Compactifications on Hyperbolic Spaces

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Collaboration with:

C.Bachas, C.Kounnas (ENS), M.Petropoulos (CPHT).

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- **1** Why are we here?
- 2 Preliminaries
- **3** The "AdS Splitting Series"
- **4** Stability
- **5** Source Brane Interpretation
- **6** Conclusions





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- Compactifications on maximally symmetric spaces
- Very nice geometric and algebraic properties
- Negative curvature hyperbolic spaces [Kehagias, Russo]
- Stability of non-supersymmetric backgrounds
- Compactification
- Dual theories
- Cosmology [Townsend]





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• Hyperbolic space solutions in M-theory

- AdS-splitting
- Hyperbolic solution are stable with respect to small perturbations even without supersymmetry
- M5-brane wrapped around an H₃ manifold





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Short i	ntroduction	n to Hyperk	oolic Mar	nifolds	

• A *n*-dimensional maximally symmetric space can be defined as a pseudosphere in *n* + 1 dimensions:

$$\epsilon_0(X^0)^2 + (X^1)^2 + \dots + (X^{n-1})^2 + \epsilon_n(X^n)^2 = \epsilon L^2$$

$$\begin{array}{c|c} (\epsilon_0, \epsilon_n, \epsilon) & --- & -++ & -+- & +++ \\ \hline \text{Space} & \text{AdS}_n & \text{dS}_n & H_n & S^n \end{array}$$

Hyperbolic (Poincaré) Spaces

• $H_n = SO(1, n) / SO(n)$ coset.



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- $H_n = SO(1, n) / SO(n)$ coset.
- Maximally symmetric.
- Constant negative curvature $R = -n(n-1)/L^2$
- Conformally flat $C_{\mu\nu\rho\sigma} = 0$.



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H_n/Γ : no moduli

- Any finite closed manifold of constant negative curvature is *H_n*/Γ, Γ ⊂ *SO*(1, *n*)
- **Rigidity theorem**: the geometry of a finite manifold H_n/Γ is determined by its fundamental group [Mostow]
 - (Algebraic) Given T₁ and T₂, lattices in SO(1, n) such as H_n/T_i is finite-volume, then if they are isomorphic then they are conjugate.

(Geometric) If M and N are complete finite-volume





Short I	ntroductio	n to Hyper	holic Mar	aifolds	
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 - (Geometric) If *M* and *N* are complete finite-volume hyperbolic and there exist an isomorphism
 - $f : \pi_1(M) \to \pi_1(N)$, then *f* is induced by a *unique* isometry.



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Short introduction to Hyperbolic Manifolds							
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H_3/Γ

- Generalization of Riemann surfaces.
- Only lower bounds on the volume *V* > 0.166





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Example: Seifert-Weber Manifold





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Example: Weeks Manifold





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M-theo					

Action and Notation

- Ansatz: direct products of symmetric spaces $M_{11} = M_0 \times M_1 \times M_2 \times ...$
- Split the Ricci tensor in blocks

$$R^{\mu}_{\nu}\big|_i = k_i \,\delta^{\mu}_{\nu}\big|_i$$

• Choose the gauge fields as:

$$F_I = Q_I \omega_I$$

where
$$\omega_I = \bigwedge_{i \in I} \omega_i$$
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Equations of motion						

Algebraic System

• The equation of motion are an algebraic system for *k_i*:

$$2k_i - R = -\sum_I \varepsilon_I(i) \varepsilon_I(0) Q_I^2$$
, for $i = 0, 1, 2, ...,$

where $R = \sum_{i} d_{i}k_{i}$ is the total Ricci scalar and

$$\varepsilon_I(i) = \begin{cases} +1 & \text{if } i \in I, \\ -1 & \text{otherwise.} \end{cases}$$

• Negative values for *k_i* are possible due to the stress-energy tensor coming from fields living in other subspaces.



• Only negative contribution on M₀: no-go for de Sitter.



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Splitting Series Solution						

AdS Splitting

- We consider Cartesian products
- The Ricci tensor has a block matrix form
- If each block is a symmetric space there is a fixed choice for the curvatures where AdS₇ is not distinguishable from AdS_d × H_{7-d}

$AdS_7 \rightarrow AdS_d \times H_{7-d}$

• Starting from the well known AdS₇ × S⁴ one gets:

 $\mathrm{AdS}_4 imes H_3 imes S^4$



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• If *n* is even there is one Killing spinor in SO(1, n - 1)

- To preserve it, $\Gamma \subset SO(1, n 3)$.
- H_n/Γ would remain infinite
- No supersymmetric compactifications for *n* even

• If *n* is odd there are two Killing spinors in SO(1, n-1)



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- No protection from supersymmetry. We can check at least the stability against scalar modes.
- Turn on dilatation of the compact subspaces
- Write the effective, dimensionally-reduced action
- Check the masses for the scalar fields against the Breitenlohner-Freedman bound
- In the AdS-splitting case there are no tachyons.



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Effective action

$$S \sim \int \mathrm{d}^d x \, \sqrt{-g^{(d)}} \left[R^{(d)} - \frac{1}{2} \partial_\mu \Phi_i \partial^\mu \Phi_i - \bar{V}(\Phi)
ight]$$

- Φ_1 is the overall volume
- Φ_2 is the ratio between the two volumes
- The effective potential is

 $\bar{V}(\Phi) = e^{-2(d_1\varphi_1 + d_2\varphi_2)/(d-2)} \times \\ \times \left(-e^{-2\varphi_1(x)} P^{(1)} - e^{-2\varphi_2(x)} P^{(2)} \right)$



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- Φ_2 is the ratio between the two volumes
- The effective potential is

$$\bar{V}(\Phi) = e^{-2(d_1\varphi_1 + d_2\varphi_2)/(d-2)} \times \\ \times \left(-e^{-2\varphi_1(x)} R^{(1)} - e^{-2\varphi_2(x)} R^{(2)} + V(\varphi_1, \varphi_2) \right)$$



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Why 00	Preliminaries	AdS Splitting 0000	Stability ○●○	M5 branes	Conclusions O
Stabili	ity				

Effective action

$$S \sim \int \mathrm{d}^d x \, \sqrt{-g^{(d)}} \left[R^{(d)} - \frac{1}{2} \partial_\mu \Phi_i \partial^\mu \Phi_i - \bar{V}(\Phi) \right]$$

- Φ_1 is the overall volume
- Φ₂ is the ratio between the two volumes
- The effective potential is

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Why 00	Preliminaries	AdS Splitting 0000	Stability ○○●	M5 branes	Conclusions O
Stabi	lity				

Four-dimensional Action for $AdS_4 \times H_3 \times S^4$

- Consider $AdS_4 \times H_3 \times S^4$.
- The stability is encoded in the Hessian matrix.
- The potential has a minimum







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- 1 Why are we here?
- 2 Preliminaries
- **3** The "AdS Splitting Series"
- ④ Stability
- **5** Source Brane Interpretation
- **6** Conclusions





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Why	Preliminaries	AdS Splitting	Stability	M5 branes	Conclusions
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Branes	5				

- Alternative description in terms of M-branes wrapping the hyperbolic space
- The stability of the AdS × *H* × *S* backgrounds results from the presence of two competing effects:
 - The tension given by the M-brane trying to shrink the internal manifold
 - The pressure due to the negative curvature that tends to blow the hyperbolic part.
- Only one magnetic charge. Back-reaction of an M5-brane





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M5 b	rane				

M5-brane Ansatz

- M5 brane wrapping a *H*₃ hyperbolic manifold
- Most simple ansatz, preserving $SO(1,2) \times SO(4) \times SO(1,3)$

$$ds^{2} = e^{2\varphi_{1}(r)} \left(-dx_{0}^{2} + dx_{1}^{2} + dx_{2}^{2} \right) + e^{2\varphi_{2}(r)} \left(dr^{2} + r^{2} d\Omega_{4}^{2} \right) + e^{2\varphi_{3}(r)} \left(\frac{dx_{8}^{2} + dx_{9}^{2} + dx_{10}^{2}}{r^{2}} \right)$$



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Why	Preliminaries	AdS Splitting	Stability	M5 branes	Conclusions	
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M5 brane						

Numerical Solution

• The system can be solved numerically



• Small-*r* regime: $AdS_4 \times S^4 \times H_3$ solution

- $\varphi_1(r) \sim \log r$ $\varphi_2(r) \sim -\log r$ $\varphi_3(r) = \text{const}$
- Asymptotically flat

$$\varphi_1(r) \sim -r^2 \qquad \qquad \varphi_2(r) \sim r^2 \qquad \qquad \varphi_3(r) \sim r^2$$

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Why 00	Preliminaries	AdS Splitting 0000	Stability 000	M5 branes	Conclusions •			
What Did We See?								

Main results

- Hyperbolic space solutions in M-theory
- AdS-splitting
- Hyperbolic solution stable with respect to small perturbations even without supersymmetry
- M5-brane wrapped around an H₃ manifold

to boldly go...

- Do supersymmetric compactifications existi?
- Closed form solution? Phase space?
- Type II solutions
- Dual theories




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- Dual theories
- Cosmological applications



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