

The String Theory Landscape: Prospects for Predictivity

Frederik Denef

Leuven

Napels, October 10, 2006

Outline

Falsifiability

Constructability

Enumerability

Complexity

Probability

Conclusions

Falsifiability

Is string theory falsifiable?

Is string theory falsifiable?

Yes.

A caricatural example

LHC measures a tower of massive scalars with $m^2 = N m_*^2$, $m_* = 124 \text{ GeV}$, and (among further supporting evidence) degeneracies $d(N)$ which perfectly fit

N	$d(N)$
1	1
2	28
3	378
4	3276
5	20503
6	99036
7	386568
8	1265940
...	...

A caricatural example

LHC measures a tower of massive scalars with $m^2 = N m_*^2$, $m_* = 124 \text{ GeV}$, and (among further supporting evidence) degeneracies $d(N)$ which perfectly fit

N	$d(N)$
1	1
2	28
3	378
4	3276
5	20503
6	99036
7	386568
8	1265940
...	...

\Rightarrow leaves little doubt we live in flat 32-dimensional space $\mathbb{R}^{1,3} \times T^{28}$ with all radii $R = 10^{-17} m$.

A caricatural example

LHC measures a tower of massive scalars with $m^2 = N m_*^2$, $m_* = 124 \text{ GeV}$, and (among further supporting evidence) degeneracies $d(N)$ which perfectly fit

N	$d(N)$
1	1
2	28
3	378
4	3276
5	20503
6	99036
7	386568
8	1265940
...	...

\Rightarrow leaves little doubt we live in flat 32-dimensional space $\mathbb{R}^{1,3} \times T^{28}$ with all radii $R = 10^{-17} m$. \rightsquigarrow falsifies string theory

Other examples

- ▶ Scattering experiments at $E \sim O(m_s)$ showing deviations from textbook string-string scattering.

Other examples

- ▶ Scattering experiments at $E \sim O(m_s)$ showing deviations from textbook string-string scattering.
- ▶ Varying fine structure constant α or other SM parameters: sometimes purported as “natural” in string theory since parameters depend on moduli and moduli could roll.

Other examples

- ▶ Scattering experiments at $E \sim O(m_s)$ showing deviations from textbook string-string scattering.
- ▶ Varying fine structure constant α or other SM parameters: sometimes purported as “natural” in string theory since parameters depend on moduli and moduli could roll.

But: [Banks-Dine-Douglas]: in our present understanding of low energy string theory, this would require excessive fine tuning (much more than c.c.) to avoid e.g. dramatic changes in cosmological constant and other disasters.

Other examples

- ▶ Scattering experiments at $E \sim O(m_s)$ showing deviations from textbook string-string scattering.
- ▶ Varying fine structure constant α or other SM parameters: sometimes purported as “natural” in string theory since parameters depend on moduli and moduli could roll.
But: [Banks-Dine-Douglas]: in our present understanding of low energy string theory, this would require excessive fine tuning (much more than c.c.) to avoid e.g. dramatic changes in cosmological constant and other disasters.
- ▶ Violations of general principles such as CPT conservation, unitarity, and so on.

Other examples

- ▶ Scattering experiments at $E \sim O(m_s)$ showing deviations from textbook string-string scattering.
- ▶ Varying fine structure constant α or other SM parameters: sometimes purported as “natural” in string theory since parameters depend on moduli and moduli could roll.
But: [Banks-Dine-Douglas]: in our present understanding of low energy string theory, this would require excessive fine tuning (much more than c.c.) to avoid e.g. dramatic changes in cosmological constant and other disasters.
- ▶ Violations of general principles such as CPT conservation, unitarity, and so on.
- ▶ Landscape vs. swampland considerations [Vafa].

Other examples

- ▶ Scattering experiments at $E \sim O(m_s)$ showing deviations from textbook string-string scattering.
- ▶ Varying fine structure constant α or other SM parameters: sometimes purported as “natural” in string theory since parameters depend on moduli and moduli could roll.
But: [Banks-Dine-Douglas]: in our present understanding of low energy string theory, this would require excessive fine tuning (much more than c.c.) to avoid e.g. dramatic changes in cosmological constant and other disasters.
- ▶ Violations of general principles such as CPT conservation, unitarity, and so on.
- ▶ Landscape vs. swampland considerations [Vafa].
- ▶ Positive FRW slices would falsify present landscape + eternal inflation ideas.

Other examples

- ▶ Scattering experiments at $E \sim O(m_s)$ showing deviations from textbook string-string scattering.
- ▶ Varying fine structure constant α or other SM parameters: sometimes purported as “natural” in string theory since parameters depend on moduli and moduli could roll.
But: [Banks-Dine-Douglas]: in our present understanding of low energy string theory, this would require excessive fine tuning (much more than c.c.) to avoid e.g. dramatic changes in cosmological constant and other disasters.
- ▶ Violations of general principles such as CPT conservation, unitarity, and so on.
- ▶ Landscape vs. swampland considerations [Vafa].
- ▶ Positive FRW slices would falsify present landscape + eternal inflation ideas.

⇒ string theory falsifiable in principle.

Other examples

- ▶ Scattering experiments at $E \sim O(m_s)$ showing deviations from textbook string-string scattering.
- ▶ Varying fine structure constant α or other SM parameters: sometimes purported as “natural” in string theory since parameters depend on moduli and moduli could roll.
But: [Banks-Dine-Douglas]: in our present understanding of low energy string theory, this would require excessive fine tuning (much more than c.c.) to avoid e.g. dramatic changes in cosmological constant and other disasters.
- ▶ Violations of general principles such as CPT conservation, unitarity, and so on.
- ▶ Landscape vs. swampland considerations [Vafa].
- ▶ Positive FRW slices would falsify present landscape + eternal inflation ideas.

⇒ string theory falsifiable in principle.

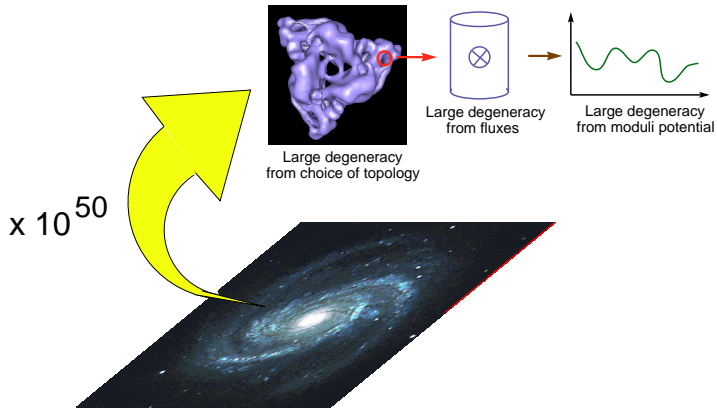
But what can we *really* hope for?

Low energy predictions from string theory in four easy steps

1. Construct/enumerate all vacua meeting rough observational constraints (4 huge dim, no massless scalars, $t_{\text{decay}} > 10 \text{ Gyr}$, ...).

Low energy predictions from string theory in four easy steps

1. Construct/enumerate all vacua meeting rough observational constraints (4 huge dim, no massless scalars, $t_{\text{decay}} > 10 \text{ Gyr}$, ...).



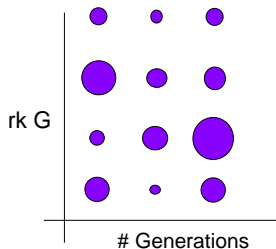
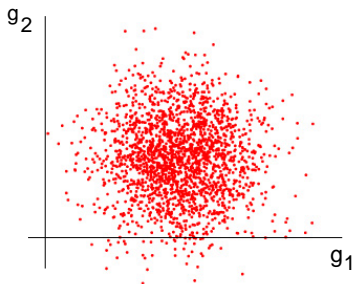
Vacua labeled by **discrete microscopic data** \vec{m} : topology, flux, critical points.

Low energy predictions from string theory in four easy steps

2. Compute low energy parameters Φ (continuous and discrete) of vacua with high accuracy.

Low energy predictions from string theory in four easy steps

2. Compute low energy parameters Φ (continuous and discrete) of vacua with high accuracy.



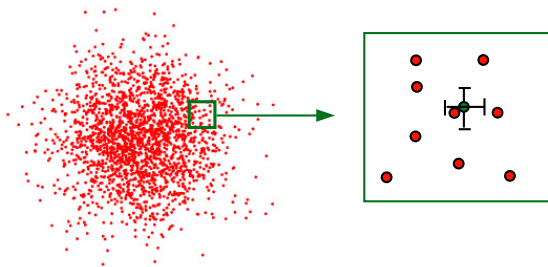
= computing map $\vec{m} \mapsto \Phi(\vec{m})$.

Low energy predictions from string theory in four easy steps

3. Find unique vacuum compatible with experiment.

Low energy predictions from string theory in four easy steps

3. Find unique vacuum compatible with experiment.



= find \vec{m} such that $\Phi(\vec{m}) = \Phi_{\text{exp}}(\vec{m})$.

Low energy predictions from string theory in four easy steps

4. Use this vacuum to predict everything we ever wanted to know.

Low energy predictions from string theory in four easy steps

4. Use this vacuum to predict everything we ever wanted to know.



$$m_* = \dots$$

$$g_* = \dots$$

$$G_* = \dots$$

= compute $\tilde{\Phi}_{\text{everything}}(\vec{m})$.

Central question

Is this a tractable problem *in principle*?

Constructability

Examples of controlled constructions which satisfy some rough observational constraints

Examples of controlled constructions which satisfy some rough observational constraints

- ▶ IIB CY_3 O3/O7 + flux + inst., of suitable topological type [Kachru-Kalosh-Linde-Trivedi, FD-Douglas-Florea-Grassi-Kachru, Lüst-Reffert-Scheidegger-Schulgin-Stieberger].

Examples of controlled constructions which satisfy some rough observational constraints

- ▶ IIB CY_3 O3/O7 + flux + inst., of suitable topological type [Kachru-Kallosch-Linde-Trivedi, FD-Douglas-Florea-Grassi-Kachru, Lüst-Reffert-Scheidegger-Schulgin-Stieberger]. **Note:** control seems to require certain simplicity in these kinds of constructions:
 1. need basis of rigid divisors \rightsquigarrow resolved orbifold models.
 2. $\log(m_{3/2}) \sim V_4 \sim C_{ab} V_2^a V_2^b \Rightarrow (V_2)_{\min} \sim (\log m_{3/2})/|C|$
small if $|C| \gg 1$

Examples of controlled constructions which satisfy some rough observational constraints

- ▶ IIB CY_3 O3/O7 + flux + inst., of suitable topological type [Kachru-Kallosch-Linde-Trivedi, FD-Douglas-Florea-Grassi-Kachru, Lüst-Reffert-Scheidegger-Schulgin-Stieberger]. **Note:** control seems to require certain simplicity in these kinds of constructions:
 1. need basis of rigid divisors \rightsquigarrow resolved orbifold models.
 2. $\log(m_{3/2}) \sim V_4 \sim C_{ab} V_2^a V_2^b \Rightarrow (V_2)_{\min} \sim (\log m_{3/2})/|C|$
small if $|C| \gg 1$
- ▶ Variant with nonsusy AdS and exponentially large volume balancing inst. with pert. corrections by “skewing” CY. [Balasubramanian-Berglund-Conlon-Quevedo-Suruliz]

Examples of controlled constructions which satisfy some rough observational constraints

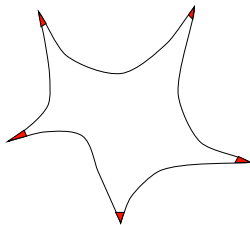
- ▶ IIB CY_3 O3/O7 + flux + inst., of suitable topological type
[Kachru-Kalosh-Linde-Trivedi, FD-Douglas-Florea-Grassi-Kachru, Lüst-Reffert-Scheidegger-Schulgin-Stieberger]. **Note:** control seems to require certain simplicity in these kinds of constructions:
 1. need basis of rigid divisors \rightsquigarrow resolved orbifold models.
 2. $\log(m_{3/2}) \sim V_4 \sim C_{ab} V_2^a V_2^b \Rightarrow (V_2)_{\min} \sim (\log m_{3/2})/|C|$
small if $|C| \gg 1$
- ▶ Variant with nonsusy AdS and exponentially large volume balancing inst. with pert. corrections by “skewing” CY.
[Balasubramanian-Berglund-Conlon-Quevedo-Suruliz]
- ▶ IIA CY_3 O6 + flux, purely classical
[DeWolfe-Giryavets-Kachru-Taylor], including metric fluxes
[Camara-Ibañez-Font,...] (but: no controlled $\Lambda > 0$ examples).

Examples of controlled constructions which satisfy some rough observational constraints

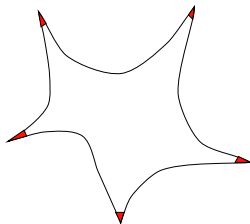
- ▶ IIB CY_3 O3/O7 + flux + inst., of suitable topological type
[Kachru-Kalosh-Linde-Trivedi, FD-Douglas-Florea-Grassi-Kachru, Lüst-Reffert-Scheidegger-Schulgin-Stieberger]. **Note:** control seems to require certain simplicity in these kinds of constructions:
 1. need basis of rigid divisors \rightsquigarrow resolved orbifold models.
 2. $\log(m_{3/2}) \sim V_4 \sim C_{ab} V_2^a V_2^b \Rightarrow (V_2)_{\min} \sim (\log m_{3/2})/|C|$
small if $|C| \gg 1$
- ▶ Variant with nonsusy AdS and exponentially large volume balancing inst. with pert. corrections by “skewing” CY.
[Balasubramanian-Berglund-Conlon-Quevedo-Suruliz]
- ▶ IIA CY_3 O6 + flux, purely classical
[DeWolfe-Giryavets-Kachru-Taylor], including metric fluxes
[Camara-Ibañez-Font,...] (but: no controlled $\Lambda > 0$ examples).
- ▶ ...

Lessons: which semi-realistic vacua can we hope to construct and control?

Lessons: which semi-realistic vacua can we hope to construct and control?

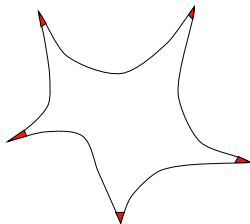


Lessons: which semi-realistic vacua can we hope to construct and control?



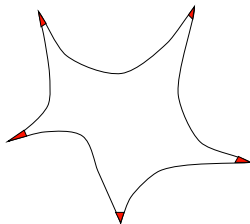
- ▶ (In some duality frame) weakly coupled, large radii.

Lessons: which semi-realistic vacua can we hope to construct and control?



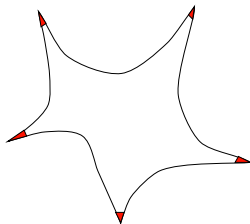
- ▶ (In some duality frame) weakly coupled, large radii.
- ▶ Small fraction of all vacua

Lessons: which semi-realistic vacua can we hope to construct and control?



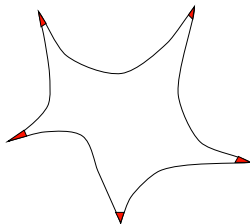
- ▶ (In some duality frame) weakly coupled, large radii.
- ▶ Small fraction of all vacua
- ▶ Physical reason to restrict to weakly coupled?

Lessons: which semi-realistic vacua can we hope to construct and control?



- ▶ (In some duality frame) weakly coupled, large radii.
- ▶ Small fraction of all vacua
- ▶ Physical reason to restrict to weakly coupled?
 \rightsquigarrow Metastability?

Lessons: which semi-realistic vacua can we hope to construct and control?



- ▶ (In some duality frame) weakly coupled, large radii.
- ▶ Small fraction of all vacua
- ▶ Physical reason to restrict to weakly coupled?

\rightsquigarrow Metastability?

$$\text{decay rate} \sim \sum e^{-c/g^2}$$

\rightsquigarrow only suff. small g can lead to suff. metastable vacua?

Enumerability

Question 0: Is there a finite number of vacua compatible with rough observational constraints?

Question 0: Is there a finite number of vacua compatible with rough observational constraints?

- Obviously infinite number without observational constraints, e.g. $[\text{AdS}_5 \times S^5]_N$.

Question 0: Is there a finite number of vacua compatible with rough observational constraints?

- ▶ Obviously infinite number without observational constraints, e.g. $[\text{AdS}_5 \times S^5]_N$.
- ▶ [Acharya-Douglas] conjecture: finite number of vacua with m_{KK} bounded below and vacuum energy bounded above.

Question 0: Is there a finite number of vacua compatible with rough observational constraints?

- ▶ Obviously infinite number without observational constraints, e.g. $[\text{AdS}_5 \times S^5]_N$.
- ▶ [Acharya-Douglas] conjecture: finite number of vacua with m_{KK} bounded below and vacuum energy bounded above.
- ▶ Evidence e.g. Cheeger's finiteness theorem:

In any sequence of Riemannian manifolds with metrics such that

- 1. the sectional curvatures are all bounded above*
- 2. the volume are bounded below*
- 3. the diameters are bounded above*

there can only be a finite number of diffeomorphism types.

Question 0: Is there a finite number of vacua compatible with rough observational constraints?

- ▶ Obviously infinite number without observational constraints, e.g. $[\text{AdS}_5 \times S^5]_N$.
- ▶ [Acharya-Douglas] conjecture: finite number of vacua with m_{KK} bounded below and vacuum energy bounded above.
- ▶ Evidence e.g. Cheeger's finiteness theorem:

In any sequence of Riemannian manifolds with metrics such that

- 1. the sectional curvatures are all bounded above*
- 2. the volume are bounded below*
- 3. the diameters are bounded above*

there can only be a finite number of diffeomorphism types.

⇒ E.g. potentially infinite number of CY manifolds does not matter to finiteness of quasi-realistic vacua.

Question 0.5: Assuming finiteness, how many quasi-realistic vacua are there?

Question 0.5: Assuming finiteness, how many quasi-realistic vacua are there?

- Popular number which got worldwide press attention is 10^{500} .

Question 0.5: Assuming finiteness, how many quasi-realistic vacua are there?

- ▶ Popular number which got worldwide press attention is 10^{500} .
- ▶ Where does this number come from?

Question 0.5: Assuming finiteness, how many quasi-realistic vacua are there?

- ▶ Popular number which got worldwide press attention is 10^{500} .
- ▶ Where does this number come from?
- ▶ Before we answer this question, let us review techniques to estimate numbers of vacua: “statistics” of string vacua

Question 0.5: Assuming finiteness, how many quasi-realistic vacua are there?

- ▶ Popular number which got worldwide press attention is 10^{500} .
- ▶ Where does this number come from?
- ▶ Before we answer this question, let us review techniques to estimate numbers of vacua: “statistics” of string vacua
- ▶ These techniques also address more refined questions, such as number distributions of observables over parameter space.

Question 0.5: Assuming finiteness, how many quasi-realistic vacua are there?

- ▶ Popular number which got worldwide press attention is 10^{500} .
- ▶ Where does this number come from?
- ▶ Before we answer this question, let us review techniques to estimate numbers of vacua: “statistics” of string vacua
- ▶ These techniques also address more refined questions, such as number distributions of observables over parameter space.
- ▶ Note: number densities in measurable parameter space matter more than total numbers.

Statistics: general idea

Statistics: general idea

Vacuum characterized by discrete (compactification) data \vec{N} and critical point of effective potential $V_N(z)$:

$$(\vec{N}, z) : V'_N(z) = 0, \quad V''_N(z) > 0$$

Statistics: general idea

Vacuum characterized by discrete (compactification) data \vec{N} and critical point of effective potential $V_N(z)$:

$$(\vec{N}, z) : V'_N(z) = 0, \quad V''_N(z) > 0$$

We want to count the number of metastable vacua in a given ensemble in a certain region of parameter space:

$$\mathcal{N}_{vac}(z \in \mathcal{S}) = \sum_{\vec{N}} \int_{\mathcal{S}} d^n z \delta^n(V'_N(z)) |\det V''_N(z)|$$

Statistics: general idea

Vacuum characterized by discrete (compactification) data \vec{N} and critical point of effective potential $V_N(z)$:

$$(\vec{N}, z) : V'_N(z) = 0, \quad V''_N(z) > 0$$

We want to count the number of metastable vacua in a given ensemble in a certain region of parameter space:

$$\begin{aligned} \mathcal{N}_{vac}(z \in \mathcal{S}) &= \sum_{\vec{N}} \int_{\mathcal{S}} d^n z \delta^n(V'_N(z)) |\det V''_N(z)| \\ &= \int_{\mathcal{S}} d^n z \rho(z) \end{aligned}$$

with

$$\rho(z) = \sum_{\vec{N}} \delta^n(V'_N(z)) |\det V''_N(z)|$$

Statistics: general idea

Vacuum characterized by discrete (compactification) data \vec{N} and critical point of effective potential $V_N(z)$:

$$(\vec{N}, z) : V'_N(z) = 0, \quad V''_N(z) > 0$$

We want to count the number of metastable vacua in a given ensemble in a certain region of parameter space:

$$\begin{aligned} \mathcal{N}_{\text{vac}}(z \in \mathcal{S}) &= \sum_{\vec{N}} \int_{\mathcal{S}} d^n z \delta^n(V'_N(z)) |\det V''_N(z)| \\ &= \int_{\mathcal{S}} d^n z \rho(z) \end{aligned}$$

with

$$\rho(z) = \sum_{\vec{N}} \delta^n(V'_N(z)) |\det V''_N(z)|$$

\rightsquigarrow not very practical; need some more structure + approx.

Statistics: general idea

If $V_N = e^K(|DW_N|^2 - 3|W_N|^2)$

$\Rightarrow V'_N(z)$ and $V''_N(z)$ can be expressed in terms of $W \equiv W_N(z)$,
 $F_A \equiv D_A W_N(z)$, $M_{AB} \equiv D_A D_B W_N(z)$, $Y_{ABC} \equiv D_A D_B D_C W_N(z)$.

Statistics: general idea

If $V_N = e^K(|DW_N|^2 - 3|W_N|^2)$

$\Rightarrow V'_N(z)$ and $V''_N(z)$ can be expressed in terms of $W \equiv W_N(z)$,
 $F_A \equiv D_A W_N(z)$, $M_{AB} \equiv D_A D_B W_N(z)$, $Y_{ABC} \equiv D_A D_B D_C W_N(z)$.

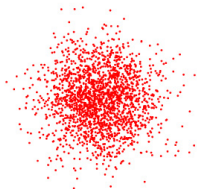
At any fixed z , varying \vec{N} will define a large discrete set in
 (W, F, M, Y) -space. The distribution of these points is given by
some measure $d\mu_0[W, F, M, Y]_z$

Statistics: general idea

If $V_N = e^K(|DW_N|^2 - 3|W_N|^2)$

$\Rightarrow V'_N(z)$ and $V''_N(z)$ can be expressed in terms of $W \equiv W_N(z)$,
 $F_A \equiv D_A W_N(z)$, $M_{AB} \equiv D_A D_B W_N(z)$, $Y_{ABC} \equiv D_A D_B D_C W_N(z)$.

At any fixed z , varying \vec{N} will define a large discrete set in
 (W, F, M, Y) -space. The distribution of these points is given by
some measure $d\mu_0[W, F, M, Y]_z$

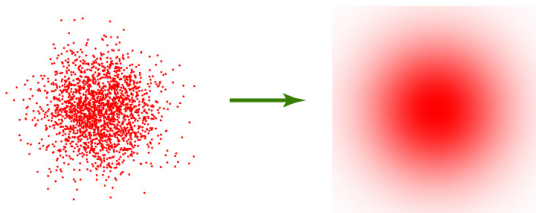


Statistics: general idea

If $V_N = e^K(|DW_N|^2 - 3|W_N|^2)$

$\Rightarrow V'_N(z)$ and $V''_N(z)$ can be expressed in terms of $W \equiv W_N(z)$,
 $F_A \equiv D_A W_N(z)$, $M_{AB} \equiv D_A D_B W_N(z)$, $Y_{ABC} \equiv D_A D_B D_C W_N(z)$.

At any fixed z , varying \vec{N} will define a large discrete set in
 (W, F, M, Y) -space. The distribution of these points is given by
some measure $d\mu_0[W, F, M, Y]_z \rightarrow$ **continuous approximation**

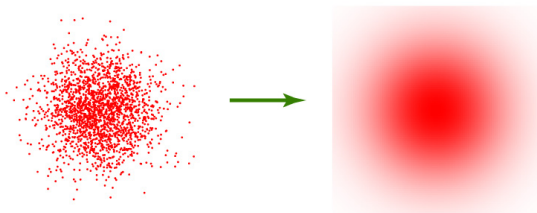


Statistics: general idea

If $V_N = e^K(|DW_N|^2 - 3|W_N|^2)$

$\Rightarrow V'_N(z)$ and $V''_N(z)$ can be expressed in terms of $W \equiv W_N(z)$,
 $F_A \equiv D_A W_N(z)$, $M_{AB} \equiv D_A D_B W_N(z)$, $Y_{ABC} \equiv D_A D_B D_C W_N(z)$.

At any fixed z , varying \vec{N} will define a large discrete set in
 (W, F, M, Y) -space. The distribution of these points is given by
some measure $d\mu_0[W, F, M, Y]_z \rightarrow$ **continuous approximation**



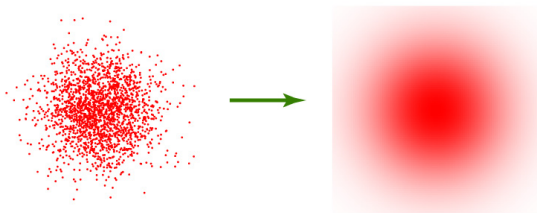
$\Rightarrow \rho(z) = \int d\mu_0[W, F, M, Y]_z f(W, F, M, Y)_z$

Statistics: general idea

$$\text{If } V_N = e^K(|DW_N|^2 - 3|W_N|^2)$$

$\Rightarrow V'_N(z)$ and $V''_N(z)$ can be expressed in terms of $W \equiv W_N(z)$,
 $F_A \equiv D_A W_N(z)$, $M_{AB} \equiv D_A D_B W_N(z)$, $Y_{ABC} \equiv D_A D_B D_C W_N(z)$.

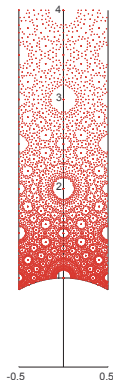
At any fixed z , varying \vec{N} will define a large discrete set in
 (W, F, M, Y) -space. The distribution of these points is given by
some measure $d\mu_0[W, F, M, Y]_z \rightarrow$ **continuous approximation**



$$\Rightarrow \rho(z) = \int d\mu_0[W, F, M, Y]_z f(W, F, M, Y)_z \rightarrow \text{finite dim. int!}$$

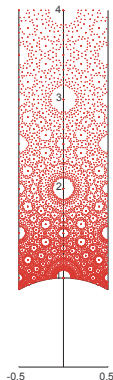
Toy example

Type IIB on rigid CY (\Rightarrow only dilaton-axion τ).



Toy example

Type IIB on rigid CY (\Rightarrow only dilaton-axion τ).



$$\begin{aligned}\mathcal{N}_{\text{vac}}(\mathcal{R}) &= \int_{\mathcal{R}} \frac{d^2\tau}{(\text{Im } \tau)^2} \int_{|W|^2 - |F|^2 \leq L_*} d^2W d^2F \delta^2(F) |W|^2 \\ &= \frac{\pi L_*^2}{2} \int_{\mathcal{R}} \frac{d^2\tau}{(\text{Im } \tau)^2}\end{aligned}$$

General distributions of susy IIB flux vacua

- Number of flux vacua in region \mathcal{S} of moduli space

$$\mathcal{N}_{\mathcal{S}}(L \leq L_*) \approx \frac{(2\pi L_*)^{b_3}}{b_3!} \int_{\mathcal{S}} \frac{1}{\pi^m} \det(R + \omega \mathbf{1})$$

where $L_* = \chi(X_4)/24$.

General distributions of susy IIB flux vacua

- Number of flux vacua in region \mathcal{S} of moduli space

$$\mathcal{N}_{\mathcal{S}}(L \leq L_*) \approx \frac{(2\pi L_*)^{b_3}}{b_3!} \int_{\mathcal{S}} \frac{1}{\pi^m} \det(R + \omega \mathbf{1})$$

where $L_* = \chi(X_4)/24$.

Example [Giryavets-Kachru-Tripathy-Trivedi]: $X_3 = \text{CY}$ hypersurface in $WP[1, 1, 1, 1, 4]$, $X_4 = \text{CY}$ hypersurface in $WP[1, 1, 1, 1, 8, 12]$. Has $\chi/24 = 972$, $b_3 = 300$, so

$$\mathcal{N}_{\text{vac}} \sim 10^{500}$$

General distributions of susy IIB flux vacua

- Number of flux vacua in region \mathcal{S} of moduli space

$$\mathcal{N}_{\mathcal{S}}(L \leq L_*) \approx \frac{(2\pi L_*)^{b_3}}{b_3!} \int_{\mathcal{S}} \frac{1}{\pi^m} \det(R + \omega \mathbf{1})$$

where $L_* = \chi(X_4)/24$.

Example [Giryavets-Kachru-Tripathy-Trivedi]: $X_3 = \text{CY}$ hypersurface in $WP[1, 1, 1, 1, 4]$, $X_4 = \text{CY}$ hypersurface in $WP[1, 1, 1, 1, 8, 12]$. Has $\chi/24 = 972$, $b_3 = 300$, so

$$\mathcal{N}_{\text{vac}} \sim 10^{500}$$

\rightsquigarrow Infamous number is just from illustrative example...

General distributions of susy IIB flux vacua

- Number of flux vacua in region \mathcal{S} of moduli space

$$\mathcal{N}_{\mathcal{S}}(L \leq L_*) \approx \frac{(2\pi L_*)^{b_3}}{b_3!} \int_{\mathcal{S}} \frac{1}{\pi^m} \det(R + \omega \mathbf{1})$$

where $L_* = \chi(X_4)/24$.

Example [Giryavets-Kachru-Tripathy-Trivedi]: $X_3 = \text{CY}$ hypersurface in $WP[1, 1, 1, 1, 4]$, $X_4 = \text{CY}$ hypersurface in $WP[1, 1, 1, 1, 8, 12]$. Has $\chi/24 = 972$, $b_3 = 300$, so

$$\mathcal{N}_{\text{vac}} \sim 10^{500}$$

\rightsquigarrow Infamous number is just from illustrative example...

Other examples can be constructed with $\mathcal{N}_{\text{vac}} \sim 10^{5000}$.

General distributions of susy IIB flux vacua

- Number of flux vacua in region \mathcal{S} of moduli space

$$\mathcal{N}_{\mathcal{S}}(L \leq L_*) \approx \frac{(2\pi L_*)^{b_3}}{b_3!} \int_{\mathcal{S}} \frac{1}{\pi^m} \det(R + \omega \mathbf{1})$$

where $L_* = \chi(X_4)/24$.

Example [Giryavets-Kachru-Tripathy-Trivedi]: $X_3 = \text{CY}$ hypersurface in $WP[1, 1, 1, 1, 4]$, $X_4 = \text{CY}$ hypersurface in $WP[1, 1, 1, 1, 8, 12]$. Has $\chi/24 = 972$, $b_3 = 300$, so

$$\mathcal{N}_{\text{vac}} \sim 10^{500}$$

\leadsto Infamous number is just from illustrative example...

Other examples can be constructed with $\mathcal{N}_{\text{vac}} \sim 10^{5000}$.

- C.c. $\Lambda = -3|W|^2$ uniformly distributed for $|\Lambda| \ll M_p^4$:

$$d\mathcal{N}[\Lambda] \sim d\Lambda$$

General distributions of susy IIB flux vacua

- ▶ Number of flux vacua in region \mathcal{S} of moduli space

$$\mathcal{N}_{\mathcal{S}}(L \leq L_*) \approx \frac{(2\pi L_*)^{b_3}}{b_3!} \int_{\mathcal{S}} \frac{1}{\pi^m} \det(R + \omega \mathbf{1})$$

where $L_* = \chi(X_4)/24$.

Example [Giryavets-Kachru-Tripathy-Trivedi]: $X_3 = \text{CY}$ hypersurface in $WP[1, 1, 1, 1, 4]$, $X_4 = \text{CY}$ hypersurface in $WP[1, 1, 1, 1, 8, 12]$. Has $\chi/24 = 972$, $b_3 = 300$, so

$$\mathcal{N}_{\text{vac}} \sim 10^{500}$$

\leadsto Infamous number is just from illustrative example...

Other examples can be constructed with $\mathcal{N}_{\text{vac}} \sim 10^{5000}$.

- ▶ C.c. $\Lambda = -3|W|^2$ uniformly distributed for $|\Lambda| \ll M_p^4$:

$$d\mathcal{N}[\Lambda] \sim d\Lambda$$

\Rightarrow smallest c.c. $\sim M_s^4/\mathcal{N}_{\text{vac}}$.

General distributions of susy IIB flux vacua

- ▶ Number of flux vacua in region \mathcal{S} of moduli space

$$\mathcal{N}_{\mathcal{S}}(L \leq L_*) \approx \frac{(2\pi L_*)^{b_3}}{b_3!} \int_{\mathcal{S}} \frac{1}{\pi^m} \det(R + \omega \mathbf{1})$$

where $L_* = \chi(X_4)/24$.

Example [Giryavets-Kachru-Tripathy-Trivedi]: $X_3 = \text{CY}$ hypersurface in $WP[1, 1, 1, 1, 4]$, $X_4 = \text{CY}$ hypersurface in $WP[1, 1, 1, 1, 8, 12]$. Has $\chi/24 = 972$, $b_3 = 300$, so

$$\mathcal{N}_{\text{vac}} \sim 10^{500}$$

\leadsto Infamous number is just from illustrative example...

Other examples can be constructed with $\mathcal{N}_{\text{vac}} \sim 10^{5000}$.

- ▶ C.c. $\Lambda = -3|W|^2$ uniformly distributed for $|\Lambda| \ll M_p^4$:

$$d\mathcal{N}[\Lambda] \sim d\Lambda$$

\Rightarrow smallest c.c. $\sim M_s^4/\mathcal{N}_{\text{vac}}$.

- ▶ String coupling g_s : uniformly distributed.

Other features of distributions of IIB flux vacua

Other features of distributions of IIB flux vacua

- Vacua cluster near conifold degenerations:

$$d\mathcal{N}[|z|] \sim \frac{d|z|}{|z|(\log |z|)^2}$$

Other features of distributions of IIB flux vacua

- Vacua cluster near conifold degenerations:

$$d\mathcal{N}[|z|] \sim \frac{d|z|}{|z|(\log |z|)^2}$$

→ Relation to dual YM coupling: $|z| \sim e^{-b/g_{YM}^2}$

Other features of distributions of IIB flux vacua

- Vacua cluster near conifold degenerations:

$$d\mathcal{N}[|z|] \sim \frac{d|z|}{|z|(\log |z|)^2}$$

→ Relation to dual YM coupling: $|z| \sim e^{-b/g_{YM}^2} \Rightarrow$ *uniform*:

$$d\mathcal{N} \sim dg_{YM}^2$$

Other features of distributions of IIB flux vacua

- Vacua cluster near conifold degenerations:

$$d\mathcal{N}[|z|] \sim \frac{d|z|}{|z|(\log |z|)^2}$$

→ Relation to dual YM coupling: $|z| \sim e^{-b/g_{YM}^2} \Rightarrow$ *uniform*:

$$d\mathcal{N} \sim dg_{YM}^2$$

Behavior near other singularities similar [Eguchi].

Other features of distributions of IIB flux vacua

- Vacua cluster near conifold degenerations:

$$d\mathcal{N}[|z|] \sim \frac{d|z|}{|z|(\log |z|)^2}$$

→ Relation to dual YM coupling: $|z| \sim e^{-b/g_{YM}^2} \Rightarrow$ *uniform*:

$$d\mathcal{N} \sim dg_{YM}^2$$

Behavior near other singularities similar [Eguchi].

- Large volumes strongly suppressed:

$$d\mathcal{N}[V] \sim e^{-cV^{2/3}} d(V^{2/3})$$

Other features of distributions of IIB flux vacua

- Vacua cluster near conifold degenerations:

$$d\mathcal{N}[|z|] \sim \frac{d|z|}{|z|(\log |z|)^2}$$

→ Relation to dual YM coupling: $|z| \sim e^{-b/g_{YM}^2} \Rightarrow$ *uniform*:

$$d\mathcal{N} \sim dg_{YM}^2$$

Behavior near other singularities similar [Eguchi].

- Large volumes strongly suppressed:

$$d\mathcal{N}[V] \sim e^{-cV^{2/3}} d(V^{2/3})$$

- F-breaking vacua, $F =: M_{susy}^2 \ll M_p^2$, for $\Lambda \sim 0$ or $\Lambda > 0$:

$$d\mathcal{N}[F, \Lambda] \sim F^5 dF d\Lambda$$

Other features of distributions of IIB flux vacua

- ▶ Vacua cluster near conifold degenerations:

$$d\mathcal{N}[|z|] \sim \frac{d|z|}{|z|(\log |z|)^2}$$

→ Relation to dual YM coupling: $|z| \sim e^{-b/g_{YM}^2} \Rightarrow$ *uniform*:

$$d\mathcal{N} \sim dg_{YM}^2$$

Behavior near other singularities similar [Eguchi].

- ▶ Large volumes strongly suppressed:

$$d\mathcal{N}[V] \sim e^{-cV^{2/3}} d(V^{2/3})$$

- ▶ F-breaking vacua, $F =: M_{susy}^2 \ll M_p^2$, for $\Lambda \sim 0$ or $\Lambda > 0$:

$$d\mathcal{N}[F, \Lambda] \sim F^5 dF d\Lambda$$

→ low breaking scale “disfavored” (but much less than naive guess dF^{2n})

Statistics of gauge groups, particle spectra, ...

Statistics of intersecting brane models [Gmeiner-Blumenhagen-Honecker-Lüst-Weigand, Gmeiner, Dijkstra-Huiszoon-Schellekens, Dienes]:

- ▶ Finite number of intersecting brane models in given background.
- ▶ No significant correlations between gauge groups, matter representations, number of generations etc.
- ▶ Number of “standard models”: about one in a billion.

Lessons: are quasi-realistic string vacua enumerable?

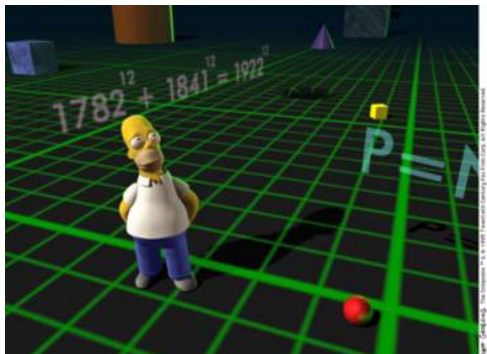
Lessons: are quasi-realistic string vacua enumerable?

We don't know, but there is no evidence against it. Nontrivial finiteness results indicate yes, in principle.

Complexity

2026: string theory under full control!

Imagine that we have a systematic classification of all vacua, and that we can compute for each vacuum every low energy quantity to very high accuracy.

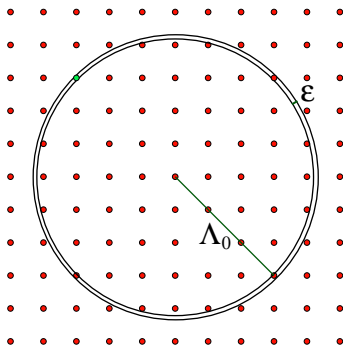


2026: string theory under full control!

Imagine that we have a systematic classification of all vacua, and that we can compute for each vacuum every low energy quantity to very high accuracy.



Simple model for matching observable data with discrete microscopic data

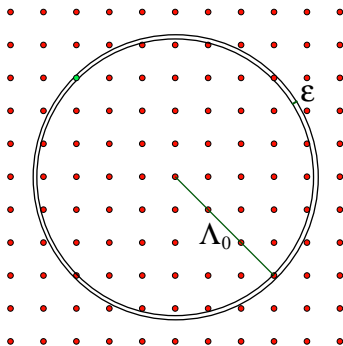


E.g. cosmological constant in Bousso-Polchinski model:

$$\Lambda(N) = -\Lambda_0 + \sum_{ij} g_{ij} N^i N^j$$

with flux $N \in \mathbb{Z}^K$.

Simple model for matching observable data with discrete microscopic data

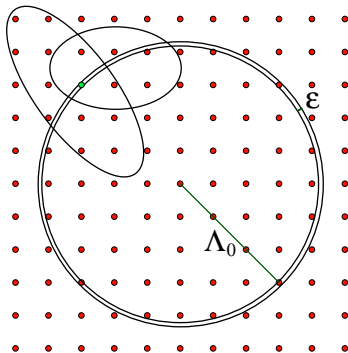


E.g. cosmological constant in Bousso-Polchinski model:

$$\Lambda(N) = -\Lambda_0 + \sum_{ij} g_{ij} N^i N^j$$

with flux $N \in \mathbb{Z}^K$. Example question: $\exists N : 0 \leq \Lambda(N) < \epsilon$?

Simple model for matching observable data with discrete microscopic data



E.g. cosmological constant in Bousso-Polchinski model:

$$\Lambda(N) = -\Lambda_0 + \sum_{ij} g_{ij} N^i N^j$$

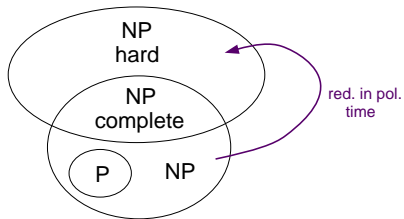
with flux $N \in \mathbb{Z}^K$. Example question: $\exists N : 0 \leq \Lambda(N) < \epsilon$?

Can be extended to more complicated models, other parameters, ...

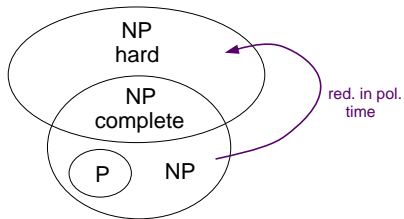
Is this a tractable problem?

- ▶ Tractable = can we solve it before the sun burns out?
- ▶ Such questions are addressed in computational complexity theory.

Basic complexity classes

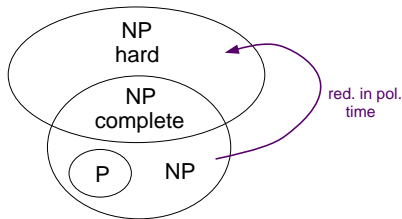


Basic complexity classes



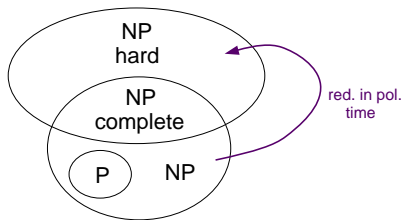
- P = yes/no problems solvable in polynomial time (e.g. is $n_1 \times n_2 = n_3$?, primality)

Basic complexity classes



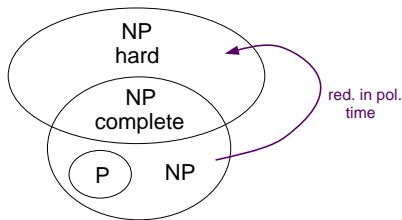
- ▶ P = yes/no problems solvable in polynomial time (e.g. is $n_1 \times n_2 = n_3?$, primality)
- ▶ NP = problems for which a candidate solution can be *verified* in polynomial time (e.g. subset sum: given finite set of integers, is there subset summing up to zero?)

Basic complexity classes



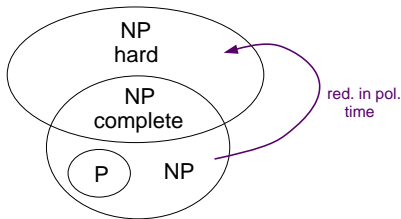
- ▶ P = yes/no problems solvable in polynomial time (e.g. is $n_1 \times n_2 = n_3?$, primality)
- ▶ NP = problems for which a candidate solution can be *verified* in polynomial time (e.g. subset sum: given finite set of integers, is there subset summing up to zero?)
- ▶ NP -hard = loosely: problem at least as hard as *any* NP problem, i.e. *any* NP problem can be reduced to it in polynomial time.

Basic complexity classes



- ▶ P = yes/no problems solvable in polynomial time (e.g. is $n_1 \times n_2 = n_3$?, primality)
- ▶ NP = problems for which a candidate solution can be *verified* in polynomial time (e.g. subset sum: given finite set of integers, is there subset summing up to zero?)
- ▶ NP-hard = loosely: problem at least as hard as *any* NP problem, i.e. *any* NP problem can be reduced to it in polynomial time.
- ▶ NP-complete = $\text{NP} \cap \text{NP-hard}$ (e.g. subset-sum, 3-SAT, traveling salesman, $n \times n$ Sudoku, ...)

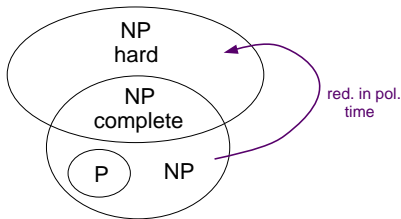
Basic complexity classes



- ▶ P = yes/no problems solvable in polynomial time (e.g. is $n_1 \times n_2 = n_3?$, primality)
- ▶ NP = problems for which a candidate solution can be *verified* in polynomial time (e.g. subset sum: given finite set of integers, is there subset summing up to zero?)
- ▶ NP -hard = loosely: problem at least as hard as *any* NP problem, i.e. *any* NP problem can be reduced to it in polynomial time.
- ▶ NP -complete = $NP \cap NP$ -hard (e.g. subset-sum, 3-SAT, traveling salesman, $n \times n$ Sudoku, ...)

So: if *one* NP -complete problem turns out to be in P , then $NP = P$.

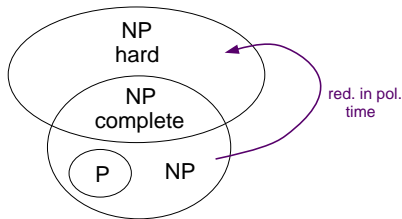
Basic complexity classes



- ▶ P = yes/no problems solvable in polynomial time (e.g. is $n_1 \times n_2 = n_3$?, primality)
- ▶ NP = problems for which a candidate solution can be *verified* in polynomial time (e.g. subset sum: given finite set of integers, is there subset summing up to zero?)
- ▶ NP -hard = loosely: problem at least as hard as *any* NP problem, i.e. *any* NP problem can be reduced to it in polynomial time.
- ▶ NP -complete = $NP \cap NP$ -hard (e.g. subset-sum, 3-SAT, traveling salesman, $n \times n$ Sudoku, ...)

So: if *one* NP -complete problem turns out to be in P , then $NP = P$.
Widely believed: $NP \neq P$, but no proof to date (Clay prize problem).

Basic complexity classes



- ▶ P = yes/no problems solvable in polynomial time (e.g. is $n_1 \times n_2 = n_3$?, primality)
- ▶ NP = problems for which a candidate solution can be *verified* in polynomial time (e.g. subset sum: given finite set of integers, is there subset summing up to zero?)
- ▶ NP -hard = loosely: problem at least as hard as *any* NP problem, i.e. *any* NP problem can be reduced to it in polynomial time.
- ▶ NP -complete = $NP \cap NP$ -hard (e.g. subset-sum, 3-SAT, traveling salesman, $n \times n$ Sudoku, ...)

So: if *one* NP -complete problem turns out to be in P , then $NP = P$.
Widely believed: $NP \neq P$, but no proof to date (Clay prize problem).
Therefore: expect no P algorithms for NP -complete problems.

Complexity of BP

- ▶ Clear: $BP \in NP$

Complexity of BP

- ▶ Clear: $BP \in NP$
- ▶ Bad news: **BP is NP-complete**

Complexity of BP

- ▶ Clear: $BP \in NP$
- ▶ Bad news: BP is NP-complete
- ▶ Proof: by mapping version of subset sum to it.

Complexity of BP

- ▶ Clear: $BP \in NP$
- ▶ Bad news: **BP is NP-complete**
- ▶ Proof: by mapping version of subset sum to it.
- ▶ Standard subset sum: Given $t, g_1, \dots, g_N \in \mathbb{Z}$,

$$\exists k_i \in \{0, 1\} : \sum_i k_i g_i = t?$$

Complexity of BP

- ▶ Clear: $BP \in NP$
- ▶ Bad news: **BP is NP-complete**
- ▶ Proof: by mapping version of subset sum to it.
- ▶ Standard subset sum: Given $t, g_1, \dots, g_N \in \mathbb{Z}$,

$$\exists k_i \in \{0, 1\} : \sum_i k_i g_i = t?$$

- ▶ Modified (bit still NPC) version: we are “promised” that

$$\sum_i k_i g_i = t \text{ with } k_i \in \mathbb{Z}^+ \quad \Rightarrow \quad k_i \in \{0, 1\}$$

Complexity of BP

- ▶ Clear: $BP \in NP$
- ▶ Bad news: **BP is NP-complete**
- ▶ Proof: by mapping version of subset sum to it.
- ▶ Standard subset sum: Given $t, g_1, \dots, g_N \in \mathbb{Z}$,

$$\exists k_i \in \{0, 1\} : \sum_i k_i g_i = t?$$

- ▶ Modified (bit still NPC) version: we are “promised” that

$$\sum_i k_i g_i = t \text{ with } k_i \in \mathbb{Z}^+ \Rightarrow k_i \in \{0, 1\}$$

- ▶ Reduction to BP: take BP with $g_{ij} = g_i \delta_{ij}$, $\Lambda_0 = t$, $\epsilon = 1$:

$$\exists N_i \in \mathbb{Z} : 0 \leq -t + \sum_i N_i^2 g_i < 1?$$

Complexity of BP

- ▶ Clear: $BP \in NP$
- ▶ Bad news: **BP is NP-complete**
- ▶ Proof: by mapping version of subset sum to it.
- ▶ Standard subset sum: Given $t, g_1, \dots, g_N \in \mathbb{Z}$,

$$\exists k_i \in \{0, 1\} : \sum_i k_i g_i = t?$$

- ▶ Modified (bit still NPC) version: we are “promised” that

$$\sum_i k_i g_i = t \text{ with } k_i \in \mathbb{Z}^+ \Rightarrow k_i \in \{0, 1\}$$

- ▶ Reduction to BP: take BP with $g_{ij} = g_i \delta_{ij}$, $\Lambda_0 = t$, $\epsilon = 1$:

$$\exists N_i \in \mathbb{Z} : 0 \leq -t + \sum_i N_i^2 g_i < 1?$$

\rightsquigarrow equivalent to modified subset sum.

Complexity of BP

- ▶ Clear: $BP \in NP$
- ▶ Bad news: **BP is NP-complete**
- ▶ Proof: by mapping version of subset sum to it.
- ▶ Standard subset sum: Given $t, g_1, \dots, g_N \in \mathbb{Z}$,

$$\exists k_i \in \{0, 1\} : \sum_i k_i g_i = t?$$

- ▶ Modified (bit still NPC) version: we are “promised” that

$$\sum_i k_i g_i = t \text{ with } k_i \in \mathbb{Z}^+ \Rightarrow k_i \in \{0, 1\}$$

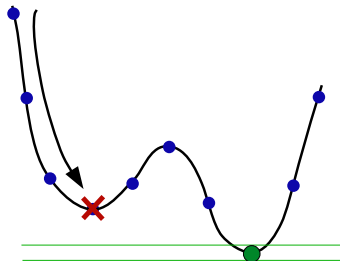
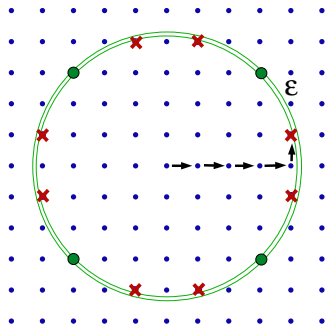
- ▶ Reduction to BP: take BP with $g_{ij} = g_i \delta_{ij}$, $\Lambda_0 = t$, $\epsilon = 1$:

$$\exists N_i \in \mathbb{Z} : 0 \leq -t + \sum_i N_i^2 g_i < 1?$$

\rightsquigarrow equivalent to modified subset sum.

- ▶ (Hard part is to show that promise version of subset sum is still NP-complete.)

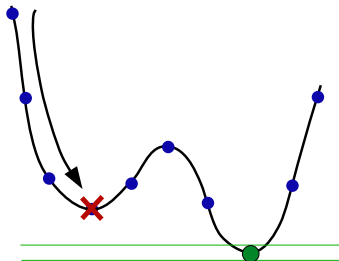
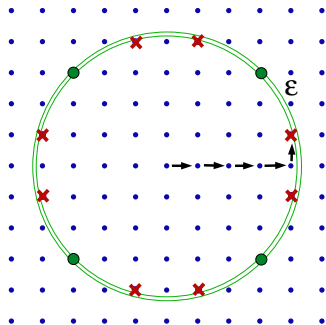
Physical intuition



Exponentially many **local minima** for local relaxation process of $|\Lambda - \epsilon/2|$ with steps $\Delta N_i = \pm \delta_{ki}$, say for $g_{ij} \equiv g_i \delta_{ij}$:

$$|\Delta \Lambda| = g_k |1 \pm 2N^k| > g_k.$$

Physical intuition

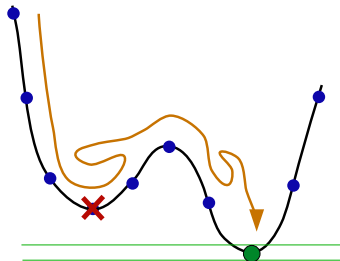
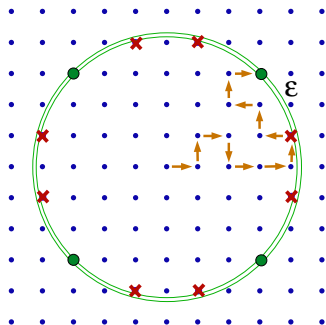


Exponentially many **local minima** for local relaxation process of $|\Lambda - \epsilon/2|$ with steps $\Delta N_i = \pm \delta_{ki}$, say for $g_{ij} \equiv g_i \delta_{ij}$:

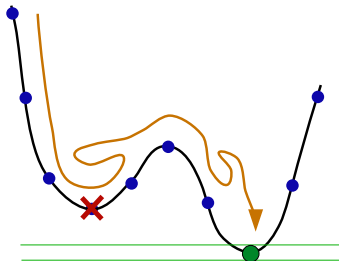
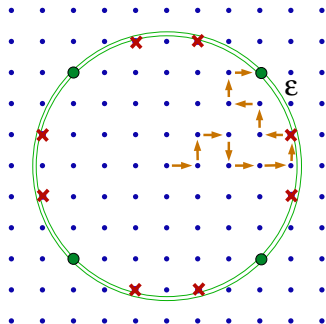
$$|\Delta \Lambda| = g_k |1 \pm 2N^k| > g_k.$$

\Rightarrow any $|\Lambda - \epsilon/2| < \min_k g_k/2$ is local minimum, but if $\epsilon \ll \min_k g_k$, one generically **gets stuck** far from target range.

Simulated annealing

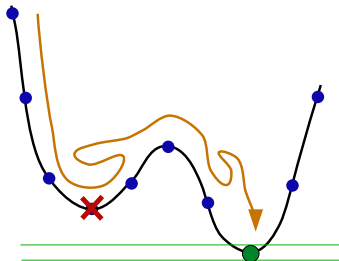
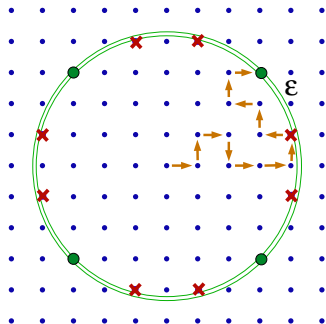


Simulated annealing



- Simulated annealing: add thermal noise to get out of local minima and gradually cool.

Simulated annealing



- ▶ Simulated annealing: add thermal noise to get out of local minima and gradually cool.
- ▶ E.g. metropolis algorithm \rightsquigarrow converges to Boltzman distribution, so will always find target range, but only guaranteed in time exponential in problem size.

Prospects for solving NP-hard problems



Ray Heine Photography

Prospects for solving NP-hard problems



- ▶ Parallel processing? (P)

Prospects for solving NP-hard problems



- ▶ Parallel processing? (P) ✗

Prospects for solving NP-hard problems



- ▶ Parallel processing? (P) ✗
- ▶ Classical polynomial time probabilistic algorithms? (BPP)

Prospects for solving NP-hard problems



- ▶ Parallel processing? (P) ✗
- ▶ Classical polynomial time probabilistic algorithms? (BPP) ✗

Prospects for solving NP-hard problems



- ▶ Parallel processing? (P) ✗
- ▶ Classical polynomial time probabilistic algorithms? (BPP) ✗
- ▶ Polynomial time quantum computing? (BQP)

Prospects for solving NP-hard problems



- ▶ Parallel processing? (P) ✗
- ▶ Classical polynomial time probabilistic algorithms? (BPP) ✗
- ▶ Polynomial time quantum computing? (BQP) ✗

Prospects for solving NP-hard problems



- ▶ Parallel processing? (P) ✗
- ▶ Classical polynomial time probabilistic algorithms? (BPP) ✗
- ▶ Polynomial time quantum computing? (BQP) ✗
- ▶ Other known physical models of computation?

Prospects for solving NP-hard problems



- ▶ Parallel processing? (P) ✗
- ▶ Classical polynomial time probabilistic algorithms? (BPP) ✗
- ▶ Polynomial time quantum computing? (BQP) ✗
- ▶ Other known physical models of computation? ✗

Lessons: is matching observable data to microscopic data tractable?

Lessons: is matching observable data to microscopic data tractable?

Well...

Caveats

Caveats

- ▶ NP-completeness is asymptotic, worst case notion. Particular instances may turn out easy. Cryptographic codes do get broken.

Caveats

- ▶ NP-completeness is asymptotic, worst case notion. Particular instances may turn out easy. Cryptographic codes do get broken.
- ▶ Some problems may be more tractable than matching continuous parameters, e.g. matching particle spectra (?) E.g. $SU(3) \times SU(2) \times U(1)$ can be deduced in polynomial time from rudimentary data [Coleman's thesis].

Caveats

- ▶ NP-completeness is asymptotic, worst case notion. Particular instances may turn out easy. Cryptographic codes do get broken.
- ▶ Some problems may be more tractable than matching continuous parameters, e.g. matching particle spectra (?) E.g. $SU(3) \times SU(2) \times U(1)$ can be deduced in polynomial time from rudimentary data [Coleman's thesis].
- ▶ String theory may have much more (as yet hidden) structure and underlying simplicity than current landscape models suggest.

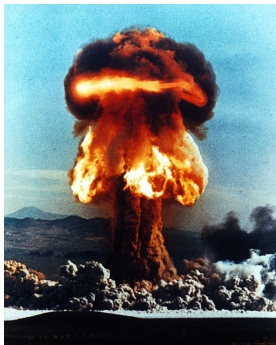
Caveats

- ▶ NP-completeness is asymptotic, worst case notion. Particular instances may turn out easy. Cryptographic codes do get broken.
- ▶ Some problems may be more tractable than matching continuous parameters, e.g. matching particle spectra (?) E.g. $SU(3) \times SU(2) \times U(1)$ can be deduced in polynomial time from rudimentary data [\[Coleman's thesis\]](#).
- ▶ String theory may have much more (as yet hidden) structure and underlying simplicity than current landscape models suggest.
- ▶ Number distributions together with experimental input could lead to exclusion of certain future measurable properties without need to determine our vacuum.

Flattening the landscape?



Flattening the landscape?



- ▶ Controlled constructions of string vacua are tricky and explicit examples sparse \rightsquigarrow maybe overestimate size?

Flattening the landscape?



- ▶ Controlled constructions of string vacua are tricky and explicit examples sparse \rightsquigarrow maybe overestimate size?
- ▶ Relatively unexplored stability issues: large number of potential decay channels [Kachru-Pearson-Verlinde, Frey-Lipper-Williams, Ceresole-Dall'Agata-Giryavets-Kallosh-Linde]

Example: thin wall bubble nucleation

Bubble nucleation rate per unit spacetime volume for a bubble with tension T , cosmological constant Λ_o outside and cosmological constant Λ_i inside [Brown-Teitelboim, Coleman-De Luccia] ($M_p \equiv 1$):

$$\Gamma \sim e^{-12\pi^2 B}$$

$$B = \frac{T\rho^3}{6} - \frac{1 - \sigma_i(1 - \frac{\Lambda_i\rho^2}{3})^{3/2}}{\Lambda_i} + \frac{1 - \sigma_o(1 - \frac{\Lambda_o\rho^2}{3})^{3/2}}{\Lambda_o}$$

$\sigma_{i,o} = \text{sign} [\pm 3 T^2 + 4(\Lambda_o - \Lambda_i)]$, ρ = bubble radius, evaluated at stationary point of $B(\rho)$:

$$\rho = \frac{12 T}{[9 T^4 + 24 T^2(\Lambda_i + \Lambda_o) + 16 (\Lambda_i - \Lambda_o)^2]^{1/2}}.$$

Example: thin wall bubble nucleation

Bubble nucleation rate per unit spacetime volume for a bubble with tension T , cosmological constant Λ_o outside and cosmological constant Λ_i inside [Brown-Teitelboim, Coleman-De Luccia] ($M_p \equiv 1$):

$$\Gamma \sim e^{-12\pi^2 B}$$

$$B = \frac{T\rho^3}{6} - \frac{1 - \sigma_i(1 - \frac{\Lambda_i\rho^2}{3})^{3/2}}{\Lambda_i} + \frac{1 - \sigma_o(1 - \frac{\Lambda_o\rho^2}{3})^{3/2}}{\Lambda_o}$$

$\sigma_{i,o} = \text{sign} [\pm 3 T^2 + 4(\Lambda_o - \Lambda_i)]$, ρ = bubble radius, evaluated at stationary point of $B(\rho)$:

$$\rho = \frac{12 T}{[9 T^4 + 24 T^2(\Lambda_i + \Lambda_o) + 16 (\Lambda_i - \Lambda_o)^2]^{1/2}}.$$

Generically $\Gamma \sim e^{-cV^2}$, but if just one out of 10^{500} or so decay channels is accidentally not suppressed, vacuum is not metastable...

Probability

Computing vacuum from cosmological selection principles?

Wait, I'm a theorist! I don't need experiment to find our vacuum!



Computing vacuum from cosmological selection principles?

Wait, I'm a theorist! I don't need experiment to find our vacuum!



- Can't we just *compute* our vacuum *ab initio* from cosmological selection principles?

Computing vacuum from cosmological selection principles?

Wait, I'm a theorist! I don't need experiment to find our vacuum!



- ▶ Can't we just *compute* our vacuum *ab initio* from cosmological selection principles?
- ▶ Example: (unrealistic) Hartle-Hawking measure selects smallest positive Λ with overwhelming probability. \Rightarrow No need to match data, just find the one with smallest Λ !

Computing vacuum from cosmological selection principles?

Wait, I'm a theorist! I don't need experiment to find our vacuum!



- ▶ Can't we just *compute* our vacuum *ab initio* from cosmological selection principles?
- ▶ Example: (unrealistic) Hartle-Hawking measure selects smallest positive Λ with overwhelming probability. \Rightarrow No need to match data, just find the one with smallest Λ !
- ▶ Problem: finding minimal $\Lambda(N)$ in BP is even harder than NP-complete!



Alternative to finding our exact microstate: probabilities on parameter space

Alternative to finding our exact microstate: probabilities on parameter space

- ▶ Eternal inflation gives in principle framework for computing probabilities on parameter space (cf. [Tegmark] review)

Alternative to finding our exact microstate: probabilities on parameter space

- ▶ Eternal inflation gives in principle framework for computing probabilities on parameter space (cf. [Tegmark] review)
- ▶ Natural measure: number of “observers” (volume, stars, ...)

Alternative to finding our exact microstate: probabilities on parameter space

- ▶ Eternal inflation gives in principle framework for computing probabilities on parameter space (cf. [Tegmark] review)
- ▶ Natural measure: number of “observers” (volume, stars, ...)
- ▶ Notorious problem: ordering ambiguity due to infinities, similar to problem: “Are there more even or odd integers?”:
 - ▶ Order $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots\}$: equally many.
 - ▶ Order $\{1, 3, 2, 5, 7, 4, 9, 11, 6, \dots\}$: twice as many odd.

Alternative to finding our exact microstate: probabilities on parameter space

- ▶ Eternal inflation gives in principle framework for computing probabilities on parameter space (cf. [Tegmark] review)
- ▶ Natural measure: number of “observers” (volume, stars, ...)
- ▶ Notorious problem: ordering ambiguity due to infinities, similar to problem: “Are there more even or odd integers?”:
 - ▶ Order $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots\}$: equally many.
 - ▶ Order $\{1, 3, 2, 5, 7, 4, 9, 11, 6, \dots\}$: twice as many odd.

Note: similar infinity problem for “black hole maximization principle” of [Smolin], moreover reasonable interpretation leads to c.c. maximization [Vilenkin].

Alternative to finding our exact microstate: probabilities on parameter space

- ▶ Eternal inflation gives in principle framework for computing probabilities on parameter space (cf. [Tegmark] review)
- ▶ Natural measure: number of “observers” (volume, stars, ...)
- ▶ Notorious problem: ordering ambiguity due to infinities, similar to problem: “Are there more even or odd integers?”:
 - ▶ Order $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots\}$: equally many.
 - ▶ Order $\{1, 3, 2, 5, 7, 4, 9, 11, 6, \dots\}$: twice as many odd.

Note: similar infinity problem for “black hole maximization principle” of [Smolin], moreover reasonable interpretation leads to c.c. maximization [Vilenkin].

- ▶ Recent progress: proposals with not manifestly absurd outcomes by [Vilenkin, Easter-Lim-Martin, Bousso].

Alternative to finding our exact microstate: probabilities on parameter space

- ▶ Eternal inflation gives in principle framework for computing probabilities on parameter space (cf. [Tegmark] review)
- ▶ Natural measure: number of “observers” (volume, stars, ...)
- ▶ Notorious problem: ordering ambiguity due to infinities, similar to problem: “Are there more even or odd integers?”:
 - ▶ Order $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots\}$: equally many.
 - ▶ Order $\{1, 3, 2, 5, 7, 4, 9, 11, 6, \dots\}$: twice as many odd.

Note: similar infinity problem for “black hole maximization principle” of [Smolin], moreover reasonable interpretation leads to c.c. maximization [Vilenkin].

- ▶ Recent progress: proposals with not manifestly absurd outcomes by [Vilenkin, Easter-Lim-Martin, Bousso]. Problems:
 - ▶ not derived from first principles, so still ambiguous.
 - ▶ highly model-dependent distributions.

Conclusions

**We need much more work, a radical breakthrough, or
another experimental shock**

We need much more work, a radical breakthrough, or
another experimental shock

