 

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 $\Gamma_M$ -matrices in null and holomorphic basis  $M = (-, +, \alpha, \bar{\alpha})$ :  $\Gamma_a \eta = \sqrt{2} e_a \wedge \eta \quad \text{for } a = (-, \alpha) \\ \Gamma_{\bar{a}} \eta = \sqrt{2} e_a \lrcorner \eta \quad \text{for } \bar{a} = (+, \bar{\alpha}) \quad \Leftrightarrow \quad \text{creation operators}$  annihilation operators

SU(4)-covariant action of  $\Gamma$ -matrices on spinor.

**IIB Killing spinor equtions** 

Arbitrary IIB spinor: (with  $a = (\alpha, 5)$ )

$$\epsilon = (f_1 + if_2)1 + (g_1^{a_1 a_2} + ig_2^{a_1 a_2})e_{a_1 a_2} + (h_1^{a_1 \cdots a_4} + ih_2^{a_1 \cdots a_4})e_{a_1 \cdots a_4}$$

$$2 \cdot 1 \qquad 2 \cdot 5$$

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#### Substitute $\epsilon$ into IIB Killing spinor eqs:

$$D_M \epsilon = (\partial_M + \frac{1}{4} \Omega_{M,N_1N_2} \Gamma^{N_1N_2} - \frac{i}{2} Q_M + \frac{i}{48} \Gamma^{N_1...N_4} F_{MN_1...N_4}) \epsilon - \frac{1}{96} (\Gamma_M^{N_1N_2N_3} G_{N_1N_2N_3} - 9\Gamma^{N_1N_2} G_{MN_1N_2}) (C\epsilon)^* = 0, A\epsilon = P_N \Gamma^N (C\epsilon)^* + \frac{1}{24} G_{N_1N_2N_3} \Gamma^{N_1N_2N_3} \epsilon = 0,$$

and expand in basis (amounts to products of  $\Gamma$ -matrices), and set all coefficients equal to zero [a].

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Problem of classifying supersymmetric solutions is reduced to parametrising the N Killing spinors and solving the linear system.

[a]: Papadopoulos et al '04, '05, Mac Conamhna '04, '05.

## Examples

- N = 1: Use Lorentz symmetry to bring Killing spinor to one of the three orbit representatives with stability subgroup G:
  - $\begin{array}{ll} G = Spin(7) \ltimes \mathbb{R}^8 & \text{with } \epsilon = (f_1 + if_2)(1 + e_{1234}) \\ G = SU(4) \ltimes \mathbb{R}^8 & \text{with } \epsilon = (f_1 + if_2)1 + (h_1 + if_2)e_{1234} \\ G = G_2 & \text{with } \epsilon = f_1(1 + e_{1234}) + ig_1(e_{15} + e_{2345}) \end{array}$

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- Maximal number of *G*-invariant spinors: simple embedding of *G* [b].

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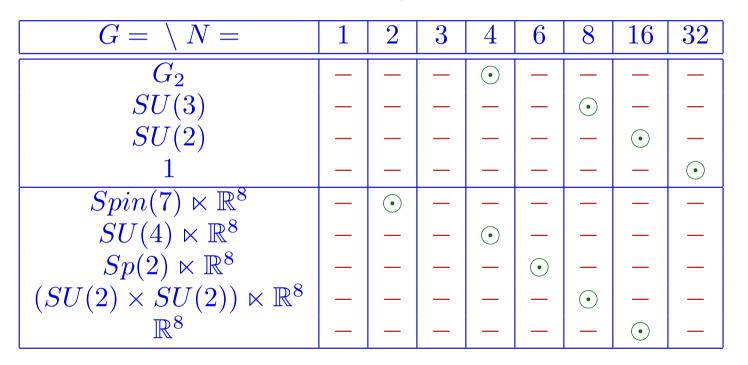
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and plug into KSE [a],

- Maximal number of *G*-invariant spinors: simple embedding of *G* [b],
- N = 31: Use Lorentz symmetry to bring orthogonal spinor to one of the three orbit representatives [c].

Maximally supersymmetric *G*-backgrounds

#### Maximal number of *G*-invariant Killing spinors [a]:



Flux deformations of gravitational solutions with same Killing spinors.

Maximally supersymmetric *G*-backgrounds

Compact  $G = G_2, SU(3), SU(2)$ : [a]

Direct product:

 $(AdS_{d/2} \times S^{d/2} \text{ or Hpp or Mink}_{1,d-1}) \times \text{(manifold } M_{10-d} \text{ of special holonomy } G\text{)}$ Fluxes constrained to d-dimensional part Maximally supersymmetric *G*-backgrounds

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Non-compact  $G = K \ltimes \mathbb{R}^8$ ,  $K = Spin(7), SU(4), Sp(2), SU(2) \times SU(2), 1$  [a]

pp-wave on 8D manifold with special holonomy Knull fluxes  $P_-, G_{-mn}, F_{-mnpq}$ 

[a]: Gran, Gutowski, Papadopoulos, D.R. '06.

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No more c.c. in the diff. KSE of IIB  $\Rightarrow N$  even  $\Rightarrow N = 32$ . NO N = 31 SOLUTIONS IN IIB!

First constraint on N in type II theories.

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Same result in IIA using moving *G*-frame method [b].

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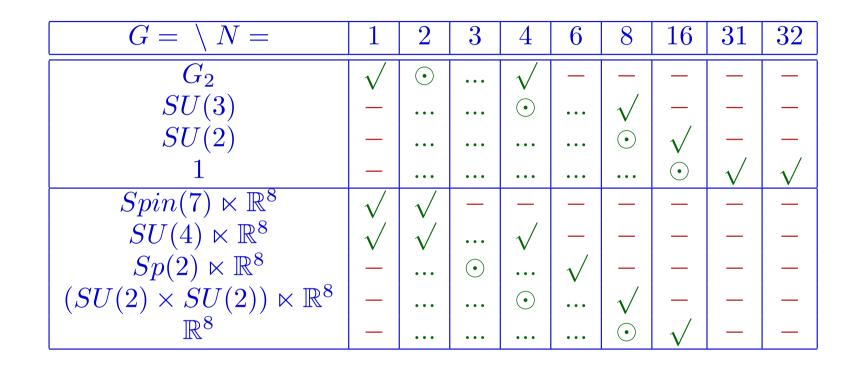
N = 31 in M-theory remains an open question.

[a]: Gran, Gutowski, Papadopoulos, Roest '06, [b]: Bandos, De Azcárraga, Varela '06.

# Outlook

- Maximal G-backgrounds of IIB  $\sqrt{}$
- $N = 31 \sqrt{}$
- Half-maximal *G*-backgrounds of IIB?
  - G = SU(3) with N = 4: AdS/CFT and flux compactifications [a]
  - all N = 16 solutions (only one case remaining: half-max. 1)
- ... etcetera! Classification of all supersymmetric solutions?
- T-duality?
- relation to Hitchin's generalised geometry? [b]

### IIB status report



- $\sqrt{}$ : Killing spinor equations have already been solved,
- $\odot$ : backgrounds with half-maximal number of *G*-invariant spinors,
- ...: have more complicated linear systems,
- -: do not occur.