TWISTOR STRING AS TENSIONLESS SUPERSTRING.

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1. Introduction. Twistor approach and MHV diagrames of (Super)Yang-Mills theory.
2. Twistor string actions:
   - i) Siegel, April 2004
   - ii) Berkovits, Febr. 2004
   - iii) E. Witten, Dec. 2003
3. Twistor string as tensionless superstring (null superstring) in twistor-like Lorentz harmonic formulation [I. B. + A. Zheltukhin, PLB 1991]
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Introduction

Earlier study: V. Nair, PLB 1986: some (MHV i.e. maximal helicity violating) tree level amplitudes of the N=4 SYM are generated by a WZNW model ≡ ***Twistor String***

- In December of 2003 [hep-th/0312171] *E. Witten* reconsidered the connection between N=4 Super-Yang-Mills and String Theories using *twistor approach.*

- *In distinction to the AdS/CFT correspondence, this connection relates week coupling limits on both sides and, hence can be checked perturbatively.*

- *This resulted in development of a new technique (Cachazo-Svrček-Witten or CSW technique) to calculate the gauge theory amplitudes*

and pushed forward the TWISTOR PROGRAMME by Penrose *aimed to express the physics in terms of twistor space rather than spacetime.*

How twistors enter the game:

The amplitude for n gauge boson scattering with 2 positive and (n-2) negative helicities (n-particle MHV amplitude) is

$$A(1,2,...,n) = 2^{n/2} i g^{n-2} Tr(t^{a_1} ... t^{a_n}) \frac{<IJ>^4}{<12> \dots <n1>}$$

$$A(1,2,...,n) \propto \frac{<IJ>^4}{<12> \dots <n1>}$$

where I,J refer to the positive helicity gluons,

$$<12> \equiv < \lambda^1 \lambda^2 > := \lambda^{1\alpha} \lambda^2_\alpha \equiv \epsilon^{\alpha\beta} \lambda^1{}_\beta \lambda^2{}_\alpha \,, \qquad [1,2] \equiv [\bar{\lambda}^1, \bar{\lambda}^2] := \bar{\lambda}^{1\dot{\alpha}} \bar{\lambda}^2_{\dot{\alpha}}$$

and the Weyl spinor λ^1 describes the momentum and helicity of the 1st gluon, *etc.*

Notice that:

- This amplitude is analytic i.e. dependent on λ but not on its c.c.
- It is the on-shell amplitude
- <12> = - <21>, etc.; hence <11>≡0.

$$A(1,2,...,n) \propto \frac{<IJ>^4}{<12>\dots<n1>}$$

Why this is possible?

Why amplitude may be expressed through λ's?

- A tree amplitude depends on the momentum and polarization vectors of the particles, *p* and ε.
- This is tantamount to saying that it depends on a bosonic spinor, say λ, because the momentum of the on-shell massless particle is light-like ($p^2=0$) and, hence, can be expressed by the *Cartan-Penrose* representation in terms of a bosonic spinor λ,

$$p_\mu = \lambda_\alpha \bar{\lambda}_{\dot{\alpha}} \tilde{\sigma}^{\dot{\alpha}\alpha}_\mu \quad \Leftrightarrow \quad p_{\alpha\dot{\alpha}} := \frac{1}{2} p_\mu \sigma^\mu_{\alpha\dot{\alpha}} = \lambda_\alpha \bar{\lambda}_{\dot{\alpha}}$$

and the polarization vector of the negative/positive helicity gluon is

$$\varepsilon^{-i}_{\alpha\dot{\alpha}} = \frac{\lambda^i_\alpha \bar{u}_{\dot{\alpha}}}{[\bar{\lambda}^{(i)}, \bar{\mu}]}, \qquad \varepsilon^{+i}_{\alpha\dot{\alpha}} = \frac{u_\alpha \bar{\lambda}^{(i)}_{\dot{\alpha}}}{<u, \lambda^{(i)}>}$$

where *u* is a fixed reference spinor (the same for all gluons in the amplitude).

- The new CSW (Cachazo-Svrček-Witten) approach (≡ MHV diagram technique) separates the Feynman diagrame on MHV pieces, considers them as vertices and connects these new vertices by scalar propogators.
- A problem: what spinor λ should be associated to the off-shell momentum *p,* such that $p^2 \neq 0$?

The answer is $\boxed{\lambda_\alpha(p) = p_{\alpha\dot{\alpha}} \bar{w}^{\dot{\alpha}}}$ with an arbitrary $\bar{w}^{\dot{\alpha}}$;
the amplitude heppens to be $\bar{w}$-idenpendent: $\frac{\partial A}{\partial \bar{w}} = 0$

Twistor string is N=4 supersymmetric theory.

- The MHV (CSW) technique have been shown to make sense not only in N=4 supersymmetric YM, but also in N=1,2 and N=0, i.e. non-supersymmetric case

  (see e.g. A. Brandhuber + G. Travaglini, hep-th/0609011).
- However, the original version was developed for N=4 and it was inspired by the ***twistor string*** which is a string model formulated in the space of *super-twistors.*
- A search for its (super)spacetime formulation is the main subject of our discussion.
- Thus the first three questions we have to address are:
- i) what is a twistor,
- ii) what is a supertwistor,
- iii) what is(are) the action(s) for twistor string

Penrose twistors and D=4 spacetime

- Twistor ≈ Dirac spinor ≈ fundamental representation of SU(2,2) (≈SO(2,4)) group which is the *conformal group* of D=4 spacetime,

$$\Upsilon^{\hat{\alpha}} = (\lambda_\alpha \, , \mu^{\dot{\alpha}}) \in \mathbb{C}^4.$$

- The space of twistor is considered as a projective space

$$\Upsilon^{\hat{\alpha}} \sim z\Upsilon^{\hat{\alpha}} \quad \Rightarrow \quad \Upsilon^{\hat{\alpha}} = (\lambda_\alpha \, , \mu^{\dot{\alpha}}) \in \mathbb{CP}^3 \, .$$

- The reason is the complex scale invariance of the ***Penrose incidence relations***

$$\boxed{\mu^{\dot{\alpha}} = x^{\dot{\alpha}\alpha}\lambda_\alpha} \qquad x^{\dot{\alpha}\alpha} := x^\mu \tilde{\sigma}_\mu^{\dot{\alpha}\alpha}$$

which define the D=4 spacetime point x^μ or, more precisely, the light-like line $\hat{x}^{\dot{\alpha}\alpha}(\tau, x) = x^{\dot{\alpha}\alpha} + \tau \, \lambda^\alpha \bar{\lambda}^{\dot{\alpha}}$ as far as the incidence relations are invariant under

$$\delta x^{\dot{\alpha}\alpha} = a \, \lambda^\alpha \bar{\lambda}^{\dot{\alpha}}$$ due to the identity $$\lambda^\alpha \lambda_\alpha := \epsilon^{\alpha\beta} \lambda_\alpha \lambda_\beta \equiv 0$$

- The ***Penrose incidence relations*** provides the general solution for the constraint

$$\bar{\Upsilon}_{\hat{\alpha}} \Upsilon^{\hat{\alpha}} := \bar{\lambda}_{\dot{\alpha}} \, \mu^{\dot{\alpha}} - \bar{\mu}^\alpha \lambda_\alpha = 0$$

Ferber supertwistor and D=4 superspace (Ferber 1978)

- Supertwistor ≈ fundamental representation of SU(2,2|N) supergroup which is the supergroup of *superconformal transformations* in D=4.

$$\Upsilon^{\Sigma} := (\Upsilon^{\hat{\alpha}}, \eta_i) = (\lambda_\alpha, \mu^{\dot{\alpha}}, \eta_i) \in \mathbb{C}^{(4|N)}, \quad \textbf{\textit{(i=1, ...N)}}$$

are considered as supercoordinates of the N-extended projective superspace, $\mathbb{CP}^{(3|N)}$.

- The Ferber-***Penrose incidence relations***

$$\boxed{\mu^{\dot{\alpha}} = x_L^{\dot{\alpha}\alpha}\lambda_\alpha \ , \ \ \eta_i = \theta_i^\alpha \lambda_\alpha} \qquad x_L^{\dot{\alpha}\alpha} := x_L^{\mu}\tilde{\sigma}_\mu^{\dot{\alpha}\alpha} := x^{\dot{\alpha}\alpha} + 2i\theta_i^\alpha \bar{\theta}^{\dot{\alpha}i}$$

involves the coordinates of N-extended D=4 superspace $Z^M := (x^\mu, \theta_i^\alpha, \bar{\theta}^{\dot{\alpha}i})$

and defines (1,N)-dimensional sub-superspace $\mathbb{R}^{(1|N)}$ in this N-extended superspace:

$$\hat{x}^{\dot{\alpha}\alpha} = x^{\dot{\alpha}\alpha} + \tau\, \lambda^\alpha \bar{\lambda}^{\dot{\alpha}} , \quad \hat{\theta}_i^\alpha = \theta_i^\alpha + \kappa_i \lambda^\alpha , \qquad \{(\tau, \kappa^i)\} = \mathbb{R}^{(1|N)}$$

This $\mathbb{R}^{(1|N)}$ is the STVZ(D. P. Sorokin, V.I. Tkach, D.V. Volkov, A.A. Zheltukhin) worldline superspace, the prototype of the worldvolume superspace of the ***superembedding approach*** to superbranes, used in the *spinning superparticle* framework by S.J. Gates, J. Lukierski, J. Kowalski-Glikman ...

Twistor string= string defined in the space of supertwistors.

There exist three *main* formulations of the twistor string model

- The original formulation by E. *Witten* (Dec.2003)
- The open string fromulation by N. *Berkovits* (Febr, 2004)
- The closed string formulation by W. *Siegel* (April 2004)

- Siegel actually proposed 2 formulations, but we will discuss one (in terms of Penrose but not ADHM twistors)
- The relation of twistor string model with N=2 spinning string defined in D=2+2 was considered by *Nietzke and Vafa* (Feb. 2004)
- The ‘two times physics’ treatment of the twistor string was considered by *Itzhak Bars* (July 2004)
- A new open twistor string model(s), the closed string sector of which contain Einstein gravity (and not the conformal one), was proposed recently by M. *Abou-Zaid, C.M. Hull and L.J. Mason (hep-th/0606272).*

Siegel's closed twistor string action (№1)[1]

Contains only *one* supertwistor $\Upsilon^{\Sigma} := (\Upsilon^{\hat{\alpha}}, \eta_i) = (\lambda_{\alpha}, \mu^{\dot{\alpha}}, \eta_i)$ and reads

$$\int_{W^2} e^{++} \wedge \nabla \Upsilon^{\Sigma} \, \bar{\Upsilon}_{\Sigma} + \int_{W^2} d^2\xi L_{YM\ current}$$

(super)twistor part of the action

The action "for current algebra of the Yang-Mills gauge group" ***G***

In the n-gluon MHV amplitude

$$A(1,2,...,n) = 2^{n/2} i g^{n-2} Tr(t^{a_1}...t^{a_n}) \frac{<IJ>^4}{<12>...<n1>}$$

The simplest choice for the YM current part is the action for free fermions in the fundamental representation of ***G.*** The Lagrangian including also vertex operator reads

$$d^2\xi L_{YM} + tr(\mathcal{J}\mathcal{A}) = \frac{1}{2} e^{++} \wedge (\bar{\psi}_I D\psi^I - D\bar{\psi}_I \, \psi^I)$$

$(D\psi^I = d\psi^I + \mathcal{A}^I{}_J \psi^J$ and $\mathcal{A}^I{}_J = \mathcal{A}^r T_r{}^I{}_J$ is the YM field).

Siegel's closed twistor string action (№1)²

$$\int_{W^2} e^{++} \wedge \bar{\Upsilon}_\Sigma \nabla \Upsilon^\Sigma + \int_{W^2} d^2\xi L_{YM\ current}$$

$\Upsilon^\Sigma := (\Upsilon^{\hat{\alpha}}, \eta_i) = (\lambda_\alpha, \mu^{\dot{\alpha}}, \eta_i)$ (action N2 contains another, ADHM supertwitor)

$$\nabla = d - i\mathcal{B} = e^{++}\nabla_{++} + e^{--}\nabla_{--}$$

U(1) connection, Lagrange multiplier for the constraint

Worldsheet zweibein forms

$$e^{++} \wedge e^{--} = d^2\xi\sqrt{|\gamma|}\,,$$
$$e^{++} \wedge \nabla = e^{++} \wedge e^{--}\nabla_{--}$$

$$\bar{\Upsilon}_\Sigma \Upsilon^\Sigma = \bar{\lambda}_{\dot{\alpha}}\mu^{\dot{\alpha}} - \bar{\mu}^\alpha \lambda_\alpha + 2i\bar{\eta}^i \eta_i = 0$$

on the surface of constraints

$$\int_{W^2} e^{++} \wedge \bar{\Upsilon}_\Sigma d\Upsilon^\Sigma + \int L_{YM}$$

Berkovits open twistor string action

$$\int_{W^2} e^{++} \wedge \bar{\Upsilon}^{-}_{\Sigma} \nabla \Upsilon^{-\Sigma} - \int_{W^2} e^{--} \wedge \bar{\Upsilon}^{+}_{\Sigma} \nabla \Upsilon^{+\Sigma} + \int_{W^2} d^2\xi L_{YM\ current}$$

contains ***two*** supertwistors: left-moving $\Upsilon^{+\Sigma}$ and right-moving $\Upsilon^{-\Sigma}$

$$\Upsilon^{\pm\Sigma} = (\Upsilon^{\pm\hat{\alpha}}, \eta_i^{\pm}) = (\lambda_{\alpha}^{\pm}, \mu^{\pm\dot{\alpha}}, \eta_i^{\pm})$$

These are glued by the boundary conditions on the bondary ∂W^2 of W^2

$$\Upsilon^{-\Sigma}|_{\partial W^2} = \Upsilon^{+\Sigma}|_{\partial W^2}, \qquad \bar{\Upsilon}^{-}_{\Sigma}|_{\partial W^2} = \bar{\Upsilon}^{+}_{\Sigma}|_{\partial W^2}.$$

The *b.c.* are also imposed on the left and right currents, $\mathcal{J}^{r--}|_{\partial W^2} = \mathcal{J}^{r++}|_{\partial W^2}$

On the surface of constraints $\bar{\Upsilon}^{-}_{\Sigma} \Upsilon^{-\Sigma} = 0$ and $\bar{\Upsilon}^{+}_{\Sigma} \Upsilon^{+\Sigma} = 0$

which appear as equations of motion for $\mathcal{B}_{--}$ and $\mathcal{B}_{++}$ in $\nabla = d - i\mathcal{B}$

the action becomes
$$\int e^{++} \wedge \bar{\Upsilon}^{-}_{\Sigma} d\Upsilon^{-\Sigma} - \int e^{--} \wedge \bar{\Upsilon}^{+}_{\Sigma} d\Upsilon^{+\Sigma} + \int L_{YM}$$

Original Witten's closed twistor string action

contains only one N=4 supertwistor $\Upsilon^{\Sigma} := (\Upsilon^{\hat{\alpha}}\,, \eta_i) = (\lambda_\alpha\,, \mu^{\dot{\alpha}}\,, \eta_i)$

$$S_W = \int_{W^2}\left[\nabla\bar{\Upsilon}_\Sigma \wedge *\nabla\Upsilon^\Sigma + d^2\xi\;\Xi(\xi)\left(\bar{\Upsilon}_\Sigma\Upsilon^\Sigma - r\right)\right]$$

$$= \int d\tau\wedge d\sigma\sqrt{|\gamma|}\;\left[\nabla_{++}\bar{\Upsilon}_\Sigma\,\nabla_{--}\Upsilon^\Sigma + \nabla_{--}\bar{\Upsilon}_\Sigma\,\nabla_{++}\Upsilon^\Sigma\right] + \int d^2\xi\,\Xi(\xi)\left(\bar{\Upsilon}_\Sigma\Upsilon^\Sigma - r\right)$$

U(1) connection in is expressed by $\nabla = d - i\mathcal{B}$

$$B = -\frac{i}{r}\bar{\Upsilon}_\Sigma d\Upsilon^\Sigma\,.$$

Lagrange multiplier for the constraint

$$\bar{\Upsilon}_\Sigma\Upsilon^\Sigma = r$$

This is a constrained sigma model action with tangent space $\mathbb{CP}^{(3|4)}$. It is important that this is Calabi-Yau supermanifold because of existence of an invariant integral 3+4 form

$$\Omega_{(3|4)} = \Omega_{(3|0)}\,\epsilon_{ijkl}\frac{\partial}{\partial\eta_i}\,\frac{\partial}{\partial\eta_j}\,\frac{\partial}{\partial\eta_k}\,\frac{\partial}{\partial\eta_l}\,, \qquad \Omega_{(3|0)} = \epsilon_{\alpha'\beta'\gamma'\delta'}\Upsilon^{\alpha'}d\Upsilon^{\beta'}\wedge d\Upsilon^{\gamma'}\wedge d\Upsilon^{\delta'}\,,$$

This guarantees the absence of anomaly in the `parity violating R-symmetry' which is used in topological twisting resulting in a ***topological B-model.*** To reproduce the MHV amplitudes of the Yang-Mills theory this has to be *enriched by D-instanton* contributions (that the Berkovits model seeks to avoid).

We have nothing to say about this formulation of twistor string below and begin from

Spacetime formulation of Siegels twistor closed string model

- (modulo YM part) it is given by the (N=4 version of the) twistor-like Lorentz- harmonic formulation tensionless superstring action of ***[I.B. and A.A.Zhelukhin, JETP Lett 1990, PLB 1991],***

$$S_S = \int_{W^2} \left[e^{++} \wedge \Pi^{\dot\alpha\alpha} \bar\lambda_{\dot\alpha} \lambda_\alpha + d^2\xi L_{YM\, current} \right] \;,$$

where $\Pi^{\dot\alpha\alpha} := dx^{\dot\alpha\alpha} - i d\theta^\alpha_i \bar\theta^{\dot\alpha i} + i\theta^\alpha_i d\bar\theta^{\dot\alpha i} \;,$ i=1,…,N, N=4.

The bosonic spinors λ is auxiliary. Its equations $e^{++} \wedge \Pi^{\dot\alpha\alpha}\lambda_\alpha = 0$ or $\Pi^{\dot\alpha\alpha}_{--}\lambda_\alpha = 0$ are non-dynamical and imply $\Pi^{\dot\alpha\alpha}_{--} \sim \bar\lambda^{\dot\alpha}\lambda^\alpha$ which solves the Virasoro constraints

$$\Pi^{\dot\alpha\alpha}_{--}\Pi_{--\dot\alpha\alpha} = 0$$

The action possesses *IRREDUCIBLE κ-symmetry*

$$\delta_\kappa x^{\dot\alpha\alpha} = i\delta_\kappa\theta^\alpha_i\,\bar\theta^{\dot\alpha i} - i\theta^\alpha_i\delta_\kappa\bar\theta^{\dot\alpha i} \,, \qquad \delta_\kappa\theta^\alpha_i = \kappa_i\lambda^\alpha \,, \qquad \delta_\kappa\bar\theta^{\dot\alpha i} = \bar\kappa^i\bar\lambda^{\dot\alpha} \,, \qquad \delta_\kappa\lambda^\alpha = \delta_\kappa\bar\lambda^{\dot\alpha} = \delta_\kappa e^{++} = 0 \,.$$

This is obtained from ∞-reducible one with $\delta_\kappa\theta^\alpha_i = \kappa_{\dot\alpha i}\Pi^{\dot\alpha\alpha}_{--}$ $\delta_\kappa\bar\theta^{\dot\alpha i} = \Pi^{\dot\alpha\alpha}_{--}\bar\kappa^i_\alpha$ by using and helps to reduce the # of degrees of freedom to the same **8 + 16/2** as in the twistor string.

From tensionless superstring to twistor string

$$S_S = \int_{W^2} \left[e^{++} \wedge (dx^{\dot\alpha\alpha} - id\theta_i^\alpha \bar\theta^{\dot\alpha i} + i\theta_i^\alpha d\bar\theta^{\dot\alpha i})\bar\lambda_{\dot\alpha}\lambda_\alpha + d^2\xi L_{YM} \right] \;,$$

$$S = \int_{W^2} \left[e^{++} \wedge \Pi^{\dot\alpha\alpha}\bar\lambda_{\dot\alpha}\lambda_\alpha + d^2\xi L_{YM} \right] \;,$$

the way to the standard twistor string is similar to the relation between different forms of the Ferber-Schirafuji (super)particle action.

By using Leibniz rule, $dx^{\dot\alpha\alpha}\,\lambda_\alpha = d(x^{\dot\alpha\alpha}\lambda_\alpha) - x^{\dot\alpha\alpha}d\lambda_\alpha$, etc. one finds

$$S = \int e^{++} \wedge \left(d\mu^{\dot\alpha}\,\bar\lambda_{\dot\alpha} - d\lambda_\alpha\,\bar\mu^\alpha - 2id\eta_i\,\bar\eta^i \right) + d^2\xi L_G$$

$$= \int e^{++} \wedge \bar\Upsilon_\Sigma d\Upsilon^\Sigma + d^2\xi L_G \;,$$

where

$$\mu^{\dot\alpha} = x_L^{\dot\alpha\alpha}\lambda_\alpha := (x^{\dot\alpha\alpha} + i\theta_i^\alpha\bar\theta^{\dot\alpha i})\lambda_\alpha \;, \quad \eta_i = \theta_i^\alpha\lambda_\alpha \;,$$

are the Ferber-Penrose incidence relations which provide the general solution for the constraint

$$\bar\Upsilon_\Sigma \Upsilon^\Sigma = 0 \;.$$

Twistor string as tensionless superstring

- The action of the Lorentz harmonic formulation of the tensionless superstring [I.B. & A. Zheltukhin, JETP Lett. 1990, PLB 1991]

$$S = \int_{W^2} d^2\xi \, \rho^{++m} \Pi_m^{\dot{\alpha}\alpha} \bar{v}_{\dot{\alpha}}^- v_\alpha^- \equiv \int_{W^2} e^{++} \wedge \Pi^{\dot{\alpha}\alpha} \bar{v}_{\dot{\alpha}}^- v_\alpha^- \, .$$

was written using the D=4 spinorial Lorentz harmonic variables [I.B. 1990]

$$(v_\alpha^-, v_\alpha^+) \in SL(2, \mathbf{C}) = Spin(1,3)$$

celestial sphere

i.e. a pair of spinors constrained by $v^{\alpha -} v_\alpha^+ = 1$

[Galperin, Delduc, Sokatchev, 1991, Galperin, Howe, Stelle 1991]

and thus providing a complexification of the SU(2)/U(1) harmonic variables [Galperin,Ivanov,Kalitzin, Ogievetski,Sokatchev,1984].

This is equivalent to the twistor string action $S = \int_{W^2} [e^{++} \wedge \Pi^{\dot{\alpha}\alpha} \bar{\lambda}_{\dot{\alpha}} \lambda_\alpha]$

as the measure with Lagrange multiplier may de redenoted as $e^{++} \wedge d := d^2\xi \, \rho^{++m} \partial_m$

and the constraint $v^{\alpha -} v_\alpha^+ = 1$ can be treated as imposed on v_α^+ which is absent in the action. → space of harmonics is $\frac{Spin(1,3)}{Spin(1,1) \otimes SO(2) \otimes K_2} = S^2$

Berkovits open twistor string model and tensionless superstring in an enlarged superspace

- On the surface of constraints the open string action by Berkovits reads

$$\int_{W^2} e^{++} \wedge \bar{\Upsilon}^{-}_{\Sigma} d\Upsilon^{-\Sigma} - \int_{W^2} e^{--} \wedge \bar{\Upsilon}^{+}_{\Sigma} d\Upsilon^{+\Sigma} + \int_{W^2} d^2\xi L_{YM}$$

- Solving the constraints

$$\bar{\Upsilon}^{-}_{\Sigma}\Upsilon^{-\Sigma} = \bar{\lambda}^{-}_{\dot{\alpha}}\mu^{-\dot{\alpha}} - \bar{\mu}^{-\alpha}\lambda^{-}_{\alpha} + 2i\bar{\eta}^{-i}\eta^{-}_{i} = 0$$

$$\bar{\Upsilon}^{+}_{\Sigma}\Upsilon^{+\Sigma} = \bar{\lambda}^{+}_{\dot{\alpha}}\mu^{+\dot{\alpha}} - \bar{\mu}^{+\alpha}\lambda^{+}_{\alpha} + 2i\bar{\eta}^{+i}\eta^{+}_{i} = 0$$

by

$$\mu^{-\dot{\alpha}} = x^{\dot{\alpha}\alpha}_{(l)L}\lambda^{-}_{\alpha} := (x^{\dot{\alpha}\alpha}_{(l)} + i\theta_{(l)}{}^{\alpha}_{i}\bar{\theta}^{\dot{\alpha}i}_{(l)})\lambda^{-}_{\alpha}\ , \quad \eta^{-}_{i} = \theta_{(l)}{}^{\alpha}_{i}\lambda^{-}_{\alpha}\ ;$$

$$\mu^{+\dot{\alpha}} = x^{\dot{\alpha}\alpha}_{(r)L}\lambda^{+}_{\alpha} := (x^{\dot{\alpha}\alpha}_{(r)} + i\theta_{(r)}{}^{\alpha}_{i}\bar{\theta}^{\dot{\alpha}i}_{(r)})\lambda^{+}_{\alpha}\ , \quad \eta^{+}_{i} = \theta_{(r)}{}^{\alpha}_{i}\lambda^{+}_{\alpha}\ ,$$

one trasnforms the Berkovits action into the action for open tensionless supersting in the direct product of two copies of the D=4 N=4 superspace

$$S = \int_{W^2} (e^{++} \wedge \hat{\Pi}^{\dot{\alpha}\alpha}_{(l)}\bar{\lambda}^{-}_{\dot{\alpha}}\lambda^{-}_{\alpha} - e^{--} \wedge \hat{\Pi}^{\dot{\alpha}\alpha}_{(r)}\bar{\lambda}^{+}_{\dot{\alpha}}\lambda^{+}_{\alpha}) + \int_{W^2} d^2\xi L_G\ ,$$

Thus the Berkovits open twistor string model is a tensionless superstring in a direct product of two copies of the D=4, N=4 superspace.

$$S = \int_{W^2} (e^{++} \wedge \hat{\Pi}^{\dot\alpha\alpha}_{(l)} \bar\lambda^{-}_{\dot\alpha} \lambda^{-}_{\alpha} - e^{--} \wedge \hat{\Pi}^{\dot\alpha\alpha}_{(r)} \bar\lambda^{+}_{\dot\alpha} \lambda^{+}_{\alpha}) + \int_{W^2} d^2\xi L_G ,$$

where

$$\Pi^{\dot\alpha\alpha}_{(l)} := dx^{\dot\alpha\alpha}_{(l)} - i d\theta_{(l)}{}^{\alpha}_{i} \bar\theta^{\dot\alpha i}_{(l)} + i \theta_{(l)}{}^{\alpha}_{i} d\bar\theta^{\dot\alpha i}_{(l)} ,$$

$$\Pi^{\dot\alpha\alpha}_{(r)} := dx^{\dot\alpha\alpha}_{(r)} - i d\theta_{(r)}{}^{\alpha}_{i} \bar\theta^{\dot\alpha i}_{(r)} + i \theta_{(r)}{}^{\alpha}_{i} d\bar\theta^{\dot\alpha i}_{(r)} .$$

Superspaces like that were first used in [E. Ivanov + A. Isaev, 88]

The boundary conditions $\Upsilon^{-\Sigma}|_{\partial W^2} = \Upsilon^{+\Sigma}|_{\partial W^2}$, $\bar\Upsilon^{-}_{\Sigma}|_{\partial W^2} = \bar\Upsilon^{+}_{\Sigma}|_{\partial W^2}$.

imposed on the twistors, identify the coordinate functions of these two superspaces up to the local symmetry transformations, the set of which includes two copies of the kappa-symmetry,

$$\delta_\kappa x^{\dot\alpha\alpha}_{(l)} = i\delta_\kappa \theta_{(l)}{}^{\alpha}_{i}\, \bar\theta^{\dot\alpha i}_{(l)} - i\theta_{(l)}{}^{\alpha}_{i} \delta_\kappa \bar\theta^{\dot\alpha i}_{(l)} , \quad \delta_\kappa \theta_{(l)}{}^{\alpha}_{i} = \kappa^{+}_{i} \lambda^{-\alpha} , \quad \delta_\kappa \bar\theta^{\dot\alpha i}_{(l)} = \bar\kappa^{+i} \bar\lambda^{-\dot\alpha} ,$$

$$\delta_\kappa x^{\dot\alpha\alpha}_{(r)} = i\delta_\kappa \theta_{(r)}{}^{\alpha}_{i}\, \bar\theta^{\dot\alpha i}_{(r)} - i\theta_{(r)}{}^{\alpha}_{i} \delta_\kappa \bar\theta^{\dot\alpha i}_{(r)} , \quad \delta_\kappa \theta_{(r)}{}^{\alpha}_{i} = \kappa^{-}_{i} \lambda^{+\alpha} , \quad \delta_\kappa \bar\theta^{\dot\alpha i}_{(r)} = \bar\kappa^{-i} \bar\lambda^{+\dot\alpha} ,$$

$$\delta_\kappa \lambda^{\pm\alpha} = \delta_\kappa \bar\lambda^{\pm\dot\alpha} = \delta_\kappa e^{\pm\pm} = 0 .$$

Open twistor string= open null-superstring in $\Sigma^{(4|4N)} \otimes \Sigma^{(4|4N)}$, $N = 4$

Open null superstring in $\Sigma^{(4+6|4)} = (x^\mu, y^{[\mu\nu]}, \theta^\alpha)$ Was studied in [Bengston & Zheltukhin 03]

Twistor string is a tensionless string²

- The fact that the action [e.g. by Siegel] $S = \int_{W^2} \left[e^{++} \wedge \Pi^{\dot{\alpha}\alpha} \bar{\lambda}_{\dot{\alpha}} \lambda_{\alpha} \right] \,,$

[equivalent to the one from I.B. & A. Zheltukhin, JETP Lett. 1990, PLB 1991], describes a ***tensionless*** object can be understood by observing its ***conformal invariance*** and, hence, the absence of a dimensional parameter (like mass or string tension).

The tesnionless nature of twistor string was noticed by Siegel (2004), who also rised the question about its tensionful prototype.

This should exist because the intrinsically tensionless string (=null-string) mass spectrum is known to be continuous [I.B. & A.A. Zheltukhin, 1989, Fort.Fis. 1993],

while the quantum state spectrum of the twistor string is assumed to include the massless fields (gauge fields).

The ***mass equal zero*** and the ***continous mass spectrum*** provide two alternative possibility to have a *conformally invariant mass spectrum*,

i.e. to have a mass spectrum which is not described by a dimensionful parameter. ⇘

Twistor string is a tensionless limit of some superstring

On a possible parent tensionful superstring for the twistor string

N=1,2

Siegel proposed that it's bosonic part is given by the so-called *QCD string* [Siegel 94], but the issues of fermions and of the κ-symmetry were not addressed in [Siegel 2004].

If we were interested in N=1 or N=2 counterparts of the twistor string, which are equivalent to N=1 and N=2 D=4 tensionless superstring of [I.B. & A.A. Zheltukhin 90], the natural answer is given by the D=4, N=1,2 Green-Schwarz superstring action in the twistor-like Lorenz-harmonic formulation of [I.B. & A.A. Zheltukhin 1991, PLB 1992],

$$S = \frac{1}{4\pi\alpha'} \int_{W^2} [e^{++} \wedge \Pi^{\dot{\alpha}\alpha} \bar{v}^-_{\dot{\alpha}} v^-_{\alpha} - e^{--} \wedge \Pi^{\dot{\alpha}\alpha} \bar{v}^+_{\dot{\alpha}} v^+_{\alpha} - e^{++} \wedge e^{--}] - \frac{1}{4\pi\alpha'} \int_{W^2} \hat{B}_2$$

where $v^{\alpha\pm}$ constrained by $v^{\alpha -} v^+_{\alpha} = 1$ are our old friend, the Lorentz harmonics, which now are both present and, hence, parametrize a noncompact coset $\frac{Spin(1,3)}{Spin(1,1)\otimes SO(2)}$

The Wess-Zumino term $\int B$ has its usual form of the pull-back of the 'flat superspace value' of the NS-NS two from superfiled B characterized by the field strength constraint

$$H_3 = dB_2 = -2i\Pi^a \wedge (d\theta^1 \wedge \sigma_a d\bar{\theta}^1 - d\theta^1 \wedge \sigma_a d\bar{\theta}^2) \qquad for\ N = 2\ ,$$

$$and \qquad H_3 = dB_2 = -2i\Pi^a \wedge d\theta \wedge \sigma_a d\bar{\theta} \qquad for\ \ N = 1\ .$$

On a possible parent tensionful superstring for the twistor string²

To derive the N=1,2 counterpart of twistor string from these N=1,2 D=4 superstring actions

$$S = \frac{1}{4\pi\alpha'}\int_{W^2}[e^{++}\wedge\Pi^{\dot\alpha\alpha}\bar v^{-}_{\dot\alpha}v^{-}_{\alpha} - e^{--}\wedge\Pi^{\dot\alpha\alpha}\bar v^{+}_{\dot\alpha}v^{+}_{\alpha} - e^{++}\wedge e^{--}] - \frac{1}{4\pi\alpha'}\int_{W^2}\hat{B}_2$$

one should take the tensionless limit $\alpha' \mapsto \infty$ while keeping e^{++}/α' finite, i.e., before setting $\alpha' \mapsto \infty$ one redefines $e^{++} \to 4\pi\alpha' e^{++}$, $e^{--} \to e^{--}/(4\pi\alpha')$.

The problem with twistor string is that it is classically equivalent to N=4 tensionless superstring and in N=4 D=4 superspace one cannot write the Wess- Zumino term $\int \boldsymbol{B}$ because the constraints for ***H=dB*** which would provide the ***κ***-symmetry is not known.

Moreover, it is known that such a constraint for N=4 H=dB does not exist [according to classification of the *Chevalley-Eilenberg cocycles* over superspaces (more exactly, on Lie superalgebras) [J.A. de Azcárraga and P.K. Townsend 1989]]

except for if one ***also enlarge the number of bosonic directions in superspace:***

$$N = 1,2 \mapsto N = 4 \quad \oplus \quad x^{\mu} \mapsto (x^{\mu}, \ldots ?)$$

On a possible parent tensionful superstring for the twistor string³

Indeed, the wanted *Chevalley-Eilenberg cocycle H=dB* (closed form which is exact in de Rahm cogomologies but is not in Cevalley-Eilenberg one as the B cannot be constructed from products of invariant forms - Cartan forms for the flat superspaces) *does exist in D=10, N=1 superspace,* which has the same number of fermionic coordinates as the D=4 N=4 superspace, but 6 additional bosonic coordinates,

$$D = 10\ ,\ N = 1\ :\qquad H_3 = dB_2 = -2i\Pi^{\underline{a}} \wedge d\Theta \wedge \Sigma_{\underline{a}} d\Theta\ .$$

where

$$\Pi^{\underline{a}} = dX^{\underline{a}} - id\Theta \wedge \Gamma^{\underline{a}} d\Theta\ , \quad X^{\underline{a}} = (x^a\ ,\ X^I)\ , \qquad \Theta^{\underline{\alpha}} = \begin{pmatrix} \theta^\alpha_i \\ \bar{\theta}^i_{\dot\alpha} \end{pmatrix} \equiv \begin{pmatrix} \theta^\alpha_i \\ (\theta_{\alpha\, i})^* \end{pmatrix}\ , \quad ,$$

$$\Sigma^{\underline{a}}_{\alpha\beta} = (\Sigma^a_{\alpha\beta}\ , \Sigma^I_{\alpha\beta})\ , \quad \Sigma^a_{\alpha\beta} = antidiag\left(\sigma^a_{\alpha\dot\beta}\delta^i_j\ , \sigma^{a\dot\alpha\beta}\delta^j_i\right)\ , \quad \ldots\ .$$

This superspace allows for existence of the heterotic superstring which, when the heterotic fermions (bosons) are neglected, can be described by the following Lorentz harmonic action

$$S = \frac{1}{4\pi\alpha'}\int_{W^2} [e^{++} \wedge \Pi^{\underline{a}} u_{\underline{a}}{}^{--} - e^{--} \wedge \Pi^{\underline{a}} u_{\underline{a}}{}^{++} - e^{++} \wedge e^{--}] - \frac{1}{4\pi\alpha'}\int_{W^2} \hat{B}_2$$

[I.B. + A. Zheltukhin, 1991, PLB 1992] [GIKOS, 1984 – SU(2)/U(1) harmonics for N=2.

E. Sokatchev, 1986 ‘Vector’ (‘light-cone′) harmonics; E.Nissimov, S. Pacheva, S. Solomon, 1987 Mixed Lorentz harmonics in superstring quantization, R. Kallosh, M. Rahmanov, 1987,…]

$$S=\frac{1}{4\pi\alpha'}\int_{W^2}[e^{++}\wedge\Pi^{\underline{a}}u_{\underline{a}}{}^{--}-e^{--}\wedge\Pi^{\underline{a}}u_{\underline{a}}{}^{++}-e^{++}\wedge e^{--}]-\frac{1}{4\pi\alpha'}\int_{W^2}\hat{B}_2$$

One can consider them as auxuliary light-like vectors 'normalized' on 2

analogos of $v_\alpha^- \bar{v}_{\dot\alpha}^-$ and $v_\alpha^+ \bar{v}_{\dot\alpha}^+$ of the D=4 action.

But, actually, these vectors u°° belong to the set of vector harmonics $U=(u_{\underline{a}}^{\pm\pm}, u_{\underline{a}}^i) \in SO(1,D\text{-}1)$ and, hence, can be expressed through the spinor harmonics, $U=(v_{\underline{\alpha}q}^-, v_{\underline{\alpha}\dot q}^+) \in Spin(1,D\text{-}1)$ highely constrained sets of spinors for D=10 (11, …)

$$u^{\underline{a}--}u_{\underline{a}}{}^{--}=0\,,$$
$$u^{\underline{a}++}u_{\underline{a}}{}^{++}=0\,,$$
$$u^{\underline{a}--}u_{\underline{a}}{}^{++}=2\,.$$

$$u_{\underline{a}}^{--}\Gamma^{\underline{a}}_{\underline{\alpha\beta}}=2v_{\underline{\alpha}q}^{-}v_{\underline{\beta}q}^{-}\,,\qquad u_{\underline{a}}^{++}\Gamma^{\underline{a}}_{\underline{\alpha\beta}}=2v_{\underline{\alpha}\dot q}^{+}v_{\underline{\beta}\dot q}^{+}\,,$$

$q=1,\dots,8,\ \dot q=1,\dots,8,\ \alpha=1,\dots,16$

This action is the p=1 representative of the twistor-like Lorentz harmonic formulations for super-p-branes [I.B. + A. Zheltukhin 1992-94]

$$S_{super-p-brane}=\int_{W^{p+1}}[\tfrac{1}{p!}\epsilon_{aa_1\dots a_p}\Pi^{\underline{b}}u_{\underline{b}}{}^{a}\wedge e^{a_1}\wedge\dots e^{a_p}-\tfrac{1}{(p+1)!}\epsilon_{a_0a_1\dots a_p}e^{a_0}\wedge\dots e^{a_p}-\int_{W^2}\hat{C}_{p+1}$$

where $U=(u_{\underline{b}}^a, u_{\underline{b}}^i) \in SO(1,D\text{-}1)$ are homogeneous coordinate of the coset $\frac{SO(1,D-1)}{SO(1,p)\otimes SO(D-p-1)}$

and, hence, can be expressed (by $U^a\Gamma=V\Gamma^aV$) through the homogeneous coordinates of the double covering coset of the Spin groups $\frac{Spin(1,D-1)}{Spin(1,p)\otimes Spin(D-p-1)}$, the spinor Lorentz harmonics

$$U=(v_{\underline{\alpha}q}^-, v_{\underline{\alpha}\dot q}^+) \in Spin(1,D\text{-}1)$$

TWISTOR STRING from D=10 G-S SUPERSTRING D=10 tensionless superstring and dimensional reduction

Taking the $\alpha' \mapsto \infty$ after the $e^{++} \to 4\pi\alpha' e^{++}$, $e^{--} \to e^{--}/(4\pi\alpha')$ redefinition in

$$S = \frac{1}{4\pi\alpha'}\int_{W^2}[e^{++}\wedge\Pi^{\underline{a}}u_{\underline{a}}{}^{--} - e^{--}\wedge\Pi^{\underline{a}}u_{\underline{a}}{}^{++} - e^{++}\wedge e^{--}] - \frac{1}{4\pi\alpha'}\int_{W^2}\hat{B}_2$$

we first arrive at D=10 tensionless superstring action

$$S = \int_{W^2} e^{++}\wedge \Pi^{\underline{a}}u_{\underline{a}}^{--}\ , \qquad u_{\underline{a}}^{--}u^{\underline{a}\,--} = 0\ ,$$

which involves only u^{--}. When u is treated as Lorentz harmonic, U=(u^{--}, …) ∈ SO(1,D-1), these are now homogeneous coordinate of the compact coset $\frac{SO(1,D-1)}{[SO(1,1)\otimes SO(8)] \otimes K_8} = S^8$.

A dimensional reduction of such an action can be done in such a manner that the D=4, N=4 null superstring appears: just consider a Lorentz frame where

$$u_{\underline{a}}^{--} = \delta_{\underline{a}}^{\,b}u_b^{--} = (u_a^{--}, 0, \ldots, 0)\ , \quad u_a^{--}u^{a--} = 0\ .$$

Tensorial superspace versus standard D=10 superspace.

- Discussing above the D=10, N=1 Green-Schwarz superstring as a tensionful parent of the twistor string, we have allowed ourselves to enlarge D=4 superspace by six additional bosonic coordinates.
- It is not clear at present whether this enlargement is unique,

even if we restrict ourselves to just six additional bosonic coordinates, these do not need being the components of the SO(6) vector X^I , but instead may be component of antisymmetric tensor $Y^{\mu\nu} = -Y^{\nu\mu}$, or equivalently,

$$x^{\dot\alpha\dot\beta} \propto \epsilon^{\dot\beta\dot\gamma} Y^{\mu\nu} \tilde\sigma_{\mu\nu}{}^{\dot\alpha}{}_{\dot\gamma}\,, \qquad x^{\alpha\beta} \propto \epsilon^{\beta\gamma} Y^{\mu\nu} \sigma_{\mu\nu\gamma}{}^{\dot\alpha}$$

- The action for a superstring in (N=4 extended) tensorial superspace reads

[I.B. + J. de Azcárraga + M. Picón +O. Varela, PRD2004] $(X^{\hat\alpha\hat\beta} = (x^{\alpha\dot\beta}, x^{\alpha\beta}, x^{\dot\alpha\dot\beta}))$

$$S_{\Sigma(10|4N)} = \frac{1}{4\pi\alpha'} \int_{W^2} \left(e^{++} \wedge \Pi^{\hat\alpha\hat\beta} \Lambda^-_{\hat\alpha} \Lambda^-_{\hat\beta} + e^{++} \wedge \Pi^{\hat\alpha\hat\beta} \Lambda^-_{\hat\alpha} \Lambda^-_{\hat\beta} - e^{++} \wedge e^{--} (\epsilon^{\hat\alpha\hat\beta} \Lambda^+_{\hat\alpha} \Lambda^-_{\hat\beta})^2 \right)\,,$$

$$\Pi^{\hat\alpha\hat\beta} = dX^{\hat\alpha\hat\beta} - 2id\Theta^{(\hat\alpha|i}\Theta^{|\hat\beta)i}\,, \qquad \Theta^{\hat\alpha i} := (\theta^\alpha_i\,,\,\bar\theta^{\dot\alpha i})\,, \quad \Lambda^\pm_{\hat\alpha} := (\lambda^\pm_\alpha, \bar\lambda^\pm_{\dot\alpha})\,.$$

The tensionless limit of this superstring

$$e^{++} \to 4\pi\alpha' e^{++}, \; e^{--} \to e^{--}/(4\pi\alpha')\,, \quad \alpha' \mapsto \infty,$$

reads

$$S_{Sp(4|4)} = \frac{1}{2}\int_{W^2} e^{++} \wedge \Pi^{\hat\alpha\hat\beta}\Lambda_{\hat\alpha}\Lambda_{\hat\beta}\,, \qquad \Lambda_{\hat\beta} := \Lambda^-_{\hat\beta}$$

- This is the w=1 representative of the family of the D=4, N=1 null superstring actions in tensorial superspace of which the Berkovits-Siegel twistor string is the w=0 case.

$$S(w) \;=\; \int_{W^2} e^{++} \wedge \left(\Pi^{\dot\alpha\alpha}\lambda_\alpha\bar\lambda_{\dot\alpha} + \frac{w}{2}\,\Pi^{\alpha\beta}\lambda_\alpha\lambda_\beta + \frac{\bar w}{2}\,\Pi^{\dot\alpha\dot\beta}\bar\lambda_{\dot\alpha}\bar\lambda_{\dot\beta}\right)\,,$$

$$\Pi^{\dot\alpha\alpha} \;=\; dx^{\dot\alpha\alpha} - id\theta^\alpha_i\bar\theta^{\dot\alpha i} + i\theta^\alpha_i d\bar\theta^{\dot\alpha i}\,, \qquad \Pi^{\alpha\beta} = dx^{\alpha\beta} - 2id\theta_i^{(\alpha}\theta_i^{\beta)} = (\Pi^{\dot\alpha\dot\beta})^*\,.$$

- For w=1 this action was first considered in [Zheltukhin & Uvarov 2002]. For general *w*, this is the an extended object counterpart of the superparticle action of [I.B.+J,Lukierski +D.Sorokin 1999] whose quantization results in a *free conformal higher spin theories.*
-The case w=1 is special [I.B. + J.Lukierski 98]): there the model possess additional κ-symmetry describes the excitation over BPS states preserving 3/4 SUSYs; for N=1 it would be all but one SUSYs, and the BPS states can be identified with D=4, N=1 BPS preon [I.B. + J.A. de Azcárraga + J.M. Izquerdo + J.Lukierski, PRL 2001].
- Thus a dimensional reduction with deformation moving from w=1 to w≠1 looks less singular as preserving the number of local symmetries.

Thus twistor string is an element of a family of tensionless super-strings in tensorial superspace.

Conclusions

- Twistor string (in its Siegel's closed string version Siegel 2004) is classically equivalent to the N=4 null-superstring in its twistor-like Lorentz harmonic formulation of [I.B. & A.A. Zheltukhin 1991-92].
- It is not intrinsically tensionless superstring but (as noticed by Siegel) a tensionless limit of some superstring.
- What is the tensionful prototype of the twistor string is not clear now, but it should be a model in superspace enlarged by some number of additional coordinates.
- The list of natural candidates include D=10 Green-Schwarz superstring (N=1 closed, the *heterotic string without(?) heterotic fermions /bosons*) as well as superstring models in tensorial superspace, the D=4, N=4 superspace enlarged by 6 tensorial coordinates.
- The Berkovits open twistor string model can be described as the open superstring moving (again) in an enlarged superspace which is given by direct product of two copies of D=4, N=4 superspaces.

Some open problems/questions

- Does the twistor string lead us outside of the standard superspaces?
- Does the N=1,2 counterparts of the twistor string make sense (like it do the N=1,2 and N=0 versions of the MHV diagramme technique).
- What one can say about original Witten's formulation?
- What is the spacetime forms of the new twistor string models by C.Hull, Abou-Zaid & Mason?
- As tensionless limit of the supersting is believed now to describe the higher spin theories (see e.g. Sagnotti+Tsulaia, Bonelli), what is the relation of quantization leading to higher spin and the one leading to the SYM (the twistor string)?
- Does it make sence the higher-dimensional generalizations of twistor string?

In particular, whether the above discussed D=10 null superstring action in Lorentz harmonic formalism,

$$S = \int_{W^2} e^{++} \wedge \Pi^{\underline{a}} u_{\underline{a}}^{--} \, , \qquad u_{\underline{a}}^{--} u^{\underline{a}\,--} = 0 \, \ldots .$$

is related with D=10 SYM? ...

Thank you for your attention!

A natural condidate for D=10 twistor string is the above D=10 null superstring

$$S = \int_{W^2} e^{++} \wedge \Pi^{\underline{a}} u_{\underline{a}}^{--} \ , \qquad u_{\underline{a}}^{--} u^{\underline{a}\,--} = 0 \ .$$

To see this one has to `extract the square root' of the light-like vector u^{--} by treating it as a part of moving frame ($u^{--} = u^{0} - u^{1}$), i.e. of the SO(1,9) valued matrix.
Then the double covering of this matrix, the 16 × 16 Spin(1,9) valued matrix, provides us with the spinor moving frame.
The double covering is defined by the condition of the invariance of gamma-matrices Γ, or sigma-matrices Σ, $\boldsymbol{u \cdot \Sigma = V \Sigma V}$ which, in particular, includes $\boldsymbol{u^{--} \cdot \Sigma = V \Sigma^{--} V}$.
This relation involve only one 16 × 8 block, v^{-}, of the matrix V and reads

$$2 v^{-}_{\underline{\alpha} q} v^{-}_{\underline{\beta} q} = u_{\underline{a}}^{--} \Sigma^{\underline{a}}_{\underline{\alpha\beta}},$$

another relation between the same variables reads

$$v^{-}_{p} \tilde{\Sigma}_{\underline{a}} v^{-}_{q} = \delta_{pq} u_{\underline{a}}^{--}$$

Thus $\boldsymbol{u}^{--}$ can be expressed through 8 higherly consrained spinors (spinorial harmonics) Parametrizing the conset of Lorentz group isomorphic to the (selestial) sphere, The action $S=\int_{W^2} e^{++}\wedge \Pi^{\underline{a}} u_{\underline{a}}^{--}$, $u_{\underline{a}}^{--}u^{\underline{a}\,--}=0$.

can thus be equivalently written as

$$S = \frac{1}{8}\int\limits_{W^2} e^{++}\wedge \Pi^{\underline{a}}\, \tilde{\Sigma}_{\underline{a}}^{\underline{\alpha\beta}}\, v_{\underline{\alpha}q}^{-} v_{\underline{\beta}q}^{-} \; , \qquad \Pi^{\underline{a}} = dX^{\underline{a}} - id\Theta\Sigma^{\underline{a}}\theta \; ,$$

This is a clear D=10 counterpart of the D=4 'spacetime' (N=4 superspace) presentation of the twistor string action, but written in terms of higherly constrained spinors, the *spinorial Lorentz harmonics* which parameterize the D=10 celestial sphere,

$$\{v_{\alpha p}^{-}\} = \frac{Spin(1,D-1)}{[Spin(1,1)\otimes Spin(8)] \subset\!\!\!\!\times K_8} = S^8 \; ,$$

The Ferber'Schirafuji transformations (using the Leibnitz rules) results in the foolowing Equivalent form of these tensionless superstring action,

$$S = \int\limits_{W^2} e^{++}\wedge (d\mu_q^{-\underline{\alpha}} v_{\underline{\alpha}q}^{-} - \mu_q^{-\underline{\alpha}} dv_{\underline{\alpha}q}^{-} - id\chi_q^{-}\chi_q^{-}) \; ,$$

$$S = \int_{W^2} e^{++} \wedge (d\mu_q^{-\underline{\alpha}} v_{\underline{\alpha}q}^{-} - \mu_q^{-\underline{\alpha}} dv_{\underline{\alpha}q}^{-} - id\chi_q^{-}\chi_q^{-}) ,$$

where

$$\mu_q^{-\underline{\alpha}} = X^{\underline{a}} \tilde{\Sigma}_{\underline{a}}^{\underline{\alpha\beta}} v_{\underline{\beta}q}^{-} - \frac{i}{2} \Theta^{\underline{\alpha}} \Theta v_q^{-} ,$$

$$\chi_q^{-} = \Theta^{\underline{\alpha}} v_{\underline{\alpha}q}^{-} .$$

These solves the constraints

$$\mu_{[q}^{-\underline{\alpha}} v_{\underline{\alpha}p]}^{-} - \tfrac{i}{2} \chi_q^{-} \chi_p^{-} = 0 ,$$

$$\mu_{(q}^{-\underline{\alpha}} v_{\underline{\alpha}p)}^{-} - \tfrac{1}{8} \delta_{qp} \mu_{p'}^{-\underline{\alpha}} v_{\underline{\alpha}p'}^{-} = 0 .$$

The complete set of constraints includes as well

$$2 v_{\underline{\alpha}q}^{-} v_{\underline{\beta}q}^{-} = u_{\underline{a}}^{--} \Sigma^{\underline{a}}_{\underline{\alpha\beta}} , \qquad v_p^{-} \tilde{\Sigma}_{\underline{a}} v_q^{-} = \delta_{pq} u_{\underline{a}}^{--}$$

Twistors in D=10,11 are higherly constrained variables.

This was also noticed in recent [I. Bars & M. Picon, 2005] in the context of two-time physics [I. Bars at al 98-2006]

Let me notice that the second order form of this action

$$S_{super-p-brane} = \frac{p}{(p+1)!}\int_{W^{p+1}} \epsilon_{a_0 a_1 \ldots a_p} E^{\underline{b}_0} u_{\underline{b}_0}{}^{a_0} \wedge e^{a_1} \wedge \ldots E^{\underline{b}_p} u_{\underline{b}_p}{}^{a_p} - \int_{W^2} \hat{C}_{p+1}$$

which is obtained from the original [I.B. + A. Zh 92-94] action

$$S_{super-p-brane} = \int_{W^{p+1}} [\frac{1}{p!}\epsilon_{a a_1 \ldots a_p} E^{\underline{b}_0} u_{\underline{b}_0}{}^{a} \wedge e^{a_1} \wedge \ldots e^{a_p} - \frac{1}{(p+1)!}\epsilon_{a_0 a_1 \ldots a_p} e^{a_0} \wedge \ldots e^{a_p} - \int_{W^2} \hat{C}_{p+1}$$

on the 'surface' of equations of motion for e^c, namely $E^{\underline{b}} u_{\underline{b}}{}^{a} = e^a$, were discussed e.g.

in I.B. hep-th/9807202=Lect.Not.Phys. 1999, and in I.B. +D.Sorokin+ M.Tonin, NPB97 (for Dp-branes, where the first order form is too complicated); very recently its gauge fixed (with respect to H=SO(1,p) × SO(D-p-1)) form, with an explicit parametrization of the coset SO(1,D-1)/H by unconstrained Goldstone fields was considered in hep-th/0607057, hep-th/0608104 by Joaquim Gomis, Peter West and K.Kamimura.

The 'twistor-like Loretz harminic' action of [I.B. + A. Zheltukhin 92-94],

$$S_{super-p-brane} = \int_{W^{p+1}} [\frac{1}{p!}\epsilon_{a a_1 \ldots a_p} E^{\underline{b}_0} u_{\underline{b}_0}{}^{a} \wedge e^{a_1} \wedge \ldots e^{a_p} - \frac{1}{(p+1)!}\epsilon_{a_0 a_1 \ldots a_p} e^{a_0} \wedge \ldots e^{a_p} - \int_{W^2} \hat{C}_{p+1}$$

Was used in [I.B. + D. Sorokin + D. V. Volkov Phys.Lett. 1995] as a basis of the generalized action principle of the superembedding approach.