DEAR ORGANIZERS

THANK YOU - HERZLICHEN DANK
GRAZIE - MERCI BEAUCOUP - etc

Endurance is one of the most difficult disciplines, but it is to the one who endures that the final victory comes. - Buddha (Gautama Buddha) Indian religious leader (563 BC - 483 BC)

Enjoy what you can, endure what you must. - Johann Wolfgang von Goethe

It is more blessed to give than to receive. - Bible, Acts (ch. XX, v. 35) - (reminder to funding agencies)

Weyl-Conformally Invariant Lightlike Branes: New Aspects in Black Hole Physics and Kaluza-Klein Dynamics

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Plan of Talk

- Introduction - Main Motivation
- New Class of p-Brane Theories – Weyl-Conformal Invariance for any p, Intrinsically Lightlike Branes for any even p (*WILL-Branes*)
- *WILL*-Brane Solutions in Various Gravitational Backgrounds – the *WILL*-Brane as a Material Event Horizon
- *WILL*-Brane as a *dynamical* source for gravity and electromagnetism – the *Membrane Paradigm*
- *WILL*-Brane Dynamics in Kaluza-Klein Spaces – trapped massless winding modes
- Conclusions

Introduction - Main Motivation

Geometrically motivated field theories (gravity, strings, branes, etc.) – their Lagrangian formulation requires reparametrization-covariant (generally-covariant) integration measure densities. Standard choice is:

- Standard Riemannian: $\sqrt{-g}$ with $g \equiv \det \|g_{\mu\nu}\|$

However, equally well-suited is the following:

- Modified non-Riemannian:

$$\Phi(\varphi) \equiv \frac{1}{D!} \varepsilon^{\mu_1 \ldots \mu_D} \varepsilon_{i_1 \ldots i_D} \partial_{\mu_1} \varphi^{i_1} \ldots \partial_{\mu_D} \varphi^{i_D}$$

Models involving Gravity with modified measure, or both standard *and* modified - *Two-Measure Gravitational Models* :

$$S = \int d^D x \, \Phi(\varphi) \, L_1 + \int d^D x \, \sqrt{-g} \, L_2$$

$$L_{1,2} = e^{\frac{\alpha\phi}{M_P}} [-\frac{1}{\kappa} R(g, \Gamma) - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + (\text{Higgs}) + (\text{fermions})]$$

Auxiliary fields φ^i are pure-gauge degrees of freedom except for the new dynamical “geometric” field:
$\zeta(x) \equiv \frac{\Phi(\varphi)}{\sqrt{-g}}$ – determined only through the matter fields locally (*i.e.*, without gravitational interaction).

Two-measure gravity models address various basic problems and provide possible solutions:

- Scale invariance and its dynamical breakdown; Spontaneous generation of dimensionfull fundamental scales;

- Cosmological constant problem;

- Geometric origin of fermionic families.

- Applications in modern brane-world scenarios.

The new Weyl-conformally invariant p-brane models will describe *light-like* branes.

Role of lightlike membranes ("shells") in General Relativity:

- Describe *impulsive lightlike signals from violent astrophysical events* [C. Barrabes, P. Hogan, *"Singular Null-Hypersurfaces in General Relativity"*] – final explosion in cataclysmic processes (supernovae, collision of neutron stars) produces burst of matter travelling with the speed of light plus gravitational radiation.

- "Membrane Paradigm" in black-hole physics [K. Thorne *et.al.*, W. Israel *et.al.*] – event horizons as membranes. Our Weyl-conformally invariant membrane ($p = 2$) model provides explicit *dynamical* realization of the "membrane paradigm".

New Class of Weyl-Invariant p-Brane Theories.

Consider the following new kind of p-brane action involving modified world-volume measure $\Phi(\varphi)$ and an auxiliary (Abelian) world-volume gauge field A_a:

$$S = -\int d^{p+1}\sigma\, \Phi(\varphi)[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \sqrt{F_{ab}(A)F_{cd}(A)\gamma^{ac}\gamma^{bd}}]$$

$$\Phi(\varphi) \equiv \frac{1}{(p+1)!}\varepsilon_{i_1\ldots i_{p+1}}\varepsilon^{a_1\ldots a_{p+1}}\partial_{a_1}\varphi^{i_1}\ldots\partial_{a_{p+1}}\varphi^{i_{p+1}}$$

where $F_{ab} = \partial_a A_b - \partial_b A_a$ and $a, b = 0, 1, \ldots, p; i, j = 1, \ldots, p+1$.

The above action is invariant under Weyl (conformal) symmetry:

$$\gamma_{ab} \longrightarrow \gamma'_{ab} = \rho\,\gamma_{ab} \quad , \quad \varphi^i \longrightarrow \varphi'^{\,i} = \varphi'^{\,i}(\varphi)$$

with Jacobian $\det\|\frac{\partial\varphi'^{\,i}}{\partial\varphi^j}\| = \rho$.

$$S = -\int d^{p+1}\sigma\, \chi\sqrt{-\gamma}[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \sqrt{F_{ab}F_{cd}\gamma^{ac}\gamma^{bd}}] \quad , \quad \chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}$$

Differences w.r.t. the standard Nambu-Goto p-branes (in the Polyakov-like formulation) :

- New integration measure density $\Phi(\varphi)$ instead of $\sqrt{-\gamma}$, and no "cosmological-constant" term
  $((p-1)\sqrt{-\gamma})$;

- Variable brane tension $\chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}$ which is Weyl-conformal *gauge dependent*: $\chi \to \rho^{\frac{1}{2}(1-p)}\chi$;

- Auxiliary world-sheet gauge field A_a in a "square-root" Maxwell (Yang-Mills) term (*Nielsen-Olesen, Spallucci* et.al.); straightforward generalization to non-Abelian A_a: $\sqrt{\mathrm{T}r(F_{ab}F_{cd})\gamma^{ac}\gamma^{bd}}$ with $F_{ab} = \partial_a A_b - \partial_b A_c + i[A_a, A_b]$;

- Possible couplings of auxiliary A_a to external world-volume ("color" charge) currents J^a;

- Weyl-invariant for *any* p; describes *intrinsically light-like* p-branes for any even p (*i.e.*, odd-dimensional world-volume).

REMARK. There are NO quantum conformal anomalies in odd dimensions!

Intrinsically Light-Like Branes

Consider the γ^{ab}-eqs. of motion:

$$\frac{1}{2}\left(\partial_a X \partial_b X\right) + \frac{F_{ac}\gamma^{cd}F_{db}}{\sqrt{FF\gamma\gamma}} = 0$$

employing short-hand notation:

$$\left(\partial_a X \partial_b X\right) \equiv \partial_a X^{\mu} \partial_b X^{\nu} G_{\mu\nu} \;, \; \sqrt{FF\gamma\gamma} \equiv \sqrt{F_{ab}F_{cd}\gamma^{ac}\gamma^{bd}}$$

Since the induced metric $g_{ab} \equiv \left(\partial_a X \partial_b X\right)$ is required to be Lorentz-type, the latter eq. is consistent for odd $(p+1)$ only. Further, F_{ab} is anti-symmetric $(p+1)\times(p+1)$ matrix, therefore, F_{ab} is *not invertible* in any odd $(p+1)$ – it has at least one zero-eigenvalue vector-field V^a $(F_{ab}V^b = 0)$.

Therefore, for any odd $(p+1)$ the induced metric $(\partial_a X \partial_b X)$ on the world-volume of the Weyl-invariant brane is *singular* (as opposed to the ordinary Nambu-Goto brane (!)) :

$$(\partial_a X \partial_b X) V^b = 0 \quad , \quad \text{i.e.} \;\; (\partial_V X \partial_V X) = 0 \;\; , \;\; (\partial_\perp X \partial_V X) = 0$$

where $\partial_V \equiv V^a \partial_a$ and $\partial_\perp$ are derivates along the tangent vectors in the complement of V^a.

Important Conclusion – An Overall Constraint on Dynamics. Every point on the world-volume of the Weyl-invariant p-brane (for odd $(p+1)$) moves with the speed of light in a time-evolution along the zero-eigenvalue vector-field V^a. Therefore, we will use the acronym *WILL*- (Weyl-Invariant Light-Like)-brane model.

Remark. We will use a natural ansatz for the world-volume electric field $F_{0i} = 0$ implying that $(V^a) = (1, \underline{0})$, *i.e.*, $\partial_V = \partial_0 \equiv \partial_\tau$.

Electrically Charged WILL-Membrane, Coupling to Rank 3 Gauge Potential

We can extend the *WILL*-brane model via couplings to external space-time electromagnetic field $\mathcal{A}_\mu$ and, furthermore, to external space-time rank 3 gauge potential $\mathcal{A}_{\mu\nu\lambda}$ keeping *manifest Weyl-invariance*:

$$S = -\int d^3\sigma\, \Phi(\varphi)[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \sqrt{F_{ab}F_{cd}\gamma^{ac}\gamma^{bd}}]$$

$$- q\int d^3\sigma\, \varepsilon^{abc}\mathcal{A}_\mu \partial_a X^\mu F_{bc} - \frac{\beta}{3!}\int d^3\sigma\, \varepsilon^{abc}\partial_a X^\mu \partial_b X^\nu \partial_c X^\lambda \mathcal{A}_{\mu\nu\lambda}$$

Physical significance of $\mathcal{A}_{\mu\nu\lambda}$: in $D=4$ when coupled to gravity its field-strength

$$\mathcal{F}_{\kappa\lambda\mu\nu} = 4\partial_{[\kappa}\mathcal{A}_{\lambda\mu\nu]} = \Lambda\sqrt{-G}\varepsilon_{\kappa\lambda\mu\nu}$$

produces dynamical (positive) cosmological constant $K = 4\pi G_N \Lambda^2$.

WILL-Membrane in Spherically-Symmetric Background

General form of spherically-symmetric gravitational background:

$$(ds)^2 = -A(r,t)(dt)^2 + B(r,t)(dr)^2 + C(r,t)[(d\theta)^2 + \sin^2(\theta)\,(d\phi)^2]$$

Schwarzschild: $A(r) = B^{-1}(r) = 1 - \frac{2GM}{r}$.

Reissner-Nordström: $A(r) = B^{-1}(r) = 1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}$.

(Anti) de Sitter: $A(r) = B^{-1}(r) = 1 - Kr^2$.

Schwarzschild-(anti)-de-Sitter: $A(r) = B^{-1}(r) = 1 - Kr^2 - \frac{2GM}{r}$.

Reissner-Nordström-(anti)-de-Sitter:
$A(r) = B^{-1}(r) = 1 - Kr^2 - \frac{2GM}{r} + \frac{GQ^2}{r^2}$.

Ansatz: $X^0 \equiv t = \tau \quad , \quad X^1 \equiv r = r(\tau,\sigma^1,\sigma^2)$

$$X^2 \equiv \theta = \theta(\sigma^1,\sigma^2) \quad , \quad X^3 \equiv \phi = \phi(\sigma^1,\sigma^2)$$

Substituting the above into the *WILL*-brane eqs. one gets:
(i) Equations for $r(\tau,\sigma^1,\sigma^2)$ from the lightlike and Virasoro-type constraints:

$$\frac{\partial r}{\partial \tau} = \pm\sqrt{\frac{A}{B}} \quad , \quad \frac{\partial r}{\partial \sigma^i} = 0$$

(ii) A restriction on the gravitational background itself (comes from $\partial_0 \left(\partial_i X^\mu G_{\mu\nu} \partial_j X^\nu\right) = 0$, which is a consequence of the constraints and X^μ eqs. of motion) :

$$\frac{dC}{d\tau} \equiv \left(\frac{\partial C}{\partial t} \pm \sqrt{\frac{A}{B}}\frac{\partial C}{\partial r}\right)\Big|_{t=\tau,\, r=r(\tau)} = 0$$

The above eq. tells us that the (squared) sphere radius $R^2 \equiv C(r,t)$ must remain constant along the *WILL*-brane trajectory. For static backgrounds $R^2 \equiv C(r)$ we have:

$$r(\tau) = r_0 \;\; (= \text{const}) \;\; , \qquad A(r_0) = 0$$

i.e., the *WILL*-brane automatically positions itself on the event horizon.

WILL-Branes and "Membrane Paradigm" in Black Hole Physics

Consider the coupled Einstein-Maxwell-*WILL*-brane system:

$$S = \int d^4x \sqrt{-G} \left[\frac{R(G)}{16\pi G_N} - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{1}{4!2}\mathcal{F}_{\kappa\lambda\mu\nu}\mathcal{F}^{\kappa\lambda\mu\nu} \right] + S_{\mathrm{WILL-brane}}$$

where $\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$, $\mathcal{F}_{\kappa\lambda\mu\nu} = 4\partial_{[\kappa}\mathcal{A}_{\lambda\mu\nu]}$, and:

$$S_{\mathrm{WILL-brane}} = -\int d^3\sigma\, \Phi(\varphi) \Big[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \sqrt{F_{ab}F_{cd}\gamma^{ac}\gamma^{bd}}\Big]$$
$$-q \int d^3\sigma\, \varepsilon^{abc}\mathcal{A}_\mu \partial_a X^\mu F_{bc} - \frac{\beta}{3!}\int d^3\sigma\, \varepsilon^{abc}\partial_a X^\mu \partial_b X^\nu \partial_c X^\lambda \mathcal{A}_{\mu\nu\lambda}$$

Eqs. of motion for the *WILL*-membrane subsystem are the same as above, the rest being:

$$R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R = 8\pi G_N \left(T^{(EM)}_{\mu\nu} + T^{(rank-3)}_{\mu\nu} + T^{(brane)}_{\mu\nu} \right)$$

$$\partial_\nu \left(\sqrt{-G} G^{\mu\kappa} G^{\nu\lambda} \mathcal{F}_{\kappa\lambda} \right) + j^\mu = 0$$

where:

$$T^{(EM)}_{\mu\nu} = \mathcal{F}_{\mu\kappa}\mathcal{F}_{\nu\lambda}G^{\kappa\lambda} - G_{\mu\nu}\frac{1}{4}\mathcal{F}_{\rho\kappa}\mathcal{F}_{\sigma\lambda}G^{\rho\sigma}G^{\kappa\lambda}$$

$$T^{(rank-3)}_{\mu\nu} = \frac{1}{3!}\left[\mathcal{F}_{\mu\kappa\lambda\rho}\mathcal{F}_{\nu}{}^{\kappa\lambda\rho} - \frac{1}{8}G_{\mu\nu}\mathcal{F}_{\kappa\lambda\rho\sigma}\mathcal{F}^{\kappa\lambda\rho\sigma}\right] = -\frac{1}{2}\Lambda^2 G_{\mu\nu}$$

$$T^{(brane)}_{\mu\nu} = -G_{\mu\kappa}G_{\nu\lambda}\int d^3\sigma \, \frac{\delta^{(4)}(x - X(\sigma))}{\sqrt{-G}} \, \chi \, \sqrt{-\gamma}\gamma^{ab}\partial_a X^\kappa \partial_b X^\lambda$$

$$j^\mu \equiv q \int d^3\sigma \, \delta^{(4)}(x - X(\sigma))\varepsilon^{abc}F_{bc}\partial_a X^\mu$$

Spherically Symmetric Static Solution

Space-time consisting of two regions separated by the *WILL*-brane as a common horizon:

$$(ds)^2 = -A_{(\mp)}(r)(dt)^2 + \frac{1}{A_{(\mp)}(r)}(dr)^2 + r^2[(d\theta)^2 + \sin^2(\theta)\,(d\phi)^2]$$

(a) Schwarzschild-de-Sitter space-time inside horizon [(-)]:

$$A(r) \equiv A_{(-)}(r) = 1 - K_{(-)}r^2 - \frac{2GM_{(-)}}{r} \;, \quad \text{for } \; r < r_0 \equiv r_{\text{horizon}}$$

(b) Reissner-Norström-de-Sitter space-time outside horizon [(+)]:

$$A(r) \equiv A_{(+)}(r) = 1 - K_{(+)}r^2 - \frac{2GM_{(+)}}{r} + \frac{GQ^2}{r^2} \;, \quad \text{for } \; r > r_0$$

where $Q^2 = 8\pi q^2 r_0^4$;

Coulomb field outside horizon:

$$\mathcal{A}_0 = \frac{\sqrt{2}\, q\, r_0^2}{r} \quad , \quad \text{for } \; r \geq r_0$$

no electric field inside horizon:

$$\mathcal{A}_0 = \sqrt{2}\, q\, r_0 = \text{const} \quad , \quad \text{for } \; r \leq r_0$$

$$\mathcal{A}_1 = \ldots = \mathcal{A}_{D-1} = 0$$

The *WILL*-membrane locates itself automatically on (“straddles”) the common event horizon at $r = r_0$:

$$X^0 \equiv t = \tau \quad , \quad \theta = \sigma^1 \quad , \quad \phi = \sigma^2$$

$$r(\tau, \sigma^1, \sigma^2) = r_0 = \text{const} \;\; \text{where} \;\; A_{(-)}(r_0) = A_{(+)}(r_0) = 0$$

Important:

WILL-brane dynamics imposes several matching conditions for the metric components and the induced cosmological constant when crossing the *WILL*-brane hypersurface:

(i) Jump of induced cosmological constant:

$$K_{(\pm)} = 4\pi G \Lambda^2_{(\pm)} \quad \text{for } r \geq r_0 \; (r \leq r_0) \quad , \quad \Lambda_{(+)} = \Lambda_{(-)} - \beta$$

(ii) From the WILL-membrane contribution to the energy-momentum tensor on the r.h.s. of Einstein eqs. :

$$\frac{\partial}{\partial r} A_{(+)}|_{r=r_0} - \frac{\partial}{\partial r} A_{(-)}|_{r=r_0} = -16\pi G \chi$$

(χ - the brane tension);

(iii) Matching the *WILL*-brane X^0 eq. of motion from "inside" and "outside":

$$\frac{\partial}{\partial r}A_{(+)}|_{r=r_0} - \frac{\partial}{\partial r}A_{(-)}|_{r=r_0} = -\frac{r_0(2q^2+\beta^2)\partial_r A_{(-)}|_{r=r_0}}{2\chi + \beta r_0 \Lambda_{(-)}}$$

The matching conditions allow to express all physical parameters in terms of 3 free parameters (q^2, β, Λ) where:

q – *WILL*-brane surface electric charge density,

β – *WILL*-brane charge w.r.t. rank 3 space-time gauge potential $\mathcal{A}_{\lambda\mu\nu}$,

$\Lambda \equiv \Lambda_{(-)}$ – vacuum expectation value of $\mathcal{F}_{\kappa\lambda\mu\nu}$ (field-strength of $\mathcal{A}_{\lambda\mu\nu}$) generating the induced cosmological constant $K_{(-)} = 4\pi G_N \Lambda^2$ inside horizon.

Brane tension: $\chi = \frac{r_0}{4}\left(2q^2 + \beta^2 - 2\beta\Lambda\right)$

Schwarzschild mass: $M_{(-)} = \frac{r_0 S(q^2,\beta,\Lambda)}{2G_N \mathcal{R}(q^2,\beta,\Lambda)}$

R.-N. mass: $M_{(+)} = M_{(-)} + \frac{r_0}{2G_N \mathcal{R}(q^2,\beta,\Lambda)}\left(2q^2 + \frac{2}{3}\beta\Lambda - \frac{1}{3}\beta^2\right)$

R.-N. charge: $Q^2 = 8\pi G_N q^2 r_0^4$

Horizon radius: $r_0^2 = \frac{1}{4\pi G_N \mathcal{R}(q^2,\beta,\Lambda)}$, where:

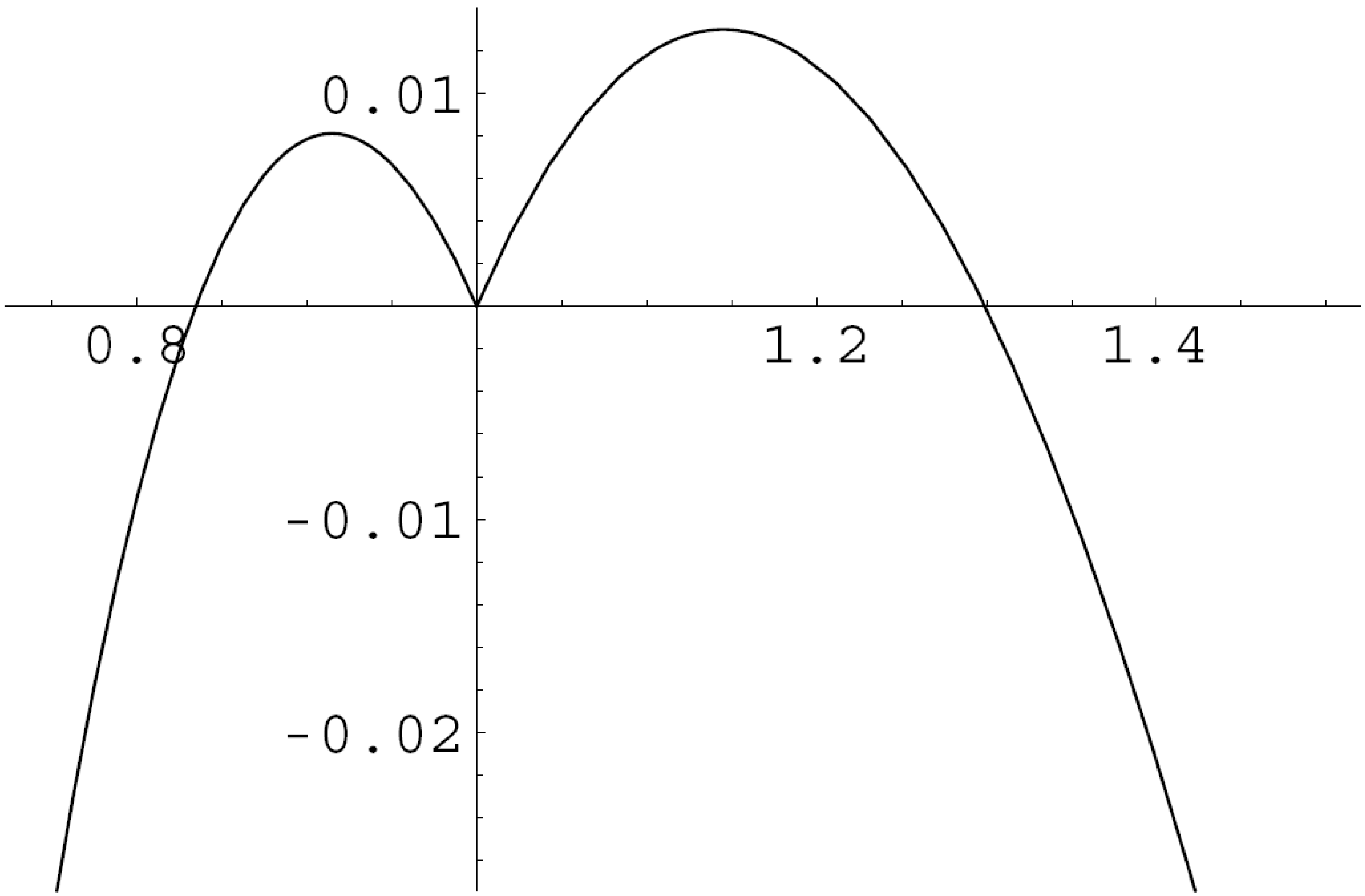
$$\mathcal{R}(q^2,\beta,\Lambda) \equiv \Lambda^2 - \beta\Lambda + q^2 + \frac{\beta^2}{2}$$
$$S(q^2,\beta,\Lambda) \equiv \frac{2}{3}\Lambda^2 - \beta\Lambda + q^2 + \frac{\beta^2}{2}$$

Common Horizon:

(a) De-Sitter type horizon from the point of view of internal Schwarzschild-de-Sitter geometry.

(b) External Reissner-Nordström horizon from the point of view of external Reissner-Nordström-de-Sitter geometry. In particular, it reduces to usual Schwarzschild horizon for $q = 0$ and $\Lambda_{(-)} = \beta$ (external geometry becomes pure Schwarzschild).

Main Result. The Einstein-Maxwell-*WILL*-brane system is the first explicit dynamical realization of the "membrane paradigm" in black hole physics.



0.01
-0.01
-0.02
0.8
1.2
1.4

Trapping Potential Well Over Common Horizon.

Consider planar motion of a (charged) test patricle with mass m and electric charge q_0 in the above background: internal Schwarzschild-de-Sitter matched with external Reissner-Norström-de-Sitter along a common event horizon materialized by Weyl-conformal invariant light-like membrane, which simultaneously is the material and charge source for gravity and electromagnetism. Conservation of energy (E) and orbital momentum (J) yields (prime indicates proper-time derivative):

$$\frac{E^2}{m^2} = r'^2 + V^2_{eff}(r)$$

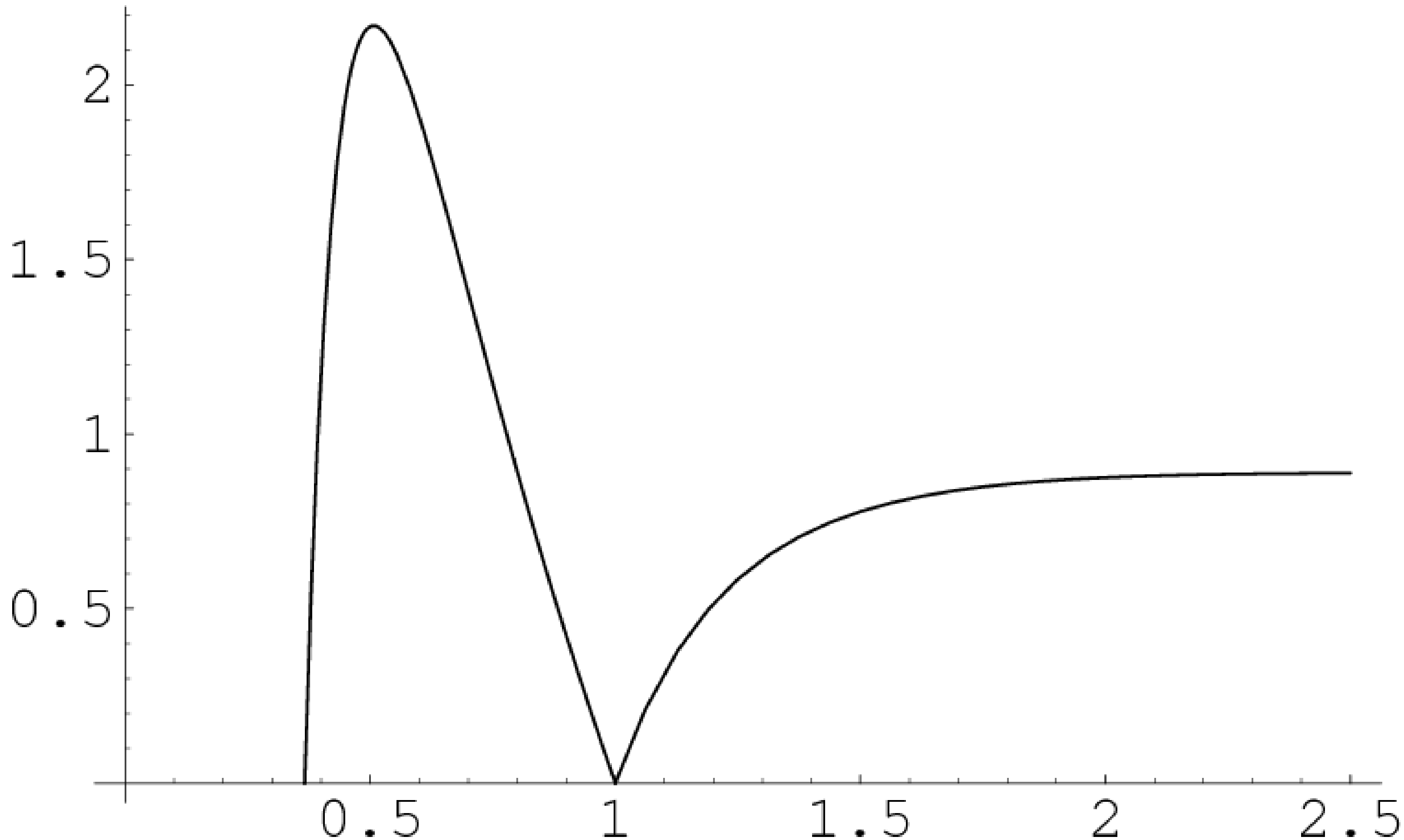
$$V^2_{eff}(r) = A_{(-)}(r)\left(1 + \frac{J^2}{m^2 r^2}\right) + \frac{2Eq_0}{m^2}\sqrt{2}qr_0 - \frac{q_0^2}{m^2}2q^2r_0^2 \qquad (r \leq r_0)$$

$$V^2_{eff}(r) = A_{(+)}(r)\left(1 + \frac{J^2}{m^2 r^2}\right) + \frac{2Eq_0}{m^2}\frac{\sqrt{2}qr_0^2}{r} - \frac{q_0^2}{m^2}\frac{2q^2r_0^4}{r^2} \qquad (r \geq r_0)$$

Analysis shows that in the parameter interval: $\Lambda_{(-)} \in \left(\frac{q^2+\frac{\beta^2}{2}}{\beta}, \infty\right)$ the effective (squared) potential $V^2_{eff}(r)$ acquires a potential "well" in the vicinity of the *WILL*-brane (the common horizon) with a minimum on the brane itself.

The simplest case is matching of interior Schwarzschild-de-Sitter (with dynamically generated cosmological constant) against pure exterior Schwarzschild (with *no* cosmological constant) along the *WILL*-brane (the common horizon), *i.e.* $\Lambda_{(-)} = \beta$, $q = 0$ and β − arbitrary).

Thus, if a test particle moving towards the common event horizon loses energy (*e.g.*, by radiation), it may fall and be trapped by the potential well, so that it never falls into the black hole.



2
1.5
1
0.5
0.5
1
1.5
2
2.5

Conclusions

Modifying of world-sheet (world-volume) integration measure – significantly affects string and p-brane dynamics.

- Acceptable dynamics in the new class of string/brane models *naturally* requires the introduction of auxiliary world-sheet/world-volume gauge fields.

- By employing square-root Yang-Mills actions for the auxiliary world-sheet/world-volume gauge fields one achieves *Weyl conformal symmetry* in the new class of p-brane theories *for any* p.

- String/brane tension - *not* a constant scale given *ad hoc*, but rather an *additional dynamical degree of freedom* beyond the ordinary string/brane degrees of freedom.

- Intrinsically *light-like* p-branes for any even p (*WILL*-branes). **NO quantum conformal anomalies** ($(p+1) =$ odd).

- When put in a gravitational black hole background, the *WILL*-membrane automatically positions itself on ("materializes") the event horizon.

- Coupled Einstein-Maxwell-*WILL*-membrane system possesses a self-consistent solution where the *WILL*-membrane serves as a source for gravity and electromagnetism. Moreover, it automatically "straddles" the common event horizon for a Schwarzschild-de-Sitter space-time (in the interior) and Reissner-Nordström-de-Sitter space-time (in the exterior). This is the first explicit dynamical realization of the "membrane paradigm" in black hole physics.

- Trapping potential "well" on common horizon "guarding" the real Scwarzschild-type horizon.