Quasinormal Modes and Meson Decay Rates

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Overview

- AdS/CFT and Quark Gluon Plasma
- D7 Brane embeddings
- Holographic Meson Melting
- Quasinormal Modes
- Meson Masses
- Comments and Outlook

AdS/CFT and QGP

Large N strong coupling limit of $\mathcal{N}=4$ SYM = IIB on AdS$_5$ $\otimes$ S^5
[Maldacena, hep-th/9711200]

$$ds^2 = r^2(d\tau^2 + d\vec{x}^2) + \frac{dr^2}{r^2} + d\Omega_5^2$$

This metric describes the Coulomb phase

AdS/CFT and QGP

But also plasma phase is possible $T \neq 0$: AdS-Black Hole

[Witten, hep-th/9803131]

$$\begin{aligned} ds^2 &= r^2(f d\tau^2 + d\vec{x}^2) + \frac{dr^2}{f r^2} + d\Omega_5^2 \\ f &= 1 - \frac{r_0^4}{r^4} \\ r_0 &= \pi T \end{aligned}$$

Euclidean time $\tau \equiv \tau + 1/T$,

Good for equilibrium thermodynamics (thermostatics)

- free energy
- phase structure
- glueball spectrum

AdS/CFT and QGP

But also plasma phase is possible $T \neq 0$: AdS-Black Hole

[Witten, hep-th/9803131]

$$\begin{aligned} ds^2 &= r^2(-f dt^2 + d\vec{x}^2) + \frac{dr^2}{f r^2} + d\Omega_5^2 \\ f &= 1 - \frac{r_0^4}{r^4} \\ r_0 &= \pi T \end{aligned}$$

Lorentzian black hole with a horizon at $r = r_0$

Good for real time phenomena out of equilibrium

- transport theory (shear viscosity)
- energy loss of quarks
- relaxation time scales: quasinormal modes

D7 Brane Embeddings

[Karch, Katz, arXiv:hep-th/0205236]

[Kruczenski, Mateos, Myers, Winters, arXiv:hep-th/0304032]

[Babington, Erdmenger, Evans, Guralnik, Kirsch, arXiv:hep-th/0306018]

[Mateos, Myers, Thomson, arXiv:hep-th/0605046]

[Albash, Filev, Johnson, Kundu, arXiv:hep-th/0605088]

[Karch, O'Bannon, arXiv:hep-th/0605120]

- D7 brane embedded in AdS (Black-hole)
- open strings = fundamental matter
- $\mathcal{N}=4$ Theory + N_f Hypermultiplets $\mathcal{N}=2$ (at finite temperature)
- D7 as probe branes $N_c \gg N_f$ ("quenched approximation")

D7 Brane Embeddings

holoSYM: $AdS_5 \otimes S^5$
D7 brane: $\mathbb{R}^4 \otimes u(\text{partially}) \otimes S^3$

$$S_{D7} \;=\; T_{D7} \int d^8\xi \sqrt{-\det P[G]}$$

$$x \;=\; 1 - \frac{r_0^2}{r^2} \quad , \qquad d\Omega_5^2 = d\Theta^2 + \cos(\Theta)^2 d\Omega_3^2 + \sin(\Theta)^2 d\phi^2$$

$$\text{e.o.m}: \quad 3\tan(\Theta)\mathcal{L}(\Theta) + \left(\frac{4[\cos(\Theta)]^6(2-x)x}{(1-x)^4\mathcal{L}(\Theta)}\Theta'\right)' = 0$$

asymptotic behaviour $\Theta(x)|_{x\to 0} \Longrightarrow$ quark mass m and chiral condensate $\langle \bar{q}q \rangle$

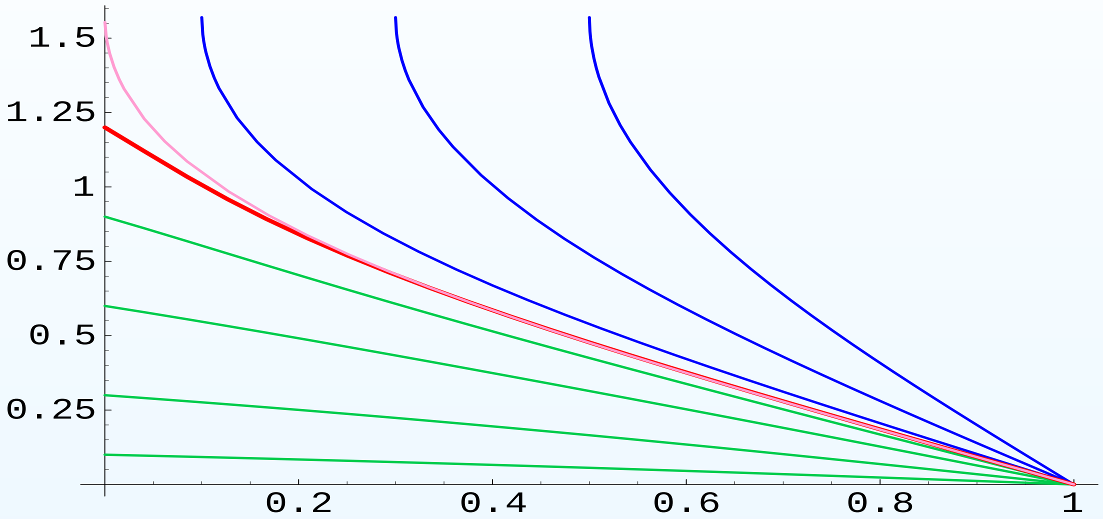
D7 Brane Embeddings

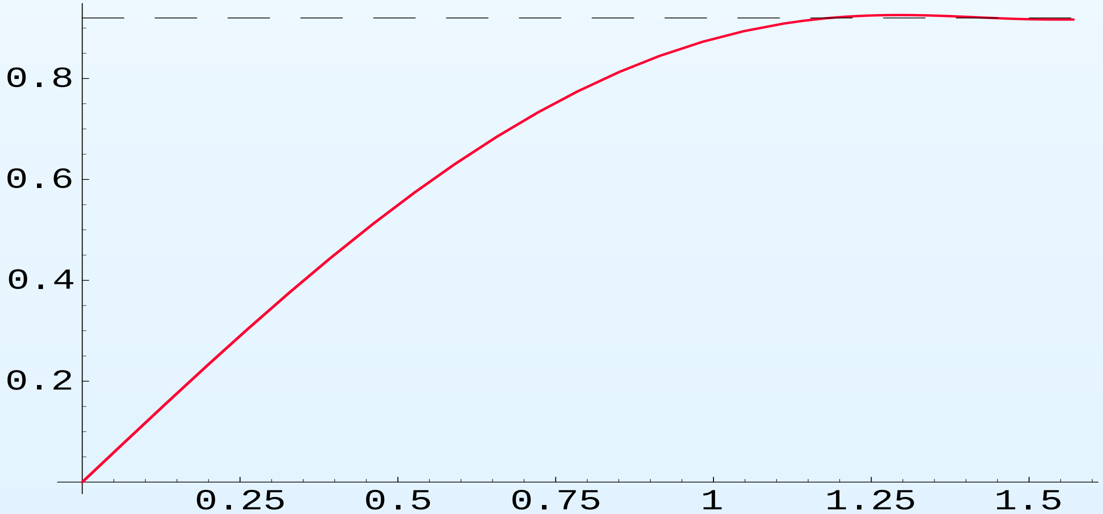
Two topologically different kinds of embeddings:

- D7-brane ends at some finite distance $r > r_0$ from Horizon
  - S^3 caps off smoothly there!
  - boundary conditions: $\Theta = \frac{\pi}{2}$ and $\Theta'|_H = \infty$
- D7-brane falls into the Black Hole
  - in Euclidean regime: S^1 caps off smoothly
  - boundary conditions: $\Theta|_H = \Theta_0$ and $\Theta'|_H = 0$

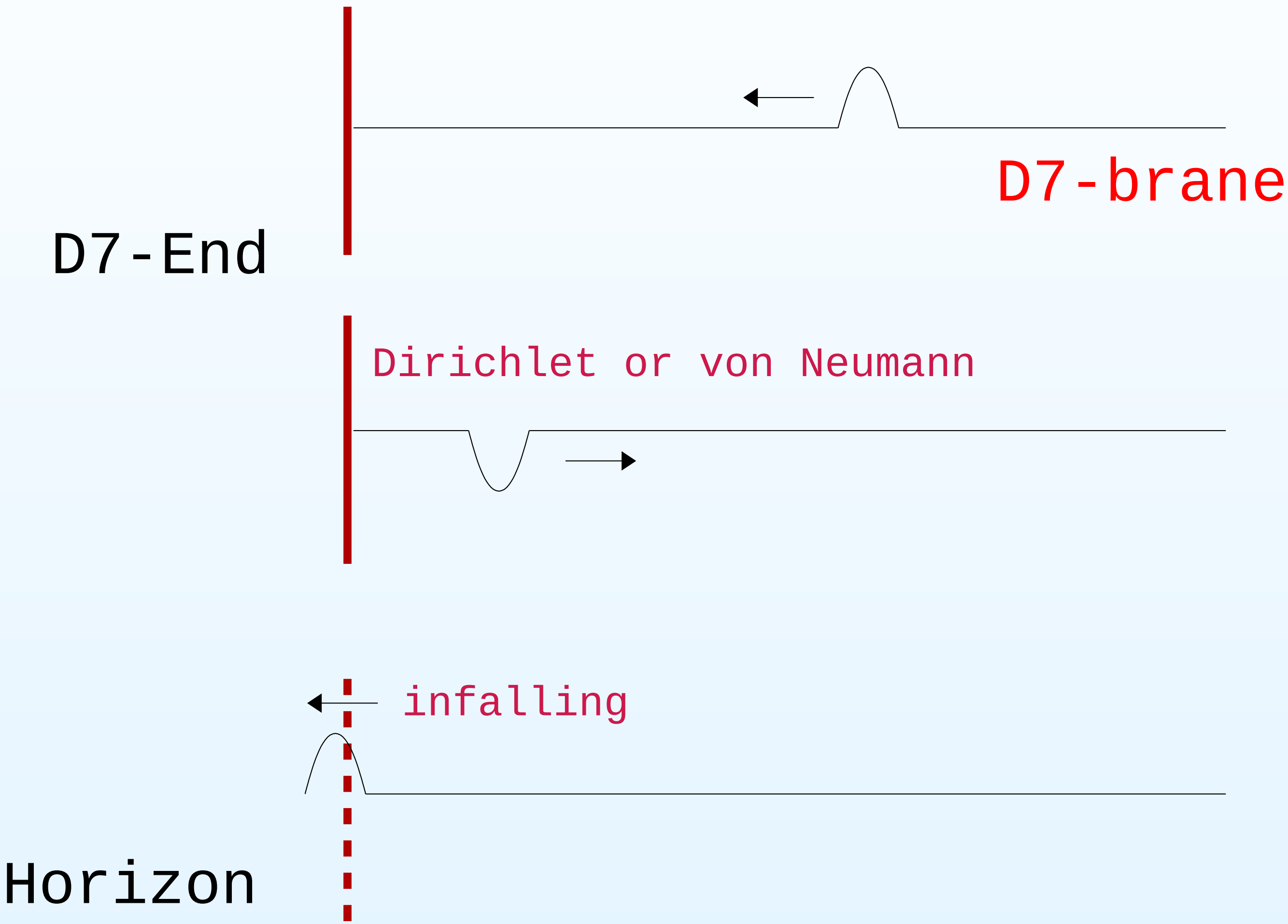
Important result: first order phase transition, chiral condensate jumps!

D7 Brane Embeddings





Meson Melting



Quasinormal Modes

- trivial background: $\Theta(x) = 0$
- fluctuations: $\vartheta(x,t) = e^{-i\omega t} h_\omega(x)$
- infalling boundary conditions: $\vartheta|_{\mathrm{Horizon}} \propto e^{-i\omega(t+r_*)}$
- Heun equation ($h_\omega \to x^{-i\frac{\omega}{4}} h_\omega$):

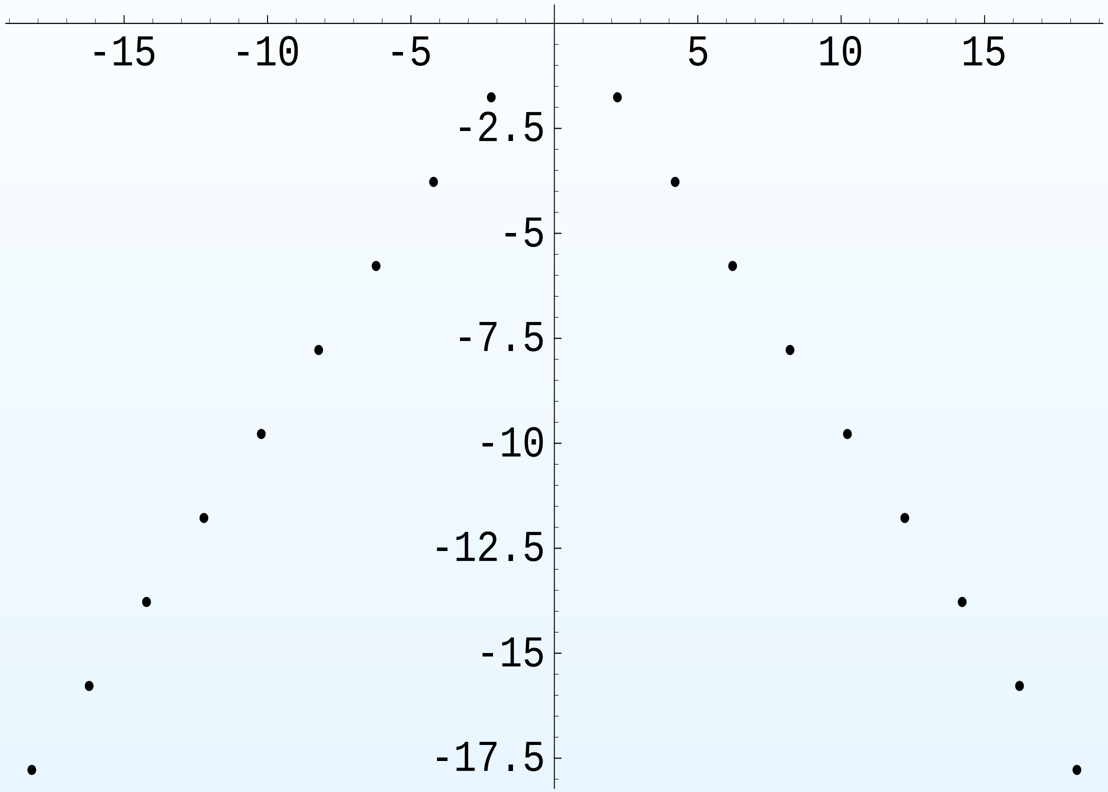
$$\frac{d^2 h_\omega(x)}{dx^2} - \left[\frac{\gamma}{x} + \frac{\delta}{x-1} + \frac{\epsilon}{2-x}\right]\frac{dh_\omega(x)}{dx} + \frac{\alpha^2 - Q}{x(x-1)(x-2)} h_\omega(x) = 0$$

- boundary conditions $h_\omega|_{\mathrm{Horizon}} = 1$ and $h'_\omega|_{\mathrm{Boundary}} = 0$ give quantization conditions for complex ω's

$$\omega_n = \Omega_n - i\frac{\Gamma_n}{2}$$

- Decay rate of n-th mode $\Longrightarrow$ meson melting

Quasinormal Modes



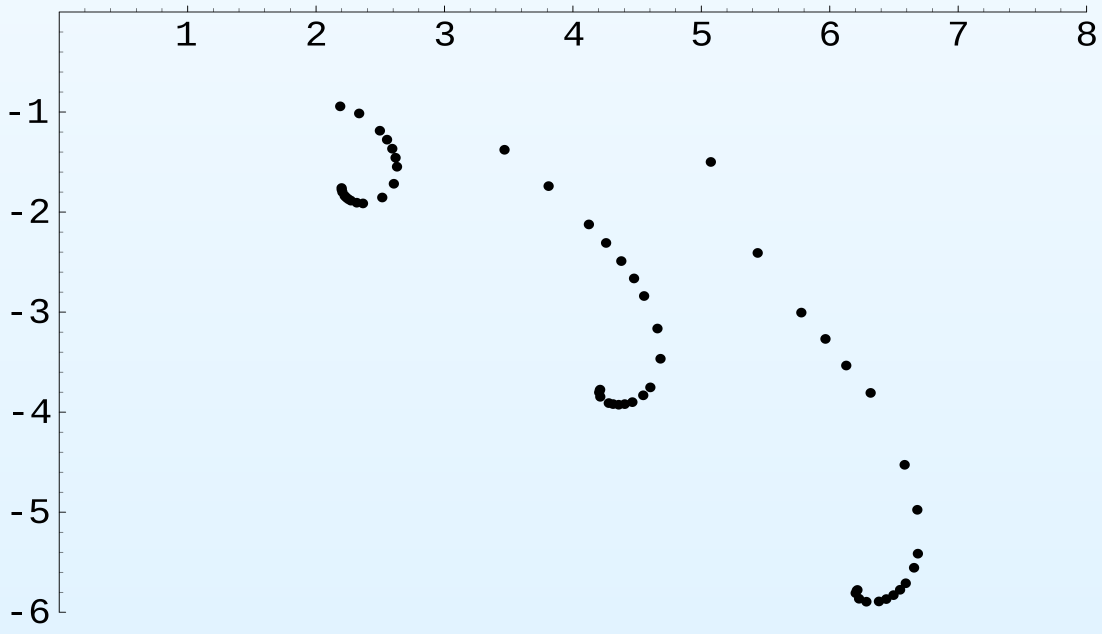
$$\Delta\omega \approx 2(\pm 1 - i)$$

lowest mode: $2.22 - 1.76i$ (lifetime $\tau = \frac{1}{2\pi T_c \Gamma} \approx 10^{-25}s$)

Quasinormal Modes

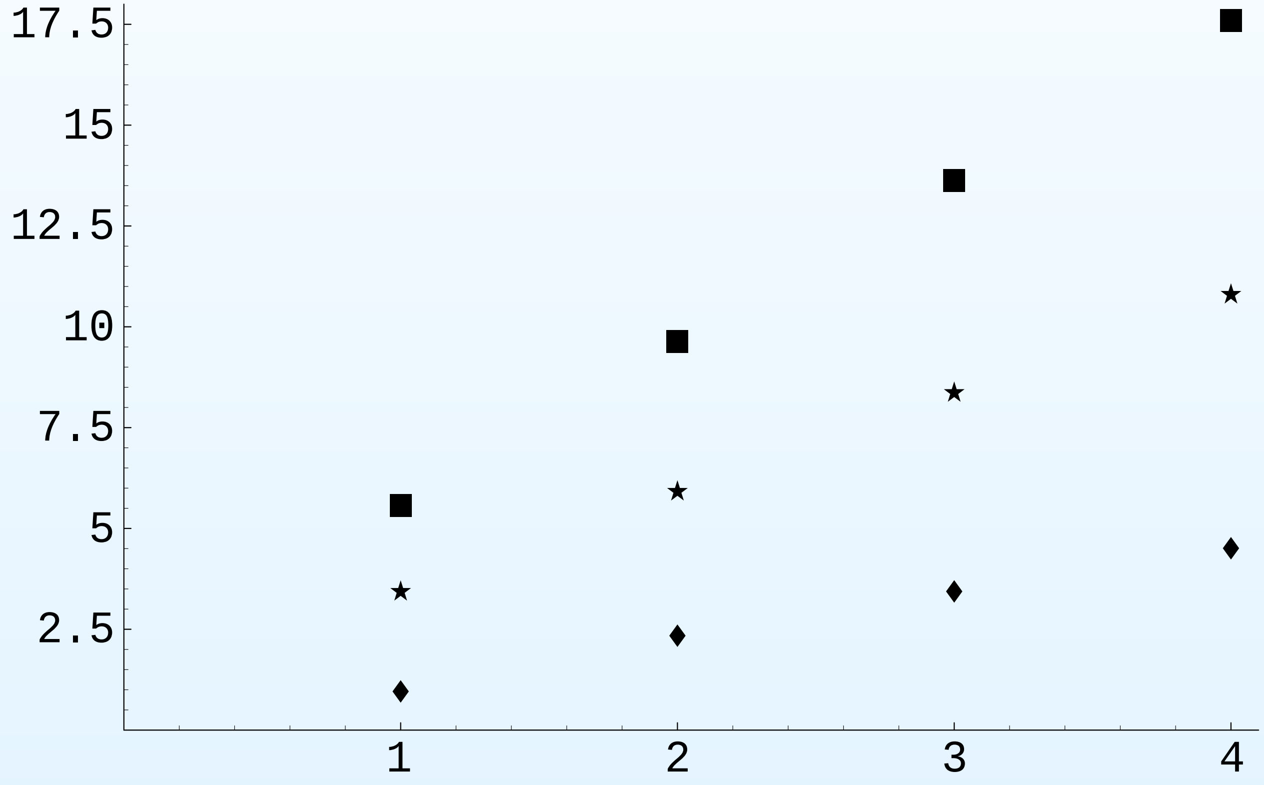
- non-trivial embeddings: $m > 0$
- lattice results: massive mesons are more stable

  [Karsch, Kharzeev, Satz, hep-lat/0512239], [Wong, hep-ph/0606200]
- numerically challenging
- three lowest quasinormal modes up to $\Theta|_H = 1.25$



Meson Masses

- embeddings not falling into horizon: $m > 0$
- stable mesons: ω real



$\omega_1 = 0.96$, $\omega_2 = 2.34$, $\omega_3 = 3.44$, $\omega_4 = 4.51$

Comments and Outlook

- We treated low spin mesons, high spin meson decay:

  [Peeters, Sonnenschein, Zamaklar, hep-th/0511044]

  [Cotrone, Martucci, Troost, hep-th/0511045]

  [Cotrone, previous talk!]

- Improve Numerics: "first shoot then relax"

  [Press, Teukolsky, Vetterling, Flannery, "Numerical Recipes"]

- Implementation of relaxation algorithm based on finite difference equations

- Apply to "realistic" models with chiral symmetry
  - holographic QCD (AdS/QCD)

    [Erlich, Katz, Son, Stephanov, hep-ph/0501128]

    [Da Rold, Pomarol, hep-ph/0501218]
  - Sakai-Sugimoto model

    [Sakai, Sugimoto, hep-th/0412141]