Compactifications on Hyperbolic Spaces

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Collaboration with: C.Bachas, C.Kounnas (ENS), M.Petropoulos (CPHT).

Outline

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Why are we here

Compactifications on Maximally Symmetric Spaces

- Compactifications on maximally symmetric spaces
- Very nice geometric and algebraic properties
- Negative curvature hyperbolic spaces [Kehagias, Russo]
- Stability of non-supersymmetric backgrounds
- Compactification
- Dual theories
- Cosmology [Townsend]

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Avant-première

Main results

- Hyperbolic space solutions in M-theory
- AdS-splitting
- Hyperbolic solution are stable with respect to small perturbations even without supersymmetry
- M5-brane wrapped around an H_3 manifold

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Short introduction to Hyperbolic Manifolds

Maximally Symmetric Spaces

- A n-dimensional maximally symmetric space can be defined as a pseudosphere in $n+1$ dimensions:

$$\epsilon_0 (X^0)^2 + (X^1)^2 + \cdots + (X^{n-1})^2 + \epsilon_n (X^n)^2 = \epsilon L^2$$

$(\epsilon_0, \epsilon_n, \epsilon)$	$- - -$	$- + +$	$- + -$	$+ + +$
Space	AdS_n	dS_n	H_n	S^n

Hyperbolic (Poincaré) Spaces

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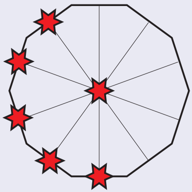
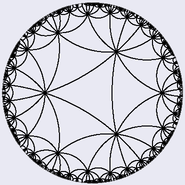
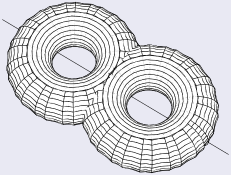
Riemann Surfaces: the Double Torus



Fundamental polygon



Tessellation of the Poincaré disk



Short Introduction to Hyperbolic Manifolds

H_n/Γ: no moduli

- Any finite closed manifold of constant negative curvature is H_n/Γ, $\Gamma \subset SO(1,n)$
- Rigidity theorem: the geometry of a finite manifold H_n/Γ is determined by its fundamental group [Mostow]
  - (Algebraic) Given Γ_1 and Γ_2, lattices in $SO(1,n)$ such as H_n/Γ_i is finite-volume, then if they are isomorphic then they are conjugate.
  - (Geometric) If M and N are complete finite-volume hyperbolic and there exist an isomorphism $f: \pi_1(M) \to \pi_1(N)$, then f is induced by a unique isometry.

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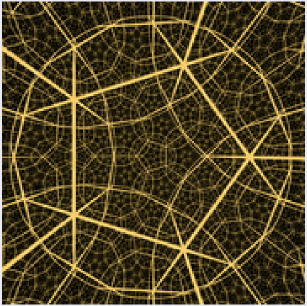
Example: Seifert-Weber Manifold



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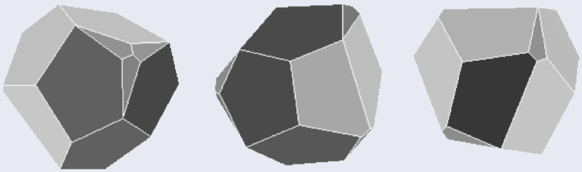

Outline

M-theory ansatz

Action and Notation

- Ansatz: direct products of symmetric spaces
  $M_{11} = M_0 \times M_1 \times M_2 \times \dots$
- Split the Ricci tensor in blocks

$$R^{\mu}{}_{\nu}\big|_i = k_i \, \delta^{\mu}{}_{\nu}\big|_i$$

- Choose the gauge fields as:

$$F_I = Q_I \omega_I$$

where $\omega_I = \bigwedge_{i \in I} \omega_i$.

Equations of motion

Algebraic System

- The equation of motion are an algebraic system for k_i:

$$2k_i - R = -\sum_I \varepsilon_I(i)\,\varepsilon_I(0) Q_I^2, \text{ for } i = 0, 1, 2, \dots,$$

where $R = \sum_i d_i k_i$ is the total Ricci scalar and

$$\varepsilon_I(i) = \begin{cases} +1 & \text{if } i \in I, \\ -1 & \text{otherwise.} \end{cases}$$

- Negative values for k_i are possible due to the stress-energy tensor coming from fields living in other subspaces.
- Only negative contribution on M_0: no-go for de Sitter.

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Splitting Series Solution

AdS Splitting

- We consider Cartesian products
- The Ricci tensor has a block matrix form
- If each block is a symmetric space there is a fixed choice for the curvatures where AdS_7 is not distinguishable from $\mathrm{AdS}_d \times H_{7-d}$

$$\mathrm{AdS}_7 \longrightarrow \mathrm{AdS}_d \times H_{7-d}$$

- Starting from the well known $\mathrm{AdS}_7 \times S^4$ one gets:

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Hyperbolic Compact Manifolds and Supergravity

Killing Spinors on H_n/Γ

- If n is even there is one Killing spinor in $SO(1, n-1)$
  - To preserve it, $\Gamma \subset SO(1, n-3)$.
  - H_n/Γ would remain infinite
  - No supersymmetric compactifications for n even
- If n is odd there are two Killing spinors in $SO(1, n-1)$

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Stability

General Approach

- No protection from supersymmetry. We can check at least the stability against scalar modes.
- Turn on dilatation of the compact subspaces
- Write the effective, dimensionally-reduced action
- Check the masses for the scalar fields against the Breitenlohner-Freedman bound
- In the AdS-splitting case there are no tachyons.

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d-dimensional Action

- Effective action

$$S \sim \int \mathrm{d}^d x \sqrt{-g^{(d)}} \left[R^{(d)} - \frac{1}{2} \partial_\mu \Phi_i \partial^\mu \Phi_i - \bar{V}(\Phi) \right]$$

- Φ_1 is the overall volume
- Φ_2 is the ratio between the two volumes
- The effective potential is

$$\bar{V}(\Phi) = e^{-2(d_1 \varphi_1 + d_2 \varphi_2)/(d-2)} \times \left(-e^{-2\varphi_1(x)} R^{(1)} - e^{-2\varphi_2(x)} R^{(2)} + V(\varphi_1, \varphi_2) \right)$$

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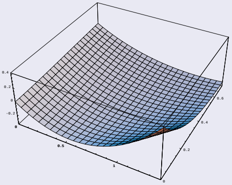
Four-dimensional Action for $\mathrm{AdS}_4 \times H_3 \times S^4$

- Consider $\mathrm{AdS}_4 \times H_3 \times S^4$.
- The stability is encoded in the Hessian matrix.
- The potential has a minimum



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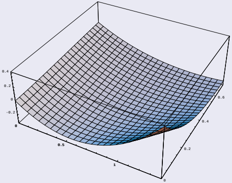

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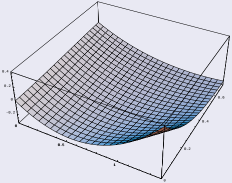

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Branes

A Brane Source Interpretation

- Alternative description in terms of M-branes wrapping the hyperbolic space
- The stability of the AdS $\times H \times S$ backgrounds results from the presence of two competing effects:
  - The tension given by the M-brane trying to shrink the internal manifold
  - The pressure due to the negative curvature that tends to blow the hyperbolic part.
- Only one magnetic charge. Back-reaction of an M5-brane

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- Alternative description in terms of M-branes wrapping the hyperbolic space
- The stability of the AdS $\times H \times S$ backgrounds results from the presence of two competing effects:
  - The tension given by the M-brane trying to shrink the internal manifold
  - The pressure due to the negative curvature that tends to blow the hyperbolic part.
- Only one magnetic charge. Back-reaction of an M5-brane

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M5 brane

M5-brane Ansatz

- M5 brane wrapping a H_3 hyperbolic manifold
- Most simple ansatz, preserving $SO(1,2) \times SO(4) \times SO(1,3)$

$$ds^2 = e^{2\varphi_1(r)}\left(-dx_0^2 + dx_1^2 + dx_2^2\right) + e^{2\varphi_2(r)}\left(dr^2 + r^2 d\Omega_4^2\right) + \\ + e^{2\varphi_3(r)}\left(\frac{dx_8^2 + dx_9^2 + dx_{10}^2}{x_8^2}\right)$$

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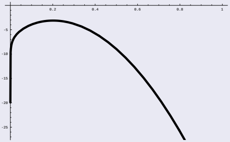
M5 brane

Numerical Solution

- The system can be solved numerically



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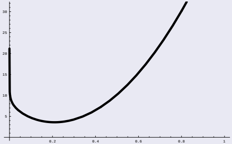
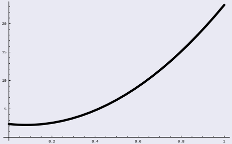


$\varphi_2(r)$



$\varphi_3(r)$

- Small-r regime: $\mathrm{AdS}_4 \times S^4 \times H_3$ solution

$$\varphi_1(r) \sim \log r \qquad \varphi_2(r) \sim -\log r \qquad \varphi_3(r) = \text{const.}$$

- Asymptotically flat

$$\varphi_1(r) \sim -r^2 \qquad \varphi_2(r) \sim r^2 \qquad \varphi_3(r) \sim r^2$$

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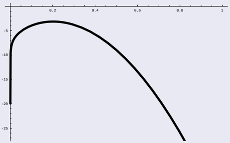

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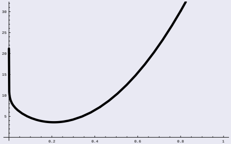

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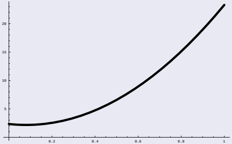

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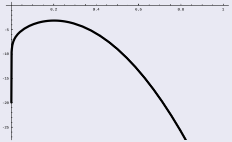

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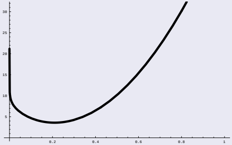

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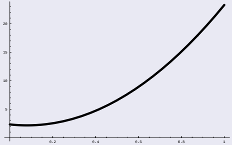

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Outline

What Did We See?

Main results

- Hyperbolic space solutions in M-theory
- AdS-splitting
- Hyperbolic solution stable with respect to small perturbations even without supersymmetry
- M5-brane wrapped around an H_3 manifold

to boldly go...

- Do supersymmetric compactifications exist?
- Closed form solution? Phase space?
- Type II solutions
- Dual theories
- Cosmological applications

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