Microstates and near-horizon D-brane probes

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- S. Kim and J.R., *Superconformal quantum mechanics of small black holes*, hep-th/0505176
- P.K. Yogendran and J.R., *Supersymmetric D-branes in the D1-D5 background*, hep-th/0607150
- J.R., work in progress

Overview

- Gaiotto-Strominger-Yin (GSY) proposal for entropy counting from superconformal quantum mechanics

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  - ‘Puffed’ (p, q) strings
- Open problems

D0-D4 black holes in type IIA

- **type IIA on** $CY_3 \Rightarrow$ N=2 in D=4.
  Consider **4D charged extremal black holes** with electric **D0-brane charge** q_0 and magnetic **D4-brane charges** p^A from wrapping D4's on CY 4-cycles $(A = 1, \ldots, b^2)$

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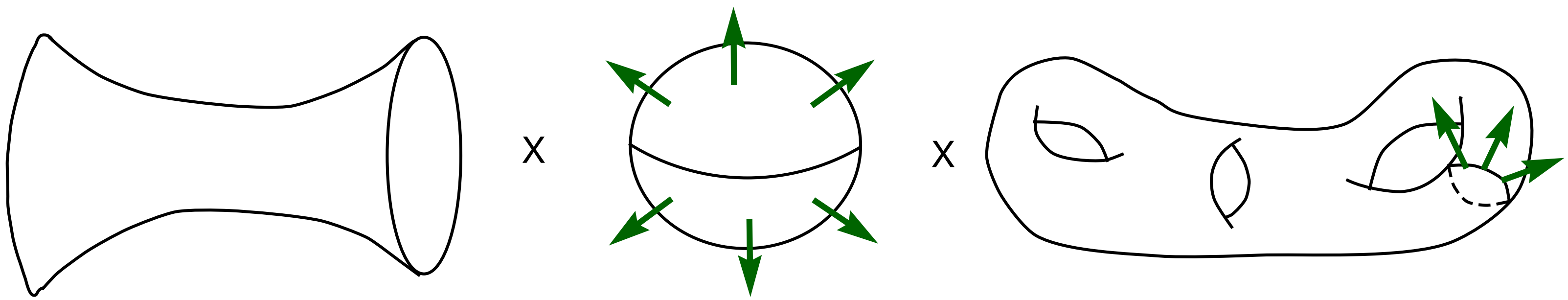
- The **near-horizon geometry** is

$$AdS_2 \times S^2 \times CY_3 \text{ with fluxes}$$



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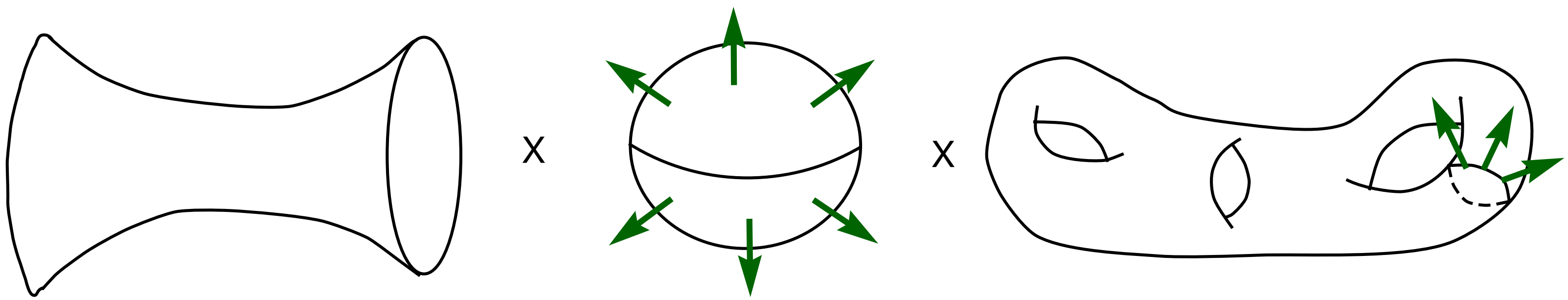

- **Attractor mechanism:** the $AdS_2 \times S^2$ **radius** R **and the** CY_3 **Kähler moduli** are fixed in terms of the charges.

GSY proposal

- As a proposal for the elusive AdS_2/CFT_1 correspondence, GSY proposed (hep-th/0412322) to consider the worldvolume **quantum mechanics living on D0 branes** in the near-horizon $AdS_2 \times S^2 \times CY_3$ attractor geometry

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- The **entropy of the black hole** is the **degeneracy of multi-particle chiral primaries** in the SCQM for given D0-charge.
- There is an **important subtlety** in implementing this proposal.

‘Puffed’ D0-branes

- A multi-D0-brane configuration in the attractor background can ‘puff up’ to form a D2-brane through the **Myers effect.** Two descriptions:
  - Fuzzy two-sphere solution of the N multi-D0-brane action in the $AdS_2 \times S^2 \times CY_3$ background.
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- These horizon-wrapping membranes **experience a magnetic field on the** CY due to the RR coupling $\int C^{(3)}$

Microstate counting

- The symmetry algebra is an **N=4 superconformal symmetry algebra** $SU(1,1|2)_Z$.

$$\left(\begin{array}{c|c} SL(2,R) & F_1 \\ F_2 & SU(2) \end{array}\right)$$

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- Chiral primaries are in 1-to-1 correspondence with **lowest Landau levels** on the CY
- The asymptotic degeneracy of multiparticle chiral primaries with total D0-charge N is ($D \equiv \frac{1}{6} C_{ABC} p^A p^B p^C \neq 0$)

$$\log d_N \simeq 2\pi \sqrt{ND} + \text{subleading}$$

Small black holes

- Subleading terms do not match with known corrections to the entropy. Previous analysis breaks down when D becomes small: higher derivative corrections become important. We will now consider **'small' black holes** with $D = 0$. These have **vanishing horizon area in the leading supergravity approximation.** Higher derivative corrections give rise to a nonzero horizon area.

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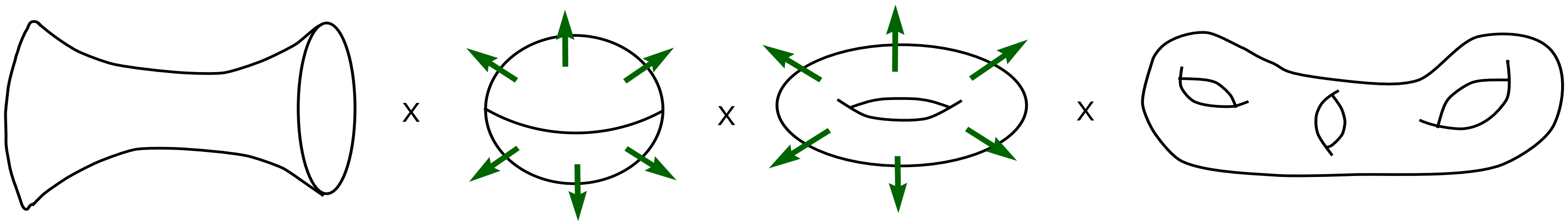
- **Prepotential**, including leading correction (for $M = K3$):

$$F = -\frac{1}{2} C_{ij} X^i X^j \frac{X^1}{X^0} - \frac{1}{64} \hat{A} \frac{X^1}{X^0}$$

Attractor geometry

The **near-horizon geometry** is now

$AdS_2 \times S^2 \times T^2 \times M$ with fluxes



$$ds^2 = R^2(-r^2 dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \sin^2\theta d\phi^2) + 2dzd\bar{z} + 2r g_{a\bar{b}} dz^a d\bar{z}^{\bar{b}}$$

$$F^{(4)} = \frac{p^1}{4\pi} \sin\theta d\theta \wedge d\phi \wedge \omega_1; \qquad F^{(2)} = \frac{R}{g_s} dr \wedge dt$$

($g_{a\bar{b}}$: metric on M; ω_1: vol. form on T^2; $R = \frac{g_s}{2\pi}\sqrt{\frac{p^1}{|q_0|}}$)

Small BH quantum mechanics

- As in GSY, we consider D2-branes with N units of D0-flux and wrapped on S^2. Terms contributing to the action:

$$S = T_2 \int d^3\sigma e^{-\phi}\sqrt{-\det(G+F)} + T_2 \int_{D2} C^{(3)} + T_2 \int_{D2} F \wedge C^{(1)}$$

This leads to the **bosonic Hamiltonian** ($\xi = 1/\sqrt{r}$):

$$H = \frac{1}{8RT} P_\xi^2 + \frac{R}{T\xi^2}(P_z - A_z)(P_{\bar{z}} - A_{\bar{z}}) + \frac{32\pi^4 R^5}{g_s^2 N \xi^2} + \frac{Q}{T} P_a g^{a\bar{b}} P_{\bar{b}}; \qquad dA \equiv 2\pi p^1 \omega_1$$

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- Note: the $AdS_2 \times S^2 \times T^2$ part and the M part decouple!

Symmetry algebra

- The full worldvolume theory includes **16 fermions.** Including fermions, the **symmetry group** splits into a product of
  - **N=4** $SU(1,1|2)_Z$ **superconformal quantum mechanics (SCQM)** involving the $AdS_2 \times S^2 \times T^2$ coordinates
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  - **N=4 supersymmetric quantum mechanics (SQM)** involving the M coordinates.
- The **ground states** are tensor products of chiral primaries of N=4 SCQM and susy ground states of N=4 SQM.

Counting ground states

- **N=4 SCQM chiral primaries** are in 1-to-1 correspondence with **lowest Landau levels** on T^2.
  $\Longrightarrow$ there are $\int_{T^2} dA = p^1$ (bosonic) chiral primaries.

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- **N=4 SQM ground states** are in 1-to-1 correspondence with **Dolbeault cohomology classes.**
  $\Longrightarrow$ on $M = K_3$: 24 bosonic susy ground states
  $\Longrightarrow$ on $M = T^4$: 8 bosonic + 8 fermionic ground states.

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$\Longrightarrow$ on $M = T^4$: 8 bosonic + 8 fermionic ground states.
- Since the number of ground states doesn't depend on the background D0-charge q_0, one can **take** $q_0 \rightarrow 0$ **so that all of the D0 charge comes from the probes** and is equal to N.

Counting multi-particle ground states

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- **Leading term agrees with black hole entropy** in both cases

The D1-D5 system

- Consider Q_1 D1-branes and Q_5 D5-branes wrapped on $M = K_3 \text{ or } T^4 \Rightarrow$ related to the D0-D4 black hole by T-duality + lift to 6 dimensions. We would like to follow a GSY-inspired approach and identify **near-horizon supersymmetric probe branes.**

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- Near horizon geometry is now $AdS_3 \times S^3 \times M$. In Poincaré coordinates:

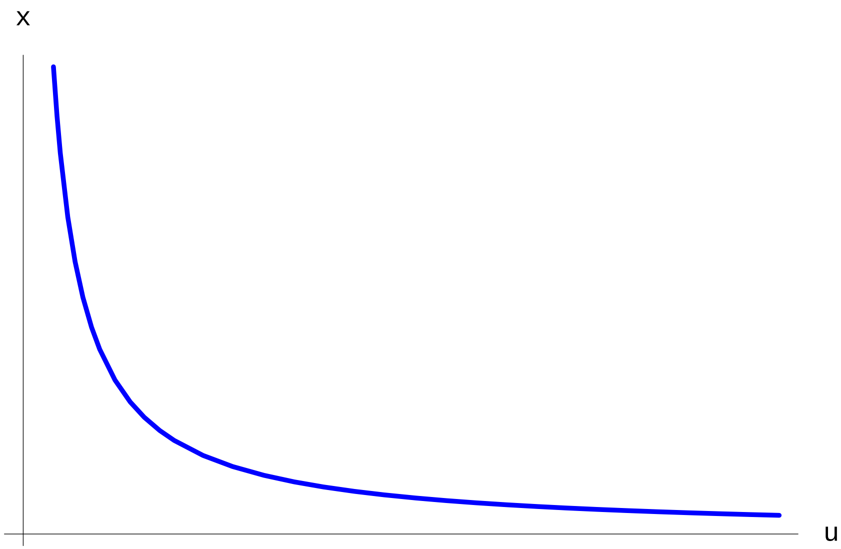
$$\begin{aligned} ds^2 &= r_1 r_5 [u^2(-dt^2 + dx^2) + \frac{du^2}{u^2} \\ &\quad + d\psi^2 + \sin^2\psi(d\theta^2 + \sin^2\theta d\phi^2)] + \frac{r_1}{r_5} ds_M^2 \\ F^{(3)} &= \frac{2r_5^2}{g}[udt \wedge dx \wedge du + \sin^2\psi \sin\theta d\psi \wedge d\theta \wedge d\phi] \end{aligned}$$

Supersymmetric AdS_2 branes

- We considered **brane configurations that preserve near-horizon supersymmetries and span an AdS_2 subspace** within AdS_3:

$$u = \frac{C}{x}$$

Such configurations are **static w.r.t. global time.**



Supersymmetric AdS_2 branes

- The near-horizon geometry preserves **16 supersymmetries. Killing spinors** ϵ come in two kinds:
  - 8 Poincaré susies: $\epsilon = \sqrt{u} R(\psi, \theta, \phi) \epsilon_+$
  - 8 enhanced susies:
    $\epsilon = \left(\frac{1}{\sqrt{u}} + \sqrt{u} (t \Gamma^{02} - x \Gamma^{12}) \right) R(\psi, \theta, \phi) \epsilon_-$

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    $\epsilon = \left(\frac{1}{\sqrt{u}} + \sqrt{u}(t\Gamma^{02} - x\Gamma^{12}) \right) R(\psi, \theta, \phi) \epsilon_-$
- Condition for brane probe to **preserve some supersymmetry**:

$$(1 - \Gamma)\epsilon = 0$$

where

  - Γ ($\mathrm{tr}\Gamma = 0, \ \Gamma^2 = 1$) is the operator entering in the κ-symmetry transformation rule on the Dp-brane
  - ϵ are the Killing spinors of the background pulled back to the world-volume.

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- Results of the analysis:

brane	AdS_3	S^3	M	restrictions
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D5	AdS_2	$\cdot$	M	
D3	AdS_2	S^2	$\cdot$	
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- All solutions preserve **half of the near horizon susies and half of the Poincaré susies**

Puffed (p, q) strings

- Let's focus on the $AdS_2 \times S^2$ **brane** in the classification above. The S^2 is contractible, hence it carries **no net D3-charge.** The S^2 is stabilized by turning on worldvolume electric flux due to the coupling $\int F \wedge C^2$.

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- Related branes have been considered
  - $p = 0$ case: Pawelczyk, Rey: hep-th/0007154
  - S-dual version: Bachas, Petropoulos: hep-th/0012234

- Equation for ψ: $\psi = \pi \frac{q}{Q_5}$ (radius of S^2 is $\sin\psi$).

  $\Rightarrow$ **'Exclusion bound'** on the number of fundamental strings: $q \leq Q_5$

  Tension: $T = 2\pi \sqrt{(pe^{-\phi})^2 + \left(\frac{Q_5}{\pi} \sin \frac{\pi q}{Q_5}\right)^2}$

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- Compare to D0-D4 black hole:

D0-D4 on $T^2 \times M$		D1-D5 on M
$AdS_2 \times S^2 \times T^2 \times M$		$AdS_3 \times S^3 \times M$
'puffed' D0 brane	$\longleftrightarrow$	'puffed' (p, q) string
(D2 on S^2)		(D3 along $AdS_2 \times S^2$)
Q_4 LLL ground states	$\longleftrightarrow$	Q_5 values of q
degenerate in energy	?	not degenerate

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- Understanding of the AdS_2 **branes in D1-D5 system** from the dual gauge theory point of view