String loop corrected hypermultiplet moduli spaces

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Introduction

- type II string theory on $\mathrm{M}_4 \times \mathrm{CY}_3$:

  $\Longrightarrow$ low energy physics: $d = 4, N = 2$ supergravity

  $\Longrightarrow$ usually: tree-level approximation for effective action (LEEA)

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supergravity

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- What makes the LEEA stringy?
  - two expansion parameters: α' and g_s

α'

- Perturbative world sheet corrections
- World sheet instantons

g_s

- String loop corrections
- Space-time instantons
  Eucl. D-branes, NS5-branes

Why should string loop effects be important?

- probe new regions of string theory moduli space
  - string dualities
  - metastable de Sitter vacua (KKLT)

- perturbative corrections can change the vacuum structure of the LEEA
  - Example: Flux potentials in type IIB orientifold compactifications
    - classically: no scale structure
      (Flux potentials independent of volume)
    - α', g_s-corrections: no-scale structure is broken

$N = 2$ (super-)multiplets in $d = 4$

multiplet	bosonic degrees of freedom	off-shell realization
supergravity multiplet	$e_\mu{}^a, A_\mu^0$	yes (Weyl multiplet)
vector multiplet (VM)	$z, \bar{z}, A_\mu$	yes
hypermultiplet (HM)	q^1, q^2, q^3, q^4	no
tensor multiplet (TM)	$v, \bar{v}, x, E_{\mu\nu}$	yes

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Remarks:

- scalar-tensor duality:
  - can always dualize TM $\Longrightarrow$ HM
  - dualization HM $\Longrightarrow$ TM requires shift symmety in one HM scalar q^*
- on-shell: Poincaré supergravity:

  gravity + n_V VM + n_H HM

  off-shell: superconformal gravity:

  Weyl + $n_V + 1$ VM + $n_H + 1$ TM

  Superconformal quotient (off-shell → on-shell)

Review of $N = 2, d = 4$ effective actions

1. coupling constants factorize in a VM and HM sector
   - α' corrections: factor containing CY_3 volume $\mathcal{V}$
   - g_s corrections: factor containing dilaton ϕ ($e^{-\phi_\infty/2} = g_s$)

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$$\text{VM IIA/B} \Longleftrightarrow \text{HM IIB/A}$$

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1. + 3. $\Longrightarrow$ String loop corrections can be captured by superconformal TM

Conformal supergravity coupled to tensor multiplets

locally superconformal TM Lagrangian (bosonic part):

$$
\begin{aligned}
e^{-1}\mathcal{L}_{\rm TM} \;=\; & \chi_T\big[\tfrac{1}{6}R+\tfrac{1}{2}D\big] \\
& -\chi_T^{-1}\big[\tfrac{1}{4}(\partial_\mu\chi_T)^2+\chi_T^2\,F_{IJ}\,\mathcal{D}_\mu\vec{L}^I\cdot\mathcal{D}^\mu\vec{L}^J-F_{IJ}\big(E_\mu{}^I\,E^{\mu J}+G^I\bar{G}^J\big)\big] \\
& +2\,F_{IJ}\,E^{\mu I}\,\vec{L}^J\,\vec{\mathcal{V}}_\mu-\tfrac{\rm i}{2}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\,\vec{F}_{IJK}\cdot(\partial_\rho\vec{L}^I\times\partial_\sigma\vec{L}^J)\,E_{\mu\nu}{}^K
\end{aligned}
$$

where $\vec{L}^I=\frac{1}{2\chi_T}\,\big[{\rm i}(v^I-\bar{v}^I),v^I+\bar{v}^I,x^I\big]$

- completely fixed by the tensor potential χ_T

$$\chi_T=F_{IJ}\big(x^Ix^J+4v^I\bar{v}^J\big)$$

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- superconformal quotient (= going to Poincaré supergravity):
  - couple $\mathcal{L}_{\rm TM}$ to compensating VM
  - fix local $SU(2)_R$, dilatational symmetries
  - integrate out superconformal gauge fields $\vec{\mathcal{V}}_\mu, D$
  - dualize tensor to scalar fields

The tensor potential χ_T

Tensor multiplet scalars as $SU(2)$ triplet: $L_{ij}\,:\;\; L_{11}=\bar{v}\,,\;L_{22}=v\,,\;L_{12}=-\tfrac{\mathrm{i}}{2}x$

Constraints on χ_T:

- local $SU(2)$, dilatations, and superconformal invariance

$$L_{ki}{}^{I}\,\frac{\partial\chi_T}{\partial L_{kj}{}^{I}}=\tfrac{1}{2}\delta^{j}{}_{i}\,\chi_T\;,\qquad \varepsilon_{kl}\,\frac{\partial^2\chi_T}{\partial L_{ik}{}^{I}\,\partial L_{jl}{}^{J}}=2\,F_{IJ}\;\varepsilon^{ij}$$

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Solving the constraints:

- Contour integral formulation

$$\mathcal{L}(v,\bar{v},x)=-\mathrm{Im}\left[\oint_{\mathcal{C}}\frac{\mathrm{d}\zeta}{2\pi\mathrm{i}\zeta}\,H(\eta^I)\right]\,,\qquad \eta=\frac{v}{\zeta}+x-\zeta\bar{v}$$

  - η = lowest component of $N=2$ tensor superfield
  - contour integral = projective superspace Lagrangian

- relation to χ_T:

$$\chi_T=-\mathcal{L}+x^I\,\frac{\partial\mathcal{L}}{\partial x^I}$$

Superconformal description of the classical c-map

(M. Roček, C. Vafa, S. Vandoren, JHEP **0602** (2006) 062)

- Contour integral formulation

$$H^{\text{class}}(\eta) = \frac{F(\eta^\Lambda)}{\eta^0}$$

  - $F(\eta^\Lambda)$ prepotential of dual VM geometry
  - η^0 additional compensating TM
  - contour $\mathcal{C}$ around one zero of $\eta^0(\zeta)$

- corresponding tensor potential:

$$\chi_T = 4\sqrt{(x^0)^2 + 4v^0\bar{v}^0}\,\mathrm{e}^{-2\phi_{10}}\,K(z,\bar{z})$$

- superconformal quotient $\Longrightarrow$ HM Lagrangians from the classical c-map

One-loop corrected hypermultiplet moduli space

- Are there deformations of H^{class} compatible with string perturbation theory?

$$H^{1-\mathrm{loop}}(\eta) = H^{\mathrm{class}} + 4\,\mathrm{i}\,c\,\eta^0\,\ln(\eta^0)$$

  - contour $\mathcal{C}$ around one zero of $\eta^0(\zeta)$
  - equals one-loop correction to the UHM (Anguelova, Roček, Vandoren)

- corresponding tensor potential:

$$\chi_T^{1-\mathrm{loop}} = 4\sqrt{(x^0)^2 + 4v^0\bar{v}^0}\,\left[\mathrm{e}^{-2\phi_{10}}\,K(z,\bar{z}) + c\right]$$

- do superconformal quotient + compare to string loop computation (Antoniadis, Ferrara, Minasian, Narain; Antoniadis, Minasian, Theisen, Vanhove)

$$c^{\mathrm{IIA}} = -\frac{\chi}{12\pi}\,, \qquad c^{\mathrm{IIB}} = \frac{\chi}{12\pi}$$

One-loop corrected hypermultiplet moduli space

- Are there deformations of $H^{\rm class}$ compatible with string perturbation theory?

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$$c^{\rm IIA} = -\frac{\chi}{12\pi}\,, \qquad c^{\rm IIB} = \frac{\chi}{12\pi}$$

> $\chi_T^{1-\rm loop}$ encodes full one-loop corrections to the HM sector
>
> arising from type II strings compactified on CY_3

One-loop exactness of the hypermultiplet moduli space

Conjecture:

- Higher string loop contributions are absorbed by coordinate transformations

Evidence:

- Evaluate contour integral in gauge $v^0 = 0, v^1 = 1$:
  $\Longrightarrow$ string loops are counted by

$$\eta^0 = x^0 \propto \mathrm{e}^{-\phi_{10}} \,,$$

- higher order contour integral contributions vanish:

$$\Delta\mathcal{L}(v, \bar{v}, x) = -\mathrm{Im}\left[(x^0)^{n-1} \oint_{\mathcal{C}_0} \frac{\mathrm{d}\zeta}{2\pi\mathrm{i}}\, \zeta^{n-3}\, F_n(\zeta\eta^\Lambda)\right] = 0\,, \quad n > 3$$

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Good evidence that the perturbative HM sector is one-loop exact

$\chi_T^{1-\mathrm{loop}}$ contains all perturbative corrections