Singleton Strings

Laura Tamassia

Institutionen för Teoretisk Fysik, Uppsala Universitet

moving to

Instituut voor Theoretische Fysica, K. U. Leuven

based on work with

J. Engquist (Utrecht) and **P. Sundell**, (SNS, Pisa), *to appear,*

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Motivation

- The **group theoretical structure** of physics in AdS backgrounds is intrinsically different with respect to the flat case.
  Exploit this in the study of string and brane dynamics in AdS spacetime.
- To make ends meet between
  - string theory in AdS spacetime (in the tensionless limit)...
  - ...and ***higher spin gauge theory***.

  This connection could provide new insight into string quantization in AdS.
- As a byproduct, exploit physical intuition from AdS brane dynamics to study a class of ***noncompact*** **WZW models**, with **negative (fractional) level)**, that are not well understood yet and are interesting in their own right.

Inspiration

- *AdS/CFT correspondence*: On the CFT side "***partonic***" description as continuous limit of a discrete **spin chain**. A ***singleton*** at each site of the spin chain. Anything similar on the string side? Beisert.

Singletons and AdS

- Singleton are **"ultra-short" unitary irreps of $\mathfrak{so}(D-1,2)$**, forming a single line in weight space with zero-point energy $\epsilon_0 = (D-3)/2$. Dirac.
- Since $SO(D-1,2)$ can be realized as the group of the **isometries of AdS_D**, singletons play an important role in the study of AdS physics.
- They also play a role in higher spin theories, since **higher spin algebras** are extensions of $\mathfrak{so}(D-1,2)$.
- Singletons **do not admit a flat space limit**. The other irreps can be classified as *massless* or *massive* according to their flat space limit.
- Singletonic particles can be naturally described in the zero radius limit of AdS (Dirac's Hypercone).
- *Compositeness theorem:* **singleton $\otimes$ singleton =$\bigoplus$ massless irrep.** Flato,Frønsdal.
- The product of more than two singletons gives massive representations.

AdS symmetries

$\mathfrak{so}(D-1,2)$ algebra:

$$[M_{AB}, M_{CD}] = i\eta_{BC} M_{AD} + 3 \text{ perm.}$$

$$M_{AB} = -M_{BA} = (M_{AB})^\dagger$$

where $A = 0', 0, 1, ..., D-1$ and $\eta_{AB} = \text{diag}(-,-,+,...,+)$

Maximal compact subalgebra $\mathfrak{so}(2) \oplus \mathfrak{so}(D-1)$:

$\mathfrak{so}(2)$ (spanned by $E = M_{0'0}$)$\rightarrow$ **AdS time translations**

$\mathfrak{so}(D-1)$ (spanned by $J_{rs} = M_{rs}$)$\rightarrow$ **AdS rotations**

The rest: **ladder operators** $L_r^\pm = M_{0r} \mp iM_{0',r}$$\rightarrow$ **AdS spin boosts**

$$[E, L_r^\pm] = \pm L_r^\pm \quad ; \quad [J_{rs}, L_t^\pm] = 2i\delta_{t[s} L_{r]}^\pm$$
$$[L_r^-, L_s^+] = 2(iJ_{rs} + \delta_{rs} E)$$
$$[J_{rs}, J_{tu}] = i\delta_{st} J_{ru} + 3 \text{ perm.}$$

The scalar singleton

A unitary irreducible lowest weight representation $\mathcal{D}(E_0, \mathbf{j})$ of $\mathfrak{so}(D-1,2)$, is characterized by its lowest weight state $|E_0, \mathbf{j}\rangle$, annihilated by L_r^-, with $\mathfrak{so}(2)$ energy eigenvalue E_0 and $\mathfrak{so}(D-1)$ LW label $\mathbf{j}$. Unitarity imposes a bound between energy and spin. In the case of scalar representations, $E_0 \geq \frac{D-3}{2}$ or $E_0 = 0$.

Consider $|\epsilon_0\rangle$ such that

$$E|\epsilon_0\rangle = \epsilon_0|\epsilon_0\rangle \quad ; \quad J_{rs}|\epsilon_0\rangle = 0 \quad ; \quad L_r^-|\epsilon_0\rangle = 0$$

Build the *generalized Verma module*

$$\mathcal{V}(\epsilon_0, 0) \equiv \{L_{r_1}^+ \dots L_{r_n}^+|\epsilon_0\rangle\}_{n=0}^{\infty}$$

One can prove that, **for the special value $\epsilon_0 = \frac{D-3}{2}$ saturating the unitarity bound**:

$L_s^+ L_s^+|\epsilon_0\rangle$ is itself a LWS (it's a *singular* vector)

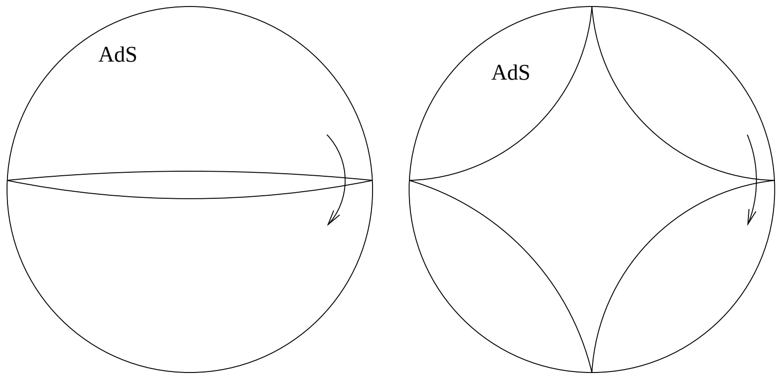
$L_s^+ L_s^+ L_{r_1}^+ \dots L_{r_n}^+|\epsilon_0\rangle$ is normal to all states (it's a *null* vector)

Singleton constructed by modding out this "trace-part" (*max. ideal*) from $\mathcal{V}(\epsilon_0, 0)$

$$\mathcal{D}(\epsilon_0, 0) \equiv \left\{L_{\{r_1}^+ \dots L_{r_n\}}^+|\epsilon_0\rangle\right\}_{n=0}^{\infty} \quad , \quad \epsilon_0 = \frac{D-3}{2}$$

The "partonic" behaviour of some branes...

Semiclassical string solution: *Folded rotating string* with $E \sim S$, (2-cusp string). It can be generalized to N cusps. Gubser, Klebanov,Polyakov;Kruczenski;Frolov,Tseytlin.



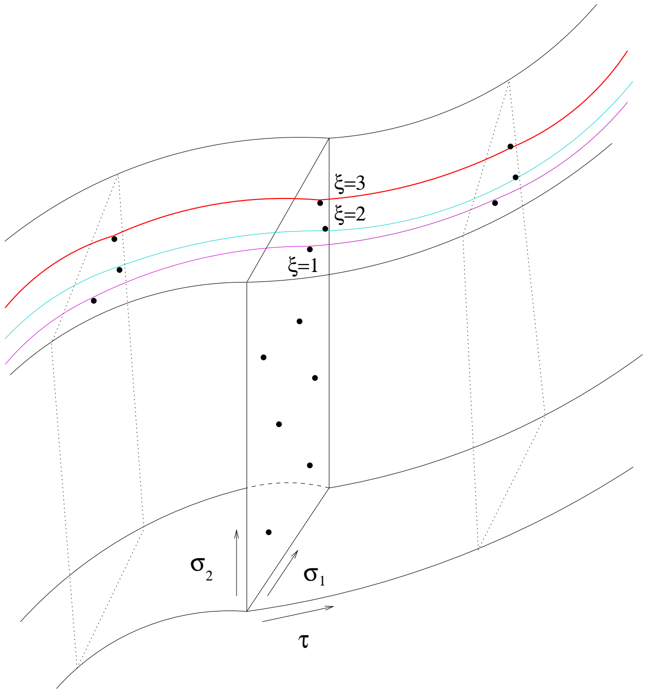
* Effective tension vanishes at the cusps

*Bound states at the endpoints

2-cusp string $\leftrightarrow$ bound state of two partons (singletons).
Deviation from being free singletons measured by the effective tension.
Arbitrary number of cusps $\leftrightarrow$ singleton gas.
Generalize to rotating p-branes $\Rightarrow$ D-dimensional singletonic degrees of freedom independent of the particular underlying p-brane.

Conjecture: The true fundamental degrees of freedom in the tensionless limit are point-like ***singletonic*** partons.

Tensionless branes in AdS



- Old idea: In the tensionless limit, extended objects fall apart.
- In particular, a p-brane in AdS can be described by the discretized $(0+1)$-dimensional Nambu-Goto action with N "pieces", or **partons**.

To uncover the **singletonic nature of the partons**,
the **tensionless** limit must be taken together with a **zero radius** limit in AdS.

$\Downarrow$

The system of N partons is described by a **singleton gas**.

The continuum limit: A coset model

Singleton gas of N brane partons $\rightarrow$ **worldline gauged sigma model**.
Target space $=$ N-singleton **phase space**.

A heuristic continuum-limit of the phase-space action describing N partons, suggests that processes involving an arbitrary number of partons can be described by a **gauged WZW model** on the coset

$$\frac{\widehat{\mathfrak{so}}(D-1,2)_{-\epsilon_0}}{\widehat{\mathfrak{h}}_{-\epsilon_0}}\,, \qquad \text{with } \epsilon_0 = (D-3)/2$$

$\mathfrak{h}$ such that the gauged model contains **only singletons and their composites**.

In general D we propose to gauge the **maximal compact subalgebra**

$$\hat{\mathfrak{h}} = \widehat{\mathfrak{so}}(D-1)_{-\epsilon_0} \oplus \widehat{\mathfrak{so}}(2)_{-\epsilon_0}$$

This leads to $c_{\text{coset}} = 0$ and to $h_{\text{singleton}} = 0$ **for any** D.
Keypoint: **the choice of the level** $-\epsilon_0$ (negative, and fractional for even D!).

The spectrum of the $\widehat{\mathfrak{so}}(D-1,2)_{-\epsilon_0}$ WZW model

Spectrum? Use a **nontrivial vacuum singular vector**! Lesage et al, Gaberdiel

For the special value $k=-\epsilon_0$, the symmetric traceless rank 2 tensor

$$V_{AB} = M_A{}^C M_{BC} - \text{trace}$$

is a **WZW primary** $\Rightarrow$ the physical WZW primaries must *decouple*

$$\langle \text{phys LWS}' | \mathrm{V_{AB}} | \text{phys LWS} \rangle = 0$$

Solved by the **set of scalars** ($P=0$ vacuum, $P=1$: singleton, $P=2$: massless, $P>2$ massive)

$$|e_0 = P\epsilon_0\rangle, \qquad P = 0, 1, 2, \ldots$$

defined by the **P-twisted** conditions

$$\left(L_r^-\right)_n |e_0\rangle = 0, \quad n \geq -P \quad ; \quad \left(L_r^+\right)_n |e_0\rangle = 0, \quad n \geq P$$
$$\left(J_{rs}\right)_n |e_0\rangle = 0, \quad E_n |e_0\rangle = \delta_{n,0}\, P\epsilon_0 |e_0\rangle, \quad n \geq 0$$

Note: for $P>1$ these are not standard WZW primaries!

In $D=4$ (and $D=3$), a *spinor* singleton and its composites are also present.

Spectral flow

An invariance of the **current algebra**

$$[J_{rs,m}, J_{tu,n}] = i(\delta_{st} J_{ru,m+n} + 3\text{ perm.}) + 2km\delta_{t[r}\delta_{s]u}\delta_{m+n,0}$$
$$[E_m, E_n] = km\delta_{m+n,0}$$
$$[L^-_{r,m}, L^+_{s,n}] = 2(iJ_{rs,m+n} + \delta_{rs}E_{m+n}) - 2km\delta_{rs}\delta_{m+n,0}$$
$$[L^\pm_{r,m}, L^\pm_{s,n}] = 0$$
$$[E_m, L^\pm_{r,n}] = \pm L^\pm_{r,m+n} \quad ; \quad [J_{rs,m}, L^\pm_{t,n}] = 2i\delta_{t[s}L^\pm_{r],m+n}$$
$$[J_{rs,m}, E_n] = 0$$

under the transformations

$$\tilde{L}^\pm_n = L^\pm_{n\mp w} \quad ; \quad \tilde{E}_n = E_n + kw\delta_{n,0} \quad ; \quad \tilde{J}_{rs,n} = J_{rs,n}$$

- The P-tupletons are all ***connected*** by spectral flow.
- The **GKO** conditions for the **maximal compact gauging**

$$E_m|\psi\rangle = 0 \quad , \quad J_{rs,m}|\psi\rangle = 0 \quad (m > 0)$$

are ***invariant*** under spectral flow.

Gaging: the one-singleton sector

$$\frac{\widehat{\mathfrak{so}}(D-1,2)_{-\epsilon_0}}{\widehat{\mathfrak{so}}(D-1)_{-\epsilon_0} \oplus \widehat{\mathfrak{so}}(2)_{-\epsilon_0}}\,, \qquad \text{with } \epsilon_0 = (D-3)/2$$

conjectured to contain only singleton and singleton-composite zero modes.

Results from GKO gauging procedure

- Zero modes: choose the sector $P = 1$ (one singleton).
- Virasoro level 1: the states that survive the gauging are singular and must be modded out. Nothing left.
- Virasoro level 2: focus on the simplest case $D = 7$. Many states checked. Gauge invariant states are singular.

Open problems:

- different behavior with the dimensionality D?
- $P > 1$ sectors (for example $P = 2$ level 1?)
- In terms of fields? (in $D = 7$, a $sp(2)$ doublet Y^{Ai}, singleton $sp(2)$-inv.).

Conclusions and Outlook

- The **maximal compact gauging** of $\widehat{\mathfrak{so}}(D-1,2)_{-\epsilon_0}$ is a good candidate for describing the singletonic degrees of freedom of tensionless branes in AdS.
- The **"magic" choice of the level** $k = -\epsilon_0$ is the key point!
- The nongauged $\widehat{\mathfrak{so}}(D-1,2)_{-\epsilon_0}$ WZW model displays striking features:
  - ***generalized*** definition of ***primary*** fields needed...
  - ... supported by the ***nontrivial*** action of the ***spectral flow***.

  (similar features in $SU(2)_{-1/2}$ model, see Lesage et al).
- $P = 2$ (massless) sector of interest for HSGT:
  - All massless primary fields generated by one LWS.
  - Hop from one massless field to another by acting with $L_1^{\pm}$.

  Affine Lie algebraic setting for HSGT?
- Gauging: BRST instead of GKO?
- ($D = 4$: free fields (symplectic bosons). "Standard" CFT setting.)

Well... a lot of work still needs to be done!