Fixing Moduli in Exact Type IIA Flux Vacua

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Outline

Calabi-Yau with fluxes approximation:
take fluxes small compared to the curvature

- Approximation in Einstein equations:
  in the large volume limit
  neglect the fluxes and their backreaction on geometry
  $\Downarrow$
  Calabi-Yau manifold
- Nothing more than supergravity limit: $L^2 \gg \alpha'$
- Villadoro,Zwirner,'05
  DeWolfe,Gyravets,Kachru,Taylor,'05: IIA Flux Vacua can have ALL moduli stabilized classically.
- Problem: even the minimum amount of flux (Dirac quantization) can change the topology!

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- Moduli fixing usually studied from a 4d point of view (effective potential V)
- Good:
  - stability issues studied more easily
  - non SUSY vacua can be studied
  - control on *exotic* vacua, e.g. non geometric vacua
- Bad:
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  - could not be sure about uplift to 10d

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The $T^6/(\mathbb{Z}_3)^2$ Orientifold Model

Type IIA SUGRA on T^6, orbifolded by $(\mathbb{Z}_3)^2$
and orientifolded by O6-planes:

(DeWolfe,Giryavets,Kachru,Taylor,'05)

- It was analysed in the large volume limit and classically.
- Superpotential:

$$W = p\,S + e_6 + e_a\,T^a + \kappa_{abc} b_a\,T^b\,T^c + \kappa_{abc}\,T^a\,T^b\,T^c$$

(Grimm,Louis,'04)

- ALL the moduli are fixed (NO massless fields)

Our main result: a full 10d description (which reproduces their results)

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Full 10d description

Strategy

We are looking for a full 10d description of these vacua.

Strategy:

Lust-Tsimpis solution ('04)

- the most general solution on AdS_4
  with internal SU(3)-structure manifold $\Rightarrow$ $\mathcal{N} = 1$ SUSY vacua
- all the fluxes compatible with 4d Poincaré switched on
- "mass parameter" $F_0 \equiv m$ (massive IIA SUGRA)

Add O6-planes

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- $SU(3)$ structure is described by Kähler 2-form J and holomorphic 3-form Ω, like on Calabi-Yau. Torsions:

$$dJ = -\frac{3}{2}\mathrm{Im}(\mathcal{W}_1^{1\oplus 1}\Omega^*) + \mathcal{W}_4^{3\oplus\bar{3}} \wedge J + \mathcal{W}_3^{6\oplus\bar{6}}$$
$$d\Omega = \mathcal{W}_1^{1\oplus 1} J \wedge J + \mathcal{W}_2^{8\oplus 8} \wedge J + \mathcal{W}_5^{3\oplus\bar{3}\,*} \wedge \Omega$$

- The only non-vanishing torsion classes are $\mathcal{W}_1^-$ and $\mathcal{W}_2^-$
- Special case of *half-flat* manifold.

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Type IIA Supergravity with O6-planes

SUGRA solution (scheme)

$$\phi\,,\,\Delta\,,\,f\;=\text{constant}$$

F_2, H_3, G_4 in terms of J, Ω, $\tilde{F}_2$.

$\tilde{F}_2$ is only constrained:

- to be in **8** of $SU(3)$
- $d\tilde{F}_2 \sim \mathbb{R}\mathrm{e}\,\Omega + \mu_6\,\delta_3$

Special case of half-flat:

$$\mathcal{W}_1^- \sim f \qquad \mathcal{W}_2^- \sim \tilde{F}_2$$

Derived conditions:

$$e^{7\phi/4} \sim \frac{\mu_6}{2\mathrm{Vol}_6(X_6)^{1/2}}$$

$$f^2 \gtrsim m^2$$

Notice:

$$m > 0 \quad \Rightarrow \quad f > 0$$

$$\Rightarrow \quad \mathcal{W}_1^- \neq 0$$

No Calabi-Yau solutions

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Smearing of O6-planes: an Exact Solution

In no case with non-vanishing mass and localized souces, is the geometry Calabi-Yau.

$\Rightarrow$ To construct explicit examples is *hard*.

Smearing of O6-planes

Much like the smearing of D-branes:

- the charge is homogeneously distributed: $\delta_3 \quad \rightarrow \quad \mathbb{R}\mathrm{e}\,\Omega$
- the orientifold projection is kept untouched.

We get:

$$m^2 > 0 \qquad f = 0 \qquad \tilde{F}_2 = F_2 = 0$$

Calabi-Yau geometry! with backreacting fluxes

In a long wavelength approximation, it describes the same physics as the localized solution.

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Is there a continuous family of vacua or a discrete set?

1. A VEV for a field-strength can be split in two pieces:
$$H_3 = H_3^{\text{flux}} + dB_2$$
2. Solution: the total H_3 is harmonic.
3. Choose H_3^{flux} harmonic (as rep. of cohomology class) and use gauge freedom to put B_2 harmonic.

There cannot be any (*physical*) non harmonic component in B_2

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they have vanishing VEV and are massive!

4. Expand H_3^{flux}, B_2 in harmonic basis and substitute in the solution.

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Moduli Stabilization

General Strategy (example of the Smeared Calabi-Yau Solution)

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This machinery applies to general Calabi-Yau solutions with general (backreacting) fluxes and smeared O6-planes.

1. Enough equations to fix (almost) everything:
   - Kähler and complex structure moduli
   - dilaton
   - all axions from B_2 and one axion from C_3

2. Some C_3 axions yet not fixed: not a problem.
   Axions take values on a compact S^1, so any other correction can lift them.

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Summary and Future Directions

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1. Solutions of IIA Supergravity with O6-planes can realize complete moduli stabilization.
2. Look for solutions of IIA with backreacting fluxes and O6's, (so going beyond the Calabi-Yau with fluxes approximation); but giving explicit examples is *hard*.
3. Smearing procedure on O6-charge: there are Calabi-Yau solutions $\rightarrow$ explicit examples.
4. Complete stabilization of moduli is confirmed.

Future directions

- Study compactifications on $SU(3)$-structure manifolds with localized sources.
- Localized solutions: describe $D2 - O6$ in massive F_0 background. Relates 2+1 gauge theories and uplift of massive IIA to M-theory.

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Topology Changings: an example

Even if the contribution of fluxes can be locally neglected
in the Equations of Motion,

the minumum amount allowed by Dirac quantization
can still produce global modifications.

Smeared vs Localized solution

Smeared source δ_3

- Calabi-Yau: $dJ = 0$
- $\delta_3 \sim \mathbb{R}\mathrm{e}\,\Omega$

$$\Downarrow$$

- δ_3 non-trivial in cohomology

Localized source δ_3

- Half-flat: $dJ \sim \mathbb{R}\mathrm{e}\,\Omega$
- $d\tilde{F}_2 \sim \mathbb{R}\mathrm{e}\,\Omega + \delta_3$

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- δ_3 is trivial in cohomology