

String loop corrected hypermultiplet moduli spaces

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D. Robles-Llana, F. S., S. Vandoren, JHEP 03 (2006) 081, hep-th/0602164

B. de Wit, F. S., JHEP 09 (2006) 062, hep-th/0606148

RTN Midterm Meeting, Napoli, October 11th, 2006

Introduction

- type II string theory on $M_4 \times CY_3$:
 - \implies low energy physics: $d = 4, N = 2$ supergravity
 - \implies usually: tree-level approximation for effective action (LEEA)

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supergravity

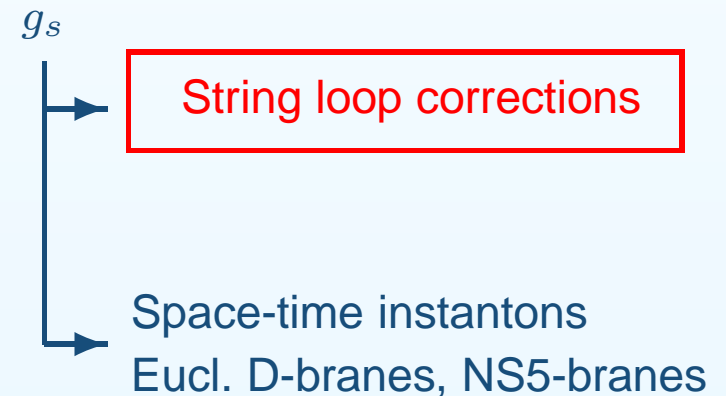
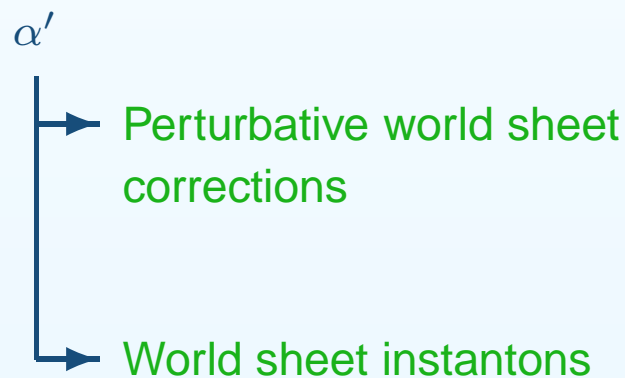
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 - \implies low energy physics: $d = 4, N = 2$ supergravity
 - \implies usually: tree-level approximation for effective action (LEEA)
- What makes the LEEA stringy?
 - two expansion parameters: α' and g_s



Why should string loop effects be important?

- probe new regions of string theory moduli space
 - string dualities
 - metastable de Sitter vacua (KKLT)
- perturbative corrections can change the vacuum structure of the LEEA
 - Example: Flux potentials in type IIB orientifold compactifications
 - classically: no scale structure
(Flux potentials independent of volume)
 - α', g_s -corrections: no-scale structure is broken

$N = 2$ (super-)multiplets in $d = 4$

multiplet	bosonic degrees of freedom	off-shell realization
supergravity multiplet	e_μ^a, A_μ^0	yes (Weyl multiplet)
vector multiplet (VM)	z, \bar{z}, A_μ	yes
hypermultiplet (HM)	q^1, q^2, q^3, q^4	no
tensor multiplet (TM)	$v, \bar{v}, x, E_{\mu\nu}$	yes

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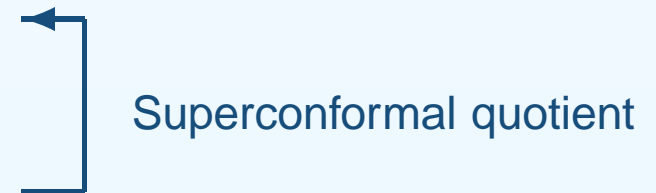
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Remarks:

- scalar-tensor duality:
 - can always dualize TM \implies HM
 - dualization HM \implies TM requires shift symmetry in one HM scalar q^*

• on-shell: Poincaré supergravity:
 gravity + n_V VM + n_H HM

off-shell: superconformal gravity:
 Weyl + $n_V + 1$ VM + $n_H + 1$ TM



Review of $N = 2, d = 4$ effective actions

1. coupling constants factorize in a VM and HM sector
 - α' corrections: factor containing CY_3 volume \mathcal{V}
 - g_s corrections: factor containing dilaton ϕ ($e^{-\phi_\infty/2} = g_s$)

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1. + 3. \implies String loop corrections can be captured by superconformal TM

Conformal supergravity coupled to tensor multiplets

locally superconformal TM Lagrangian (bosonic part):

$$\begin{aligned} e^{-1} \mathcal{L}_{\text{TM}} = & \chi_T \left[\frac{1}{6} R + \frac{1}{2} D \right] \\ & - \chi_T^{-1} \left[\frac{1}{4} (\partial_\mu \chi_T)^2 + \chi_T^2 F_{IJ} \mathcal{D}_\mu \vec{L}^I \cdot \mathcal{D}^\mu \vec{L}^J - F_{IJ} (E_\mu^I E^{\mu J} + G^I \bar{G}^J) \right] \\ & + 2 F_{IJ} E^{\mu I} \vec{L}^J \vec{\mathcal{V}}_\mu - \frac{i}{2} e^{-1} \varepsilon^{\mu\nu\rho\sigma} \vec{F}_{IJK} \cdot (\partial_\rho \vec{L}^I \times \partial_\sigma \vec{L}^J) E_{\mu\nu}^K \end{aligned}$$

where $\vec{L}^I = \frac{1}{2\chi_T} [i(v^I - \bar{v}^I), v^I + \bar{v}^I, x^I]$

- completely fixed by the tensor potential χ_T

$$\chi_T = F_{IJ} (x^I x^J + 4v^I \bar{v}^J)$$

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- superconformal quotient (= going to Poincaré supergravity):
 - couple \mathcal{L}_{TM} to compensating VM
 - fix local $SU(2)_R$, dilatational symmetries
 - integrate out superconformal gauge fields $\vec{\mathcal{V}}_\mu, D$
 - dualize tensor to scalar fields

The tensor potential χ_T

Tensor multiplet scalars as $SU(2)$ triplet: $L_{ij} : L_{11} = \bar{v}, L_{22} = v, L_{12} = -\frac{i}{2}x$

Constraints on χ_T :

- local $SU(2)$, dilatations, and superconformal invariance

$$L_{ki}{}^I \frac{\partial \chi_T}{\partial L_{kj}{}^I} = \frac{1}{2} \delta^j{}_i \chi_T, \quad \varepsilon_{kl} \frac{\partial^2 \chi_T}{\partial L_{ik}{}^I \partial L_{jl}{}^J} = 2 F_{IJ} \varepsilon^{ij}$$

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Solving the constraints:

- Contour integral formulation

$$\mathcal{L}(v, \bar{v}, x) = -\text{Im} \left[\oint_{\mathcal{C}} \frac{d\zeta}{2\pi i \zeta} H(\eta^I) \right], \quad \eta = \frac{v}{\zeta} + x - \zeta \bar{v}$$

- $\eta =$ lowest component of $N = 2$ tensor superfield
- contour integral = projective superspace Lagrangian
- relation to χ_T :

$$\chi_T = -\mathcal{L} + x^I \frac{\partial \mathcal{L}}{\partial x^I}$$

Superconformal description of the classical c-map

(M. Roček, C. Vafa, S. Vandoren, JHEP 0602 (2006) 062)

- Contour integral formulation

$$H^{\text{class}}(\eta) = \frac{F(\eta^\Lambda)}{\eta^0}$$

- $F(\eta^\Lambda)$ prepotential of dual VM geometry
 - η^0 additional compensating TM
 - contour \mathcal{C} around one zero of $\eta^0(\zeta)$
- corresponding tensor potential:

$$\chi_T = 4 \sqrt{(x^0)^2 + 4v^0 \bar{v}^0} e^{-2\phi_{10}} K(z, \bar{z})$$

- superconformal quotient \implies HM Lagrangians from the classical c-map

One-loop corrected hypermultiplet moduli space

- Are there deformations of H^{class} compatible with string perturbation theory?

$$H^{1\text{-loop}}(\eta) = H^{\text{class}} + 4i c \eta^0 \ln(\eta^0)$$

- contour \mathcal{C} around one zero of $\eta^0(\zeta)$
- equals one-loop correction to the UHM (Anguelova, Roček, Vandoren)
- corresponding tensor potential:

$$\chi_T^{1\text{-loop}} = 4 \sqrt{(x^0)^2 + 4v^0 \bar{v}^0} [e^{-2\phi_{10}} K(z, \bar{z}) + c]$$

- do superconformal quotient + compare to string loop computation
(Antoniadis, Ferrara, Minasian, Narain; Antoniadis, Minasian, Theisen, Vanhove)

$$c^{\text{IIA}} = -\frac{\chi}{12\pi}, \quad c^{\text{IIB}} = \frac{\chi}{12\pi}$$

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$$c^{\text{IIA}} = -\frac{\chi}{12\pi}, \quad c^{\text{IIB}} = \frac{\chi}{12\pi}$$

$\chi_T^{1\text{-loop}}$ encodes full one-loop corrections to the HM sector
arising from type II strings compactified on CY_3

One-loop exactness of the hypermultiplet moduli space

Conjecture:

- Higher string loop contributions are absorbed by coordinate transformations

Evidence:

- Evaluate contour integral in gauge $v^0 = 0, v^1 = 1$:
 \implies string loops are counted by

$$\eta^0 = x^0 \propto e^{-\phi_{10}},$$

- higher order contour integral contributions vanish:

$$\Delta\mathcal{L}(v, \bar{v}, x) = -\text{Im} \left[(x^0)^{n-1} \oint_{\mathcal{C}_0} \frac{d\zeta}{2\pi i} \zeta^{n-3} F_n(\zeta \eta^\Lambda) \right] = 0, \quad n > 3$$

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Good evidence that the perturbative HM sector is one-loop exact

$\chi_T^{1\text{-loop}}$ contains all perturbative corrections