Non-linear Realizations, Superbranes and Kappa symmetry

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Motivation

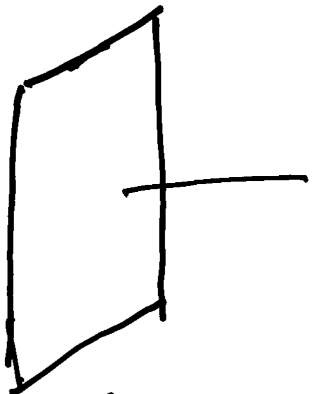
- Goldstone fields – Brane actions

- Gauge symmetries in NLR.
  Kappa symmetry

- E_{10}, E_{11} ..

Superbranes in flat space-time

p-brane



$a' = p+1, \ldots, D-1$

p+1 longitudinal directions
$a = 0, \ldots p$

$\underline{a} = (a, a')$

G = Super Poincaré group

Generators, $P_{\underline{a}}, J_{\underline{ab}}, Q_{\underline{\alpha}}$

$$[J_{\underline{ab}}, J_{\underline{cd}}] = -i \eta_{\underline{bc}} J_{ad} + \ldots$$

$$[J_{\underline{ab}}, P_{\underline{c}}] = -i \eta_{\underline{bc}} P_{\underline{a}} + \ldots$$

$$[Q_{\underline{\alpha}}, J_{\underline{ab}}] = -\frac{i}{2} (\Gamma_{\underline{ab}})_{\underline{\alpha}}{}^{\underline{\beta}} Q_{\underline{\beta}}$$

$$\{Q_{\underline{\alpha}}, Q_{\underline{\beta}}\} = 2 (\Gamma^{\underline{a}} C^{-1})_{\underline{\alpha\beta}} P_{\underline{a}}$$

Coset $\frac{G}{H}$

$$H = SO(1,p) \otimes SO(D-p-1)$$

$$g = \underbrace{e^{i x^{\underline{a}} P_{\underline{a}}} \, e^{\bar{\theta}^{\underline{\alpha}} Q_{\underline{\alpha}}}}_{g_0} \, \underbrace{e^{i \phi_a{}^{b'} J^a{}_{b'}}}_{U}$$

g_0 ↑ Superspace

$$\bar{\theta}^{\underline{\alpha}} = \theta_{\underline{\beta}} C^{\underline{\beta}\underline{\alpha}}$$

Fields $x^a(\xi)$, $\theta_{\underline{\alpha}}(\xi)$, $\phi_a{}^{b'}(\xi)$ depend only on the world-sheet parameters ξ^i, $i = 0, \ldots, p$

No Goldstone Superfields

MC one form associated to g_0

$$\Omega_0 = -i g_0^{-1} d g_0 = \Pi^{\underline{a}} P_{\underline{a}} - i \bar{\Pi}^{\underline{\alpha}} Q_{\underline{\alpha}}$$

$$\begin{cases} \Pi^{\underline{a}} = dx^{\underline{a}} + i \bar{\theta} \Gamma^{\underline{a}} d\theta \\ \Pi^{\underline{\alpha}} = d\bar{\theta}^{\underline{\alpha}} \end{cases}$$

Complete MC one form

$$\Omega = U^{-1} \Omega_0 U - i U^{-1} dU$$

$$= \underbrace{e^{\underline{a}} P_{\underline{a}} - i \bar{e}^{\underline{\alpha}} Q_{\underline{\alpha}}}_{\text{supervielbein}} + \frac{1}{2} \underbrace{\omega^{\underline{ab}}}_{\text{spin connection}} J_{\underline{ab}}$$

MC equation $d\Omega + i \Omega \wedge \Omega = 0$

$$de^{\underline{a}} + \omega^{\underline{a}}{}_{\underline{b}} e^{\underline{b}} = i \bar{e} \Gamma^{\underline{a}} e$$

$$d\bar{e}^{\underline{\alpha}} - \frac{1}{4} \bar{e}^{\underline{\beta}} \omega^{\underline{ab}} (\Gamma_{\underline{ab}})_{\underline{\beta}}{}^{\underline{\alpha}} = 0$$

$$d\omega^{\underline{ab}} + \omega^{\underline{ac}} \omega_{\underline{c}}{}^{\underline{b}} = 0$$

In order to have explicit expressions supervielbein, spin connection

$$U^{-1} P_{\underline{a}} U = \Phi_{\underline{a}}{}^{\underline{b}} P_{\underline{b}}$$

vector Lorentz Transformation

$$U^{-1} Q_{\underline{\alpha}} U = \tilde{\Phi}_{\underline{\alpha}}{}^{\underline{\beta}} Q_{\underline{\beta}}$$

spinor Lorentz Transformation

$$e^{\underline{a}} = \Pi^{\underline{b}} \Phi_{\underline{b}}{}^{\underline{a}}$$

$$\bar{e}^{\underline{\alpha}} = \bar{\Pi}^{\underline{\beta}} \tilde{\Phi}_{\underline{\beta}}{}^{\underline{\alpha}}$$

$$\omega_{\underline{a}}{}^{\underline{b}} = (U^{-1} d U)_{\underline{a}}{}^{\underline{b}}$$

A possible invariant action

$$A^{NG}[x, \theta, \phi] = -T \int \mathcal{L}^{NG}$$

$$\mathcal{L}^{NG} = -\frac{1}{(p+1)!} \varepsilon_{a_0 \cdots a_p} e^{a_0} \wedge \cdots \wedge e^{a_p}$$

Parametrization Lorentz Transformation

$$\Phi = \begin{pmatrix} \mathbb{1}_1 & \varphi \\ -\varphi^T & \mathbb{1}_2 \end{pmatrix} \begin{pmatrix} B_1 & 0 \\ 0 & B_2 \end{pmatrix} =$$

$$\begin{pmatrix} B_1 & \varphi B_2 \\ -\varphi^T B_1 & B_2 \end{pmatrix}$$

B_1 — $(p+1)\times(p+1)$ matrix

B_2 — $(D-(p+1))\times(D-(p+1))$ matrix

φ — $(p+1)\times(D-(p+1))$ matrix

$$B_1 = \frac{1}{\sqrt{\mathbb{1}_1 + \varphi\varphi^T}}$$

$$B_2 = \frac{1}{\sqrt{\mathbb{1}_2 + \varphi^T\varphi}}$$

B_1, B_2, φ are known functions of $\phi_a{}^{b'}$

$$(B_1)_a{}^b = (\cosh V)_a{}^b$$

$$(B_2)_{a'}{}^{b'} = (\cosh \tilde{V})_{a'}{}^{b'}$$

$$(\varphi)_a{}^{b'} = \phi_a{}^{c'}\left(\frac{\sinh \tilde{V}}{\tilde{V}\cosh \tilde{V}}\right)_{c'}{}^{b'}$$

$$(V^2)_a{}^b = -\phi_a{}^{c'}\phi^b{}_{c'} = -(\phi\phi^T)_a{}^b$$

$$(\tilde{V}^2)_{a'}{}^{b'} = -\phi^c{}_{a'}\phi_c{}^{b'} = -(\phi^T\phi)_{a'}{}^{b'}$$

Constrained form of Lorentz harmonics has been used to construct brane actions

Bandos, Pasti, Sorokin, Tonin, Volkov, Zheltukhin, ...

Relation To NG action

$$e^a = d\xi^i \left(\Pi_i^c - \Pi_i^{b'} (\varphi^T)_{b'}{}^c\right) (B_2)_a{}^c$$

Consider as variables $\varphi_a{}^{b'}$ instead of $\phi_a{}^{b'}$

φ is not a dynamical field, on-shell

$$\varphi_a{}^{b'} = -(\Pi^{-1})_b{}^i \, \Pi_i{}^{b'}$$

$$A^{NG} = -T \int d^{p+1}\xi \sqrt{-\det G_{ij}}$$

$$G_{ij} = \Pi_i^{\underline{a}} \Pi_j^{\underline{b}} \, h_{\underline{ab}}$$

WZ Term

Consider $p+2$ form

$$h = -\frac{i}{p!}\left(e^{a_1}\wedge\cdots\wedge e^{a_p}\right)\wedge \bar{e}^{\alpha}\wedge\bar{e}^{\beta}\,(\Gamma_{\underline{a_1\cdots a_p}}C^{-1}|$$

in $p=1,2 \bmod 4$ is closed.

Using

$$\tilde{\Phi}\,\Gamma_{\underline{a}}\,\tilde{\Phi}^{-1} = (\Phi^{-1})_{\underline{a}}{}^{\underline{b}}\,\Gamma_{\underline{b}}$$

$$h = -\frac{i}{p!}\left(\pi^{a_1}\wedge\cdots\wedge\pi^{a_p}\right)\wedge\bar{\pi}^{\alpha}\wedge\bar{\pi}^{\beta}\,(\Gamma_{\underline{a_1}\cdots\underline{a_p}}C^{-1})$$

$$h = d\,\mathcal{L}^{WZ}$$

Super p-brane action

$$\mathcal{L} = \mathcal{L}^{NG} + b\,\mathcal{L}^{WZ}$$

$b = \pm 1$ Kappa symmetry

Kappa symmetry as a Local SUSY right action on $\frac{G}{H}$

I.N. McArthur

D0 particle

$$A = -\int e^0 + i b \bar{\theta} \Gamma_{11} d\theta$$ Azcarraga-Lukierski

Consider the local right action generated by the unbroken SUSY Q_*

$$e^{\bar{\kappa}_* Q^*}$$ → 16 independent spinors

Transformation of Goldstone fields

$$\delta x^{\underline{a}} = i \delta\bar{\theta} \Gamma^{\underline{a}} \theta$$

$$\delta \bar{\theta}^{\underline{\alpha}} = \bar{\kappa}_*^{\beta} \tilde{\Phi}^{-1}{}_{\underline{\beta}}{}^{\underline{\alpha}}$$ Bandos, Zheltukhin,

$$\delta \phi_0{}^{b'} = 0$$

$$\delta_\kappa \mathcal{L} = 2i\left(\bar\kappa_*(1+b\Gamma^0\Gamma_{11})\Gamma^0 e\right)$$

If $b=-1$

$$\bar\kappa_* = \bar\kappa\,\frac{1}{2}(1+\Gamma^0\Gamma_{11})$$

κ 32 component spinor

$$\delta_\kappa \mathcal{L} \simeq 0$$

If we write the local invariance in terms of 32 component spinor

$$\delta\bar\Theta = \bar{\tilde\kappa}\,\frac{1}{2}(1-\Gamma_\kappa(\varphi))$$

$$\Gamma_\kappa(\varphi) = \hat{\Phi}(\varphi)\Gamma_0\Gamma_{11}\tilde{\Phi}^{-1} =$$

$$= \Phi_0{}^{\underline{a}}\,\Gamma_{\underline{a}}\Gamma_{11} \qquad \tilde\kappa = \tilde\phi\,\kappa$$

If we eliminate the nondynamical Goldstone fields $\varphi^{a'}$ we get

$$\Gamma_\kappa(x,\theta) = \frac{\Pi_0{}^{\underline{a}}\,\Gamma_{\underline{a}}\,\Gamma_{11}}{\sqrt{-\Pi_0{}^{\underline{b}}\,\Pi_0{}^{\underline{c}}\,\eta_{\underline{bc}}}}$$

Super p-brane case

Action invariant under local unbroken susy c6l compensating "boost" transformation is introduced

$$h = e^{\bar{\kappa}_* Q^* + i r_a{}^{b'} J^a{}_{b'}}$$

$r_a{}^{b'}$ can be written in terms of $\bar{\kappa}_*$.

$$\bar{\kappa}_* = \bar{\kappa}\,\frac{1}{2}(1+\Gamma_*)$$

$$\Gamma_* = \frac{1}{(p+1)!}\,\epsilon^{a_0 \cdots a_p}\,\Gamma_{a_0}\cdots\Gamma_{a_p}$$

The transformation of θ is

$$\delta\bar{\theta} = \tilde{\bar{\kappa}}\,\frac{1}{2}(1+\Gamma_k(\varphi))$$

$$\Gamma_k(\varphi) = \frac{1}{(p+1)!}\,\epsilon^{a_0 \cdots a_p}\,\phi^{-1 b_0}_{a_0}\cdots\phi^{-1 b_p}_{a_p}\,\underline{\Gamma_{b_0 \cdots b_p}}$$

Once φ is eliminated

$$\Gamma_k(x,\theta) = \frac{1}{(p+1)!\sqrt{-\det G_{ij}}}\, \epsilon^{i_0 \cdots i_p}$$

$$\Pi_{i_0}{}^{b_0} \cdots \Pi^{b_p}_{i_p} \Gamma_{\underline{b_0 \cdots b_p}}$$

Conclusions

- We can construct super-p-brane actions from NLR using ordinary Goldstone fields
- Gauge symmetries can be understood as local right actions associated to unbroken translations generators
- Extension to general curved backgrounds
- D branes
- E_{10}, E_{11}