Testing AdS/CFT at string loops

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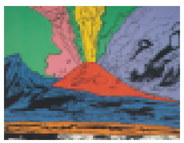


Constituents, Fundamental Forces and Symmetries of the Universe

2nd Workshop and Midterm Meeting, Napoli, October 9-13, 2006

G.Grignani, M.Orselli, B. Ramadanovich, G.W.Semenoff and D. Young, "*Divergence cancellation and loop corrections in string field theory on a plane wave background*", JHEP 0512:017,2005 e-Print Archive:hep-th/0508126

G.Grignani, M.Orselli, B. Ramadanovich, G.W.Semenoff and D. Young, "*AdS/CFT versus string loops*", JHEP 06:040,2006, e-Print Archive:hep-th/0605080

AdS/CFT

Maximally supersymmetric Yang-Mills theory on 4D flat space with gauge group SU(N) and coupling constant $\mathbf{g_{YM}}$

is exactly equivalent to

type IIB superstring theory on background $AdS_5 X S^5$ with N units of 5-form flux through S^5 . The radius of curvature is

$$\boxed{R_{AdS_5} = R_{S^5} = (4\pi g_s N)^{1/4}\sqrt{\alpha'} = \left(g^2_{\mathrm{YM}} N\right)^{1/4}\sqrt{\alpha'}}$$

and $\boxed{g^2_{\mathrm{YM}} = 4\pi g_s}$ $\boxed{\lambda \equiv g^2_{\mathrm{YM}} N}$ ‘t Hooft coupling

String state $\longleftrightarrow$ gauge invariant composite operator

Energy $\longleftrightarrow$ conformal dimension of composite operator

This is a weak coupling – strong coupling duality

•The Penrose limit of AdS_5XS^5 geometry gives a pp-wave space

$$ds^2 = -4dx^+dx^- - \mu^2 \sum_{i=1}^{8} (x^i)^2 (dx^+)^2 + \sum_{i=1}^{8} (dx^i)^2$$

$F_{+2345} = F_{+6789} = \mu$ transverse SO(4)xSO(4) symmetry

Free strings are exactly solvable in the light-cone gauge

light-cone Hamiltonian

$$\boxed{p^- = \sum_{n=-\infty}^{\infty} \left(a_n^{i\dagger} a_n^i + b_n^{a\dagger} b_n^a \right) \sqrt{\mu^2 + (\alpha' p^+)^{-2} n^2}}$$

level matching condition

$$\sum_{n=-\infty}^{\infty} n \left(a_n^{i\dagger} a_n^i + b_n^{a\dagger} b_n^a \right) = 0$$

•The analogous limit of Super Yang-Mills can be taken (BMN) → detailed matching of BMN operators in the large N, planar limit and free string states

planar limit ⟷ free strings

non-planar corrections ⟷ string interactions

→ (ongoing) matching interactions in light-cone string field theory on a pp-wave background and Yang-Mills theory computations

Non planar corrections in Yang-Mills Theory

String loop corrections to the energy of the 2-oscillator (2-impurity) states

Symmetric traceless: $|[9,1]\rangle^{(ij)} = \left(\alpha_n^{\dagger\, i}\alpha_{-n}^{\dagger\, j} + \alpha_n^{\dagger\, j}\alpha_{-n}^{\dagger\, i} - \frac{1}{2}\delta^{ij}\alpha_n^{\dagger\, k}\alpha_{-n}^{\dagger\, k}\right)|0\rangle$

Trace: $|[1,1]\rangle \equiv \alpha_n^k \alpha_{-n}^k |0>$ $\qquad i,j,k = 1,\ldots,4$

In N=4 SYM there are 6 scalar fields

Select a combination of them $\qquad Z = \phi^5 + i\phi^6$
which is charged under a U(1) subgroup of the R Symmetry

$$(\phi_1, \phi_2, \phi_3, \phi_4)\,, (D_\mu Z) \qquad\qquad SO(4)\times SO(4)$$

Dual BMN operator:

$$\sum_{p=0}^{J} Tr\left[\phi_i Z^p \phi_j Z^{J-p}\right] Tr\left[Z^{J_1}\right]\ldots Tr\left[Z^{J_k}\right] e^{\frac{2\pi i n p}{J}}$$

$$\boxed{p^- = \mu(\Delta - J)}$$

$$\boxed{p^+ = \frac{\Delta + J}{2\mu R^2} = \frac{\Delta + J}{2\mu\alpha'\sqrt{g^2_{\rm YM}N}}}$$

$$\Delta - J = 2\left(1 + \tfrac{1}{2}\lambda' n^2 - \tfrac{1}{8}\lambda'^2 n^4 + \ldots\right) + \frac{g_2^2}{4\pi^2}\left(\frac{1}{12} + \frac{35}{32\pi^2 n^2}\right)\left(\lambda' - \frac{1}{2}\lambda'^2 n^2\right) + \ldots$$

$$\frac{1}{\left(\mu\alpha' p^+\right)^2} = \frac{g^2_{YM}N}{J^2} \equiv \lambda'$$

$$4\pi g_s\left(\mu\alpha' p^+\right)^2 = \frac{J^2}{N} \equiv g_2 \quad , \quad N, J \to \infty$$

$$\begin{aligned}\frac{p^-}{\mu} \approx\ & 2\sqrt{1 + (\mu\alpha' p^+)^{-2} n^2} \\ & + \frac{g_2^2}{4\pi^2}\left[\left(\frac{1}{12} + \frac{35}{32\pi^2 n^2}\right)\frac{1}{\left(\mu\alpha' p^+\right)^2} - \frac{n^2}{2}\left(\frac{1}{12} + \frac{35}{32\pi^2 n^2}\right)\frac{1}{\left(\mu\alpha' p^+\right)^4} + \ldots\right] + \ldots\end{aligned}$$

Has not been computed using string theory!

The only available formulation of string theory in which interactions can be computed is

pp-wave light-cone string field theory

Light-cone String Field Theory

string field operator Φ : fundamental object in light-cone string field theory

creates or annihilates complete strings $\Phi : \mathbf{H}_m \to \mathbf{H}_{m\pm 1}$

m-string Hilbert space

Φ is a functional of x^+, p^+ and the worldsheet coordinates $X^I(\sigma), \theta_a(\sigma), \theta_{\dot{a}}(\sigma)$
where $X^I(\sigma)=X^I(\sigma,\tau=0)$ and likewise for other fields.

In the momentum-space representation Φ is a functional of $P^I(\sigma), \lambda_a(\sigma), \lambda_{\dot{a}}(\sigma)$

where λ is – i times the momentum conjugate to θ, i.e.: $\lambda_a = \frac{1}{2\pi\alpha'}\theta_a^\dagger$

in the momentum-space representation

$$X^I(\sigma) = i\frac{\delta}{\delta P_I(\sigma)}, \qquad \theta_a(\sigma) = i\frac{\delta}{\delta\lambda^a(\sigma)}$$

The light-cone string field theory dynamics of Φ is governed by the non-relativistic Schroedinger equation

light-cone string field theory Hamiltonian

$$\mathcal{H}_{SFT}\mathbf{\Phi} = i\frac{\partial}{\partial x^+}\mathbf{\Phi}$$

In terms of the string coupling the light-cone SFT Hamiltonian has the expansion

$$\mathcal{H}_{SFT} = \mathcal{H}^{(2)}_{l.c.} + g_s\mathcal{H}^{(3)}_{l.c.} + g_s^2\mathcal{H}^{(4)}_{l.c.} + \ldots$$

In the free string theory limit it should be equal to the Hamiltonian coming from the string theory σ-model $\mathcal{H}^{(2)}_{l.c.}$

We will use perturbation theory to compute the energy of the states.

$$\boxed{\delta E_n^{(2)} = g_s^2\langle\phi_n|\mathcal{H}^{(3)}\frac{P}{E_n^{(0)} - \mathcal{H}^{(2)}}\mathcal{H}^{(3)}|\phi_n\rangle + g_s^2\langle\phi_n|\mathcal{H}^{(4)}|\phi_n\rangle}$$

free strings

To quantize the string field we promote the fields to operators acting on the string Fock space where it creates or destroys a complete string with given excitation number

$$\mathbf{\Phi} : \mathbf{H}_m \to \mathbf{H}_{m\pm 1}$$

In the bosonic case

where $$\widehat{\mathcal{H}}^{(2)} = \int d\alpha D^8 P(\sigma) \mathbf{\Phi}^\dagger \mathcal{H}^{(2)}_{\mathrm{l}.c.} \mathbf{\Phi}$$

$$\alpha = \alpha' p^+$$

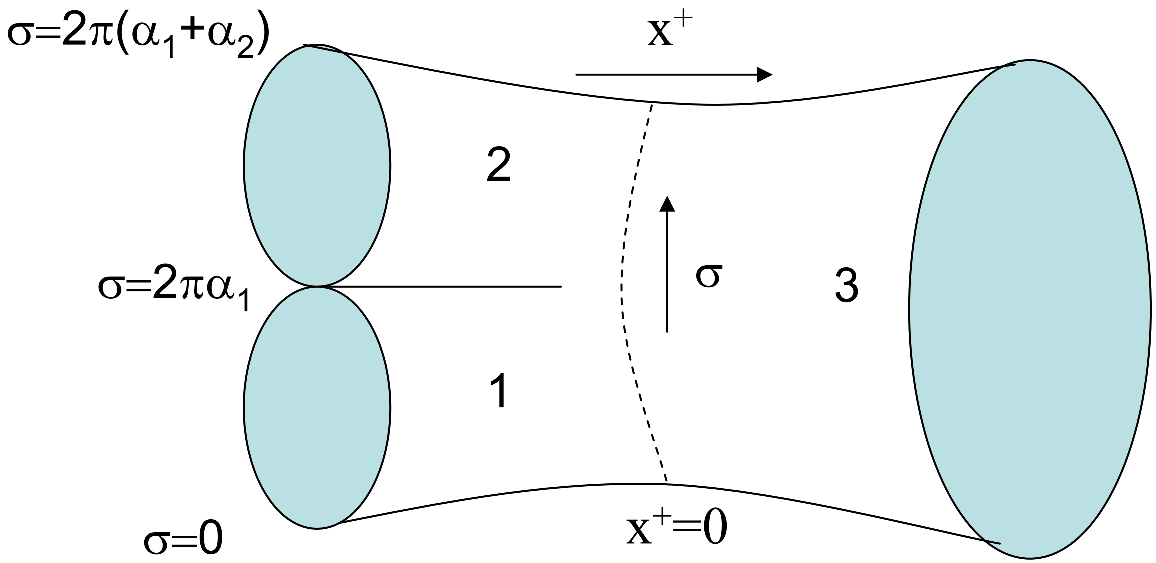
$$D^8 P(\sigma) = \prod_{n=-\infty}^{\infty} dp_n \qquad p_n = \sqrt{\frac{\omega_n}{2\alpha'}}(a_n^\dagger + a_n)$$

$$\mathcal{H}^{(2)}_{l.c.} = \frac{1}{\alpha' p^+} \sum_{n=-\infty}^{\infty} \omega_n a_n^\dagger a_n \qquad \omega_n = \sqrt{n^2 + (\mu\alpha' p^+)^2}$$

Turning on interactions: the cubic vertex

The corrections to superalgebra generators, once interactions are turned on, are obtained following some guiding principles.

1. The interaction should couple the string worldsheet in a continuos way



The interaction vertex for the scattering of 3 strings is constructed with a δ-functional enforcing worldsheet continuity.

In pp-wave superstring however the situation is slightly more complicated but the basic principle governing the interaction is very simple:

2. The superalgebra has to be realized in the full interacting theory

Supercharges that square to the Hamiltonian receive corrections when adding interactions

The picture remains geometric but in addition to a delta-functional enforcing continuity in superspace one has to insert local operators at the interaction point.

<u>These operators represent functional generalizations of derivative couplings</u>

→ **<u>prefactors</u>**

There are two different set of superalgebra generators.

1. <u>Kinematical generators</u> :

$$P^+, \quad P^I, \quad J^{+I}, \quad J^{ij}, \quad J^{i',j'}, \quad Q^+, \quad \bar{Q}^+$$

They are not corrected by interactions. The symmetries they generate are not affected by adding higher order terms to the action

→ These generators remain quadratic in the string field Φ in the interacting field theory and act diagonally on $\mathbf{H_m}$

2. Dynamical generators:

$$\mathcal{H}, \; Q^{-}, \; \bar{Q}^{-}$$

They receive corrections in the presence of interactions and couple different number of strings

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}^{(2)} + g_s \hat{\mathcal{H}}^{(3)} + g_s^2 \hat{\mathcal{H}}^{(4)} + \ldots$$

$$\hat{Q}^{-}_{\dot{a}} = \hat{Q}^{-(2)}_{\dot{a}} + g_s \hat{Q}^{-(3)}_{\dot{a}} + g_s^2 \hat{Q}^{-(4)}_{\dot{a}} + \ldots$$

cubic
in the string field

quartic
in the string field

$$\hat{\mathcal{H}}^{(3)}, \hat{Q}^{-(3)}_{\dot{a}}, \hat{\bar{Q}}^{-(3)}_{\dot{a}} : \mathbf{H}_m \rightarrow \mathbf{H}_{m\pm 1}$$

$$\hat{\mathcal{H}}^{(4)}, \hat{Q}^{-(4)}_{\dot{a}}, \hat{\bar{Q}}^{-(4)}_{\dot{a}} : \mathbf{H}_m \rightarrow \mathbf{H}_m \bigcup \mathbf{H}_{m\pm 2}$$

the corrections are such that $\hat{\mathcal{H}}$, $\hat{Q}^{-}$ and $\hat{\bar{Q}}^{-}$ still satisfiy the superalgebra

The requirement that the superalgebra is satisfied in the interacting theory now gives rise to two kind of constaints:

Kinematical constraint

(anti)-commutation relation of kinematical and dynamical generators

leads to continuity condition in superspace

Dynamical constraint

anti-commutation relation of dynamical generators alone

requires insertion of interaction point operators: the prefactors

To solve the constraints we use perturbation theory.

Kinematical constraint

$\mathcal{H}^{(3)}$ and $Q^{(3)}$ must be proportional to

$$\Delta^8\left[\sum_{r=1}^{3} P_r^I(\sigma)\right]\Delta^8\left[\sum_{r=1}^{3} \lambda_r^a(\sigma)\right]\delta\left(\sum_{r=1}^{3}\alpha_r\right)$$

Dynamical constraints

Supersymmetry algebra

$$\{Q^-_{\dot a},\bar Q^-_{\dot b}\} = 2\delta_{\dot a\dot b}\mathcal{H} - i\mu(\gamma_{ij}\Pi)_{\dot a\dot b}J^{ij} + i\mu(\gamma_{i'j'}\Pi)_{\dot a\dot b}J^{i'j'}$$

Expanding the Hamiltonian and the supercharges at the order g_s: the dynamical constraint on $Q^{(3)}$ and $\mathcal{H}^{(3)}$ is

$$\left\{Q^{(2)-}_{\dot a},\bar Q^{(3)-}_{\dot b}\right\} + \left\{Q^{(3)-}_{\dot a},\bar Q^{(2)-}_{\dot b}\right\} = 2\delta_{\dot a\dot b}\mathcal{H}^{(3)}$$

the same as in flat space

the constraints are solved by inserting **prefactors** in the interaction vertex that do not depend on the string field

$$\hat h_3(\alpha_r, P_r(\sigma), \lambda_r(\sigma)) \qquad \hat q_3(\alpha_r, P_r(\sigma), \lambda_r(\sigma))$$

Even imposing all the constraints the vertex is not uniquely fixed: three known solutions and overall unknown function f(μ,p$^+$)

We can write

$$|H_3\rangle = \hat{h}_3 |V\rangle$$

prefactor

kinematical part of the vertex, common to all dynamical generators

$$|V\rangle = |E_a\rangle |E_b\rangle \delta\left(\sum_{r=1}^{3} \alpha_r\right)$$

$$|E_a\rangle \sim \exp\left[\frac{1}{2}\sum_{r,s=1}^{3}\sum_{m,n\in\mathbf{Z}} a^{\dagger}_{m(r)} \bar{N}^{rs}_{mn} a^{\dagger}_{n(s)}\right] |0\rangle_{123}$$

$$\bar{N}^{3r}_{np} \sim -e(n)\frac{\sin(|n|\pi r)}{2\pi\sqrt{\omega^{(3)}_n \omega^{(r)}_p}}\frac{(\omega^{(r)}_p + \beta_r \omega^{(3)}_n)}{p - \beta_r n} \qquad r = 1, 2$$

$$\bar{N}^{rs}_{np} = \frac{\sqrt{\beta_r\beta_s}\left(\sqrt{\omega^{(r)}_n - \beta_r\mu\alpha_3}\sqrt{\omega^{(s)}_p - \beta_s\mu\alpha_3} + e(np)\sqrt{\omega^{(r)}_n + \beta_r\mu\alpha_3}\sqrt{\omega^{(s)}_p + \beta_s\mu\alpha_3}\right)}{4\pi\sqrt{\omega^{(r)}_n \omega^{(s)}_p}(\beta_s\omega^{(r)}_n + \beta_r\omega^{(s)}_p)}$$

and similar expressions for the fermionic part $|E_b\rangle$

The bosonic part of the prefactor is given by

$$\hat{h}_3 \sim \sum_{r=1}^{3}\left[\sum_{n=0}^{\infty}\frac{\omega_{n(r)}}{\alpha_r} a^{i\dagger}_{n(r)} a^{i}_{n(r)} + \sum_{n=1}^{\infty}\frac{\omega_{n(r)}}{\alpha_r} a^{i'\dagger}_{-n(r)} a^{i'}_{-n(r)}\right]$$

Contact Terms

We want to compute the energy of string states using a quantum mechanical perturbation theory.

The dynamical generators are expanded in terms of g_s. Up to the $O(g_s^2)$ we have

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}^{(2)} + g_s \hat{\mathcal{H}}^{(3)} + g_s^2 \hat{\mathcal{H}}^{(4)} + \dots$$

$$Q = Q^{(2)} + g_s Q^{(3)} + g_s^2 Q^{(4)} + \dots$$

The susy algebra up to the $O(g_s^2)$

$$\boxed{\left\{Q_{\dot a}^{(3)}, Q_{\dot b}^{(3)}\right\} + \left\{Q_{\dot a}^{(2)}, Q_{\dot b}^{(4)}\right\} + \left\{Q_{\dot a}^{(4)}, Q_{\dot b}^{(2)}\right\} = 2\delta_{\dot a \dot b}\mathcal{H}^{(4)}}$$

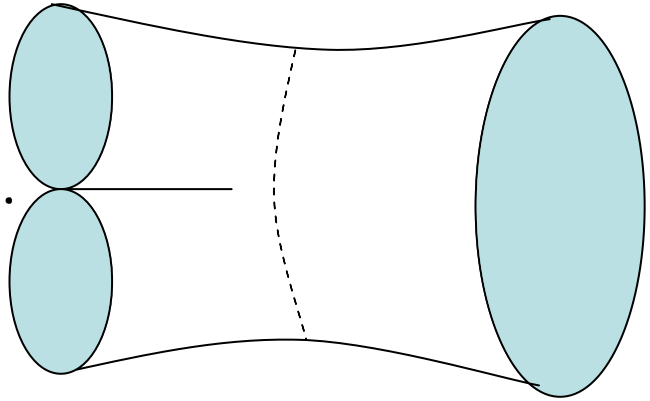
Necessarily there is an $\mathcal{H}^{(4)}$ $\Longrightarrow$ the so-called **contact term**

Light-cone string field perturbation theory on a single string in a 2 oscillator state $|\phi_n\rangle$ uses QM perturbation theory to compute the correction to its light-cone energy

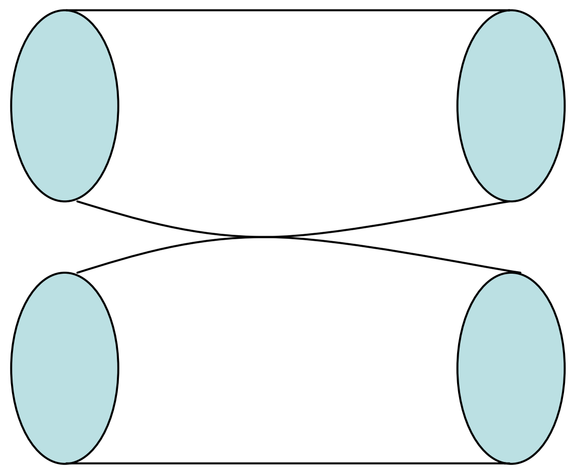
$$\mathcal{H}^{(2)}|\phi_n\rangle = E_n^{(0)}|\phi_n\rangle \qquad \delta E_n^{(1)} = g_s \langle\phi_n|\mathcal{H}^{(3)}|\phi_n\rangle = 0$$

$$\delta E_n^{(2)} = g_s^2 \langle\phi_n|\mathcal{H}^{(3)} \frac{P}{E_n^{(0)} - \mathcal{H}^{(2)}} \mathcal{H}^{(3)}|\phi_n\rangle + g_s^2 \langle\phi_n|\mathcal{H}^{(4)}|\phi_n\rangle$$

$$\mathcal{H}^{(3)} = g_s \;\cdot$$



$$\mathcal{H}^{(4)} = g_s^2 \;\cdot$$



The string join/split points of each vertex coincide

In type IIA/B superstring on Minkowski space-time, it is known that the contact terms are necessary to cancel certain singularities in the integrations over parameters of light-cone SFT diagrams. In the conformal field theory they are also seen to arise as additional contributions needed to cancel certain singular surface terms which arise in the integration of correlators of vertex operators over the modular parameters of Riemann surfaces.

J.Greensite and F.R.Klinkhamer NPB 304 (1988)

A little history

$$\delta E_n^{(2)} = g_s^2 \langle \phi_n | \mathcal{H}^{(3)} \frac{P}{E_n^{(0)} - \mathcal{H}^{(2)}} \mathcal{H}^{(3)} | \phi_n \rangle + g_s^2 \langle \phi_n | \mathcal{H}^{(4)} | \phi_n \rangle \qquad |\phi_n\rangle = |[9,1]\rangle$$

Roiban, Spradlin and Volovich, hep-th/0211220

$$\frac{\delta E^{(2)}}{\mu} = \frac{g_2^2}{4\pi^2}\left(\frac{1}{12} + \frac{35}{32\pi^2 n^2}\right)\lambda' + \ldots$$

It matches! But obtained from

$$\delta E_n^{(2)} = \frac{1}{2} g_s^2 < \phi_n | \mathcal{H}^{(3)} \frac{P}{\mathcal{H}^{(2)} - E_0} \mathcal{H}^{(3)} | \phi_n > + g_s^2 < \phi_n | \mathcal{H}^{(4)} | \phi_n >$$

Reflection symmetry of the one-loop light-cone string diagram ??

P.Gutjhar and A. Pankiewicz, **hep-th/0407098**, no ½

$$\frac{\delta E^{(2)}}{\mu} = \frac{g_2^2}{4\pi^2}\left[\left(\frac{1}{12} + \frac{65}{64\pi^2 n^2}\right)\lambda' - \left(\frac{3\lambda'^{3/2}}{16\pi^2}\right) + \frac{n^2}{4}\left(\frac{1}{24} + \frac{89}{64\pi^2 n^2}\right)\lambda'^2 + \ldots\right] + \ldots$$

It does not match!

- Expansion in half integer powers of λ'
- Uses truncation to 2-impurities
- Uses Spradlin-Volovich vertex
- Fixes the pre-factor f=1
- Sets $Q^{(4)}=0$

Results: for any vertex

$$\delta E^{(2)} = <\phi_n|\mathcal{H}^{(3)}\frac{P}{E_0-\mathcal{H}^{(2)}}\mathcal{H}^{(3)}|\phi_n> + <\phi_n|\mathcal{H}^{(4)}|\phi_n>$$

1. We confirmed Gutjhar and Pankiewicz result, the natural expansion parameter is $\sqrt{\lambda'} = \frac{1}{\mu\alpha' p^+}$

2. In the trace state **|[1, 1]i** the leading order is $\sqrt{\lambda'}$ and it **diverges**

   Each term in the equation above diverges individually. With the ½ of the "reflection symmetry" no cancelation of divergences

   ⬇

   no 1/2

3. Same for the **|[9, 1]i** state and 4 impurity intermediate states

$$\delta E^{\text{GAUGE}} = \frac{g_2^2}{4\pi^2}\left(\frac{1}{12}+\frac{35}{32\pi^2 n^2}\right)\left(\lambda' - \frac{1}{2}\lambda'^2 n^2\right)+\dots$$

4. Dobashi-Yoneya vertex

$$\delta E^{\text{DY}} = \frac{g_2^2}{4\pi^2}\left(\frac{1}{12}+\frac{35}{32\pi^2 n^2}\right)\lambda' + \dots$$

Divergences cancel and it works at 1-loop!

$$\begin{aligned}\delta E^{\text{DY}} &= \frac{g_2^2}{4\pi^2}\left[\left(\frac{1}{12}+\frac{35}{32\pi^2 n^2}\right)\left(\lambda' - \frac{4}{3}\frac{n^2}{2}\lambda'^2\right) + \frac{n^4}{24}\left(1+\frac{255}{16\pi^2 n^2}\right)\lambda'^3\right.\\ &+ \left.\frac{n^4}{384}\left(\frac{9}{\pi}+\frac{142}{5\pi^2}\right)\lambda'^{7/2} + \mathcal{O}(\lambda'^4)\right]\end{aligned}$$

Best matching of this quantity so far

DY vertex is an improvement over its predecessors

- No half integer powers up to $\lambda'^{7/2}$
- Uses truncation to 2-impurities
- Fixes the pre-factor f=4/3
- Sets $Q^{(4)}=0$

Conclusions

1. **Dobashi Yoneya vertex seems to be the most promising**

2. **Possible sources of discrepancies**

   a) number of impurities in the intermediate states.

   b) $Q^{(4)} \neq 0$
   a contact term which does not diverge?

3. **The problem of matching non-planar YM and string interactions in the context of AdS/CFT is still open**