Seven-branes in Type IIB String Theory

Jelle Hartong

University of Groningen

In collaboration with Eric A. Bergshoeff, Tomás Ortín and Diederik Roest

The D7 brane

- The world-volume action for a D7 brane in Einstein frame:

$$pe^{\phi}\sqrt{-g_8} + pC_8$$

  where C_8 is the dual potential of the axion and $p \in \mathbb{Z}$. (All world-volume fields are put to zero as well as the 2-forms)
- It can generally be proven that from the Wess-Zumino term it follows that $\tau = \chi + ie^{-\phi}$ has the following monodromy $\tau \to \tau + p$.
- The moduli space of the axion is S^1: identify τ and $T\tau \equiv \tau + 1$.
- The transverse space to the D7 brane is two-dimensional. The total monodromy of τ has to be trivial.
- $\tau = \frac{p}{2\pi i} \log(z - z_{i\infty})$.

We have to use S-duality

- To realize that the total monodromy is trivial we need to add another brane to the solution.
- We cannot add an anti-D7 brane for this would break supersymmetry.
- We are forced to make use of the S-duality of type IIB in order to construct solutions whose total monodromy is trivial.
- We must therefore consider solutions in which τ transforms under $PSL(2,\mathbb{Z})$ (generated by T and S) or possibly some subgroup.

- Existential problem: type IIB string theory is conjectured to be $SL(2,\mathbb{Z})$ duality invariant but the actual theory is not known.
- Question: what is the meaning of a supergravity solution whose monodromy group is $\subseteq PSL(2,\mathbb{Z})$?
- We must take a step in the dark.
- In some cases one can a posteriori justify the approach. This is done by setting τ equal to some other τ which *is* understood. Example WV theories on D3-brane probes which have τ as a coupling constant.

Short historical overview

- 7-brane solutions so far:
  [Greene, Shapere, Vafa, Yau, Nucl. Phys. B337 (1990)], [Gibbons, Green, Perry, hep-th/9511080], [Sen, hep-th/9605150]
  - Global description
  - susy via uplift over $SU(2)$ holonomy surface
  - all objects have monodromy Λ with Tr$\Lambda = 2$

  [Bergshoeff, Gran, Roest, hep-th/0203202], [Bergshoeff, de Roo, Kerstan, Ortín, Riccioni, hep-th/0601128]
  - Local description
  - susy via 10d Killing spinor
  - objects with monodromies Tr$\Lambda <, =, > 2$
- There is disagreement
- Globally well-defined 7-brane solutions have never been studied from the $d = 10$ point of view.

Source terms

$$S = \int d^{10}x\sqrt{-g}\left(R - \frac{\partial_\mu\tau\partial^\mu\bar\tau}{2\,(\mathrm{Im}\tau)^2} - \int_\Sigma d^8\sigma\sqrt{-g_{(8)}}\frac{\delta(x-X(\sigma))}{\sqrt{-g}}\frac{1}{\mathrm{Im}\tau}\left(p + q|\tau|^2 + r\frac{\tau+\bar\tau}{2}\right)\right)$$

$$Q = \begin{pmatrix} \frac{r}{2} & p \\ -q & -\frac{r}{2} \end{pmatrix}$$

- $SL(2,\mathbb{R})$-invariant tension
- No Wess-Zumino term
- Scalar d.o.f. in the Nambu-Goto term are dual to the 8-form d.o.f. in the Wess-Zumino term (special for 7-branes)
- To obtain this action one can use the PST formalism in which the scalars and 8-forms appear as independent fields. PST applied to the bulk type IIB action: [Dall'Agata, Lechner, Tonin, hep-th/9806140]
- $\det Q = \begin{cases} = 0 & \text{D7} \\ > 0 & \text{new} \\ < 0 & \text{excluded} \end{cases}$

The functions τ and f

$$ds^2 = -dt^2 + d\vec{x}_7^2 + (\text{Im}\,\tau)|f|^2 dz d\bar{z}\,,$$

$$\tau = \tau(z)\,, \quad f = f(z)\,,$$

$$\epsilon = \left(\frac{f}{\bar{f}}\right)^{1/4} \epsilon_0\,,$$

- τ and f are analytic functions and completely determine the solution
- Holonomy of Killing spinor ϵ = monodromy of f:

$$f \to e^{-2\pi k i}(c\tau + d) f$$

- Spin structure: $k = 0, 1$. This talk $k = 0$.
- $f = F(\tau) h(z)$

$$F \to \delta(a,b,c,d)(c\tau + d) F \quad \text{whenever} \quad \tau \to \frac{a\tau + b}{c\tau + d}$$

- Monodromy of h such as to cancel the roots of unity $\delta(a,b,c,d)$.

Constructing solutions

- From source terms in the τ e.o.m. it follows that τ agrees with local expansions of modular functions.
- The source terms in the Einstein e.o.m. determine the order of the zeros/poles of f.
- Any solution is constructed as follows:
  1. Pick a monodromy group, e.g. $\Gamma = PSL(2,\mathbb{Z})$ or $\Gamma_0(2)$, etc.
  2. Construct its fundamental domain. Associate objects with the orbifold points.
  3. Find a 1-1 automorphic mapping of this fundamental domain to the transverse space of 7-branes. Gives τ.
  4. Find a cusp form $F(\tau)$ and cancel its roots of unity $\delta(a,b,c,d)$ by $h(z)$. Gives f.

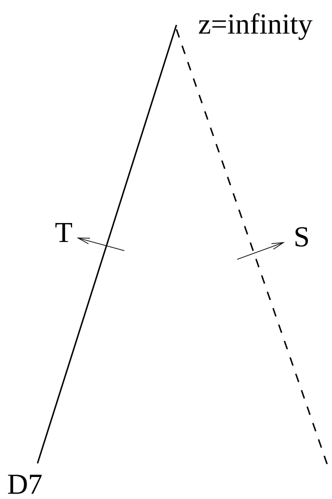
Examples

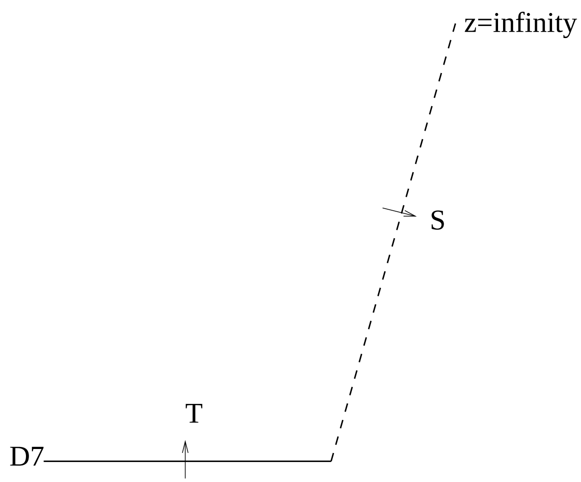
$$j(\tau) = \frac{(z_i - z_{i\infty})}{(z - z_{i\infty})}\,,$$

$$f = \eta^2 (z - z_{i\infty})^{-1/12} (z - z_i)^{-1/4}\,,$$

$$j(\tau) = \frac{(z - z_\rho)}{(z - z_{i\infty})}$$

$$f = \eta^2 (z - z_{i\infty})^{-1/12} (z - z_\rho)^{-1/6}$$





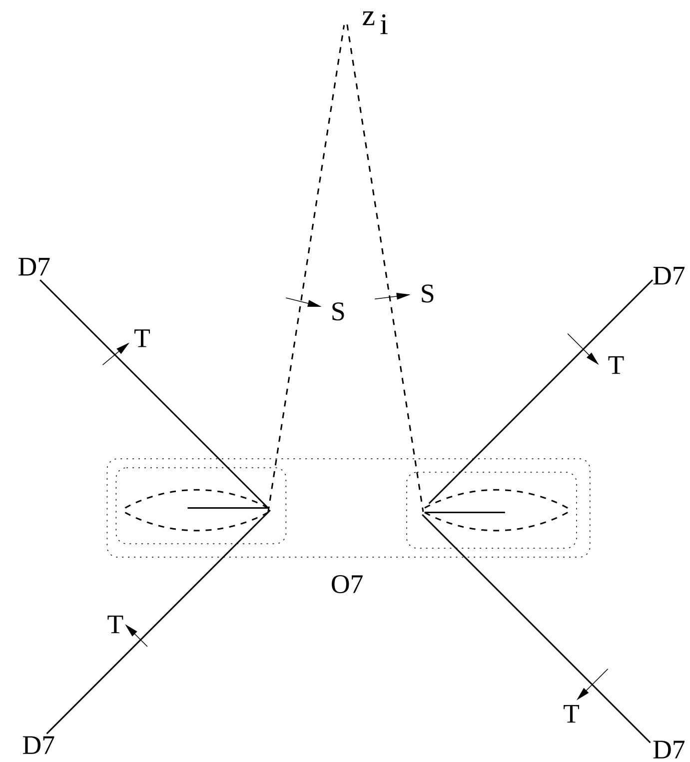
- $T\tau = \tau + 1$ and $S\tau = -1/\tau$
- At z_i a $-S$ object and at z_ρ a $-T^{-1}S$ object is located.
- These solutions are different from what is called the one D7 brane solution in the literature.

Properties of solutions

$$m = \frac{1}{16\pi G_3}\left(\text{area fundamental domain} + \text{sign}(q)\, 2\sqrt{\det Q}\right)$$

- At the location of the $-S$ and $-T^{-1}S$ objects the metric has a deficit angle.
- BPS relation: $m = -\,\text{sign}(q_\infty)\,\frac{\sqrt{\det Q_\infty}}{8\pi G_3}$ with e^{Q_∞} the monodromy of τ around infinity. (different from what was stated in the literature)
- Black deformations are probably not possible. Another example of such a circumstance is given in [Howe, Izquierdo, Papadopoulos, Townsend, hep-th/9505032]
- Solutions with 6,12,24 D7 branes are special because for those the $-S$ and $-T^{-1}S$ objects can cancel each other out.

The 6 D7 brane solution



- This solution only contains 6 D7 branes.
- When all points coincide $\tau =$ cst: O7 plane (monodromy: $-I$)
- Weak coupling limit: 4 D7's sticking out of an O7 plane.
- Beyond weak coupling the O7 plane splits into two non-perturbative pieces [Sen, hep-th/9605150].
- The non-perturbative structure of the split orientifold is immediate from picture

The monodromy group $\Gamma_0(2)$

$$j_{\Gamma_0(2)}(\tau) \equiv \frac{1}{(1+i)^{12}}\left(\frac{\eta(\tau)}{\eta(2\tau)}\right)^{24} = \frac{z-z_0}{z-z_{i\infty}}\,, \quad f = \eta(\tau)\eta(2\tau)(z-z_{i\infty})^{-3/24}(z-z_0)^{-3/24}$$

- The group $\Gamma_0(2)$ is generated by T and ST^2S.
- The solution contains a D7 brane and an S-dual D7 brane and nothing else.
- The S-dual D7 brane: $\tau \to ST^qS\tau$ where $q \in \mathbb{Z}$.

$$\tau = -\frac{2\pi i}{q}\frac{1}{\log(z-z_0)}$$

- The monodromy around infinity is ST^2ST which is of second order.

Future

- Similar solutions appear as strings in $N = 2$, $d = 4$ supergravity (see talk by Patrick Meessen). They are interesting because they strongly depend on the moduli space of the theory which in the case of $N = 2$, $d = 4$ can be a CY3 compactification, [Bergshoeff, JH, Hübscher, Ortín].
- D3-brane probes. What are the D3-brane world-volume theories when probing these new 7-brane backgrounds? [Bergshoeff, Chemissany, JH, Ploegh]. Beyond the probe approximation: globally well-defined D3/D7 systems.