Twistor Strings and Supergravity

Mohab Abou Zeid

Theoretical Particle Physics Group, Vrije Universiteit Brussel

&

The International Solvay Institutes

E-mail: mohab.abouzeid@cern.ch

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Based On

- Collaboration with Christopher Hull (Imperial College London) and Lionel Mason (University of Oxford)

  - M. Abou Zeid and C. M. Hull, *A chiral perturbation expansion for gravity*, JHEP 0602:057,2006, [arXiv:hep-th/0511189]

  - M. Abou Zeid, C. M. Hull and L. J. Mason, *Einstein supergravity and new twistor string theories*, [hep-th/0606272], to appear in CMP

  - M. Abou Zeid, C. M. Hull and L. J. Mason, in progress

Introduction and Motivation

- Twistor string theory was developed by `Witten,` who hoped to find a string theoretic description of $N = 4$ SYM theory for weak 't Hooft coupling $\tau = g_{YM}^2 N_C$
- Witten defined twistor string theory as a topological B-model with target the Calabi-Yau supermanifold $\mathbb{CP}^{3|4}$, aka $N = 4$ projective supertwistor space
- The B-model on $\mathbb{CP}^{3|4}$ (or any Calabi-Yau target) describes – for open strings – holomorphic bundles and more general sheaves, together with their moduli...

Via the Penrose transform, the open strings (for 'space-filling branes') reproduce the perturbative spectrum of $N = 4$ SYM!

The interactions can also be reproduced, although this is somewhat harder to see...

- For closed strings the vertex operator for the physical $(0, 1)$-form on $\mathbb{CP}^{3|4}$ describes linearised deformations of the complex structure of a suitable region of $\mathbb{CP}^{3|4}$ (e. g. a neighbourhood $\mathbb{PT}_0$ of a projective line in $\mathbb{CP}^{3|4}$)

  The twistor space field theory action has a term with a Lagrange multiplier imposing $N(J) = 0$, where $N(J)$ is the Nijenhuis tensor of the complex structure J of the deformed region

- Via the Penrose transform and `'nonlinear graviton'` construction, integrable complex structure deformations of e. g. $\mathbb{PT}_0$ describe solutions of the ASD Weyl equations in spacetime:

$$W_{ABCD} = 0$$

where W_{abcd} denotes the Weyl tensor with SD and ASD parts W_{ABCD} and $W_{A'B'C'D'}$

This describes one helicity of conformal gravity, and leads to an action of the form

$$\int d^4x \sqrt{g} U^{ABCD} W_{ABCD}$$

with U symmetric in all its indices and of Lorentz spin $(2, 0)$

- In addition there is a term

$$\int d^4x \, \sqrt{g} U^{ABCD} U_{ABCD}$$

which arises from D-instantons in Witten's topological B-model

Integrating out U gives an action equivalent to the conformal gravity action

$$\int d^4x \, \sqrt{g} W^{abcd} W_{abcd}$$

Thus one gets both the spectrum and the interactions of $N = 4$ conformal SUGRA!

- Similar results can be derived using an alternative formulation of the twistor string due to Berkovits

  The Berkovits model reproduces the correct tree-level SYM amplitudes using ordinary string tree amplitudes as opposed to D-instanton contributions

  The construction uses the fact that in split spacetime signature $++--$ there is a 3- real dimensional submanifold $P\mathcal{T}_{\mathbb{R}}$ of complex twistor space, e. g. the standard embedding $\mathbb{RP}^3 \subset \mathbb{CP}^3$ in the flat case, and the data about deformations of the complex structure is encoded in an analytic vector field f on $P\mathcal{T}_{\mathbb{R}}$

- The Berkovits model is a theory of open strings with boundaries on $\mathbb{PT}_\mathbb{R}$ and action

$$S = \int d^2\sigma \left(Y_I \tilde{\partial} Z^I + \tilde{Y}_J \partial \tilde{Z}^J - \tilde{A} J - A \tilde{J} \right) + S_C$$

$N = 4$ conformal SUGRA physical states are created by an open string vertex operator constructed from a vector field f defined on $\mathbb{PT}_\mathbb{R}$, corresponding to deformations of the embedding of $\mathbb{PT}_\mathbb{R}$ in $\mathbb{PT}$, together with a vertex operator constructed from a 1-form $g_I dZ^I$ on $\mathcal{T}$:

$$V_f = Y_I f^I(Z) \qquad V_g = g_I(Z) \partial Z^I$$

The physical state conditions are

$$\partial_I f^I = 0 \qquad Z^I g_I = 0$$

and the gauge invariances are

$$\delta f^I = Z^I \Lambda \qquad \delta g_I = \partial_I \chi$$

The world-sheet theory includes a left and right-moving current algebra S_C reminiscent of heterotic string constructions, and this plays a key role both in the quantum consistency of the model and in determining its spacetime symmetries

- However one derives it, the emergence of CSUGRA is disappointing from the point of view of Witten's original goal of describing pure $N = 4$ SYM: e. g. at tree level the topological B-model computes the amplitudes of $N = 4$ SYM as desired, but at loops the string theory computes the amplitudes of $N = 4$ `SYM conformally coupled to` $N = 4$ `CSUGRA`
- Moreover there does not seem to be any obvious limits which one could take in the B-model or in the Berkovits model in order to decouple the gauge and gravitational sectors...
- So CSUGRA seems unavoidable in the Witten and Berkovits models, but this theory is generally considered to be inconsistent: it leads to 4th order PDE for the fluctuations of the metric, and thus to a lack of unitarity

- On the other hand, $N = 4$ SYM makes sense without CSUGRA, and it would be desirable to find a perturbative string theory description of it...

- A twistor string that gave `Einstein SUGRA (with 2nd order field equations for the graviton) coupled to SYM` would be much more useful, and might have a limit in which the gravity could be decoupled

  The spacetime conformal invariance would be broken in such a theory, and in particular this would introduce a `dimensionful parameter` which could be used to define the decoupling limit

- In fact there is an important variant of the Penrose construction that applies to conformally SD spaces that are also Ricci-flat, so that the full Riemann tensor is self-dual:

$$W_{A'B'C'D'} = 0, \qquad R_{A'B'} = 0 \qquad \Rightarrow \qquad R_{A'B'C'D'} = 0$$

- The corresponding twistor spaces $P\mathcal{T}$ then have extra structure In particular, they have a fibration $P\mathcal{T} \to \mathbb{CP}^1$

  The holomorphic 1-form on $\mathbb{CP}^1$ pulls back to give a holomorphic 1-form on $P\mathcal{T}$ which takes the form $I_{\alpha\beta}Z^\alpha dZ^\beta$ in homogeneous coordinates Z^α, for some $I_{\alpha\beta}(Z) = -I_{\beta\alpha}(Z)$ (which are the components of a closed 2-form on the non-projective twistor space $\mathcal{T}$

- The dual bi-vector $I^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta} I_{\gamma\delta}$ defines a Poisson structure and is called the *infinity twistor*

- Choosing a `point at infinity`, corresponding to such an infinity twistor, breaks the `conformal group` down to the `Poincaré group`

  e. g. on Minkowski space, the infinity twistor determines the light-cone at infinity in the conformal compactification

  A similar situation obtains more generally: `the infinity twistor breaks conformal invariance`

- Self-dual space-times are obtained by seeking deformations of the complex structure of twistor space as before, but now Ricci-flatness in space-time places further restrictions on the deformations allowed

  The vector field f on $\mathbb{RP}^3$ is required to be a Hamiltonian vector field with respect to the infinity twistor:

$$f^\alpha = I^{\alpha\beta} \frac{\partial h}{\partial Z^\beta}$$

  for some function h of homogeneity degree 2 on $\mathbb{RP}^3$

- In the linearised theory, such a function h corresponds to a positive-helicity graviton in space-time via the Penrose transform, and the non-linear graviton construction gives the generalisation of this to the non-linear theory

- A Dolbeault representative of the complex structure deformation is given by a $(0,1)$-form j^α of the form

$$j^\alpha = I^{\alpha\beta} \frac{\partial h}{\partial Z^\beta}$$

where h is a $(0,1)$-form representing an element of $H^1(\mathbb{PT}', \mathcal{O}(2))$

- This suggests seeking twistor strings that are modifications of the Berkovits or the Witten model with explicit dependence on the infinity twistor, such that there are extra constraints on the vertex operators imposing that the deformation of the complex structure be of the form given above
- Then the leading term in the action analogous to

$$\int d^4x \sqrt{g} U^{ABCD} W_{ABCD}$$

should have a multiplier imposing self-duality, not just conformal self-duality, and further terms quadratic in the multiplier (from instantons in Witten's approach) could then give Einstein gravity

- A formulation of Einstein gravity of just this form was discussed in my paper with Hull, and an $N = 8$ supersymmetric version was constructed some time ago by Siegel

- I will now survey our recent construction of new twistor string models which give Einstein (super)gravity coupled to (super) Yang-Mills

- The new theories are constructed by gauging certain symmetries of the Berkovits twistor string which are defined by the infinity twistor

  Their structure is very similar to that of the Berkovits model, but the gauging adds new terms to the BRST operator so that the vertex operators have new constraints and gauge invariances

I will describe the spectra and some of the interactions of 2 classes of theories for which the `world-sheet anomalies` cancel

- The corresponding `target space theories` can be expected to be anomalous in general, with the anomalies arising from inconsistencies in the corresponding twistor string model, though the mechanism for this is as yet unknown

  This may rule out some of the models we construct, or restrict the choice of gauge group G

  The situation is similar to that of the Berkovits and Witten models, which give target space theories that are anomalous in general, with the anomalies canceling only for the 4-dimensional groups $G = U(1)^4$ or $G = SU(2) \times U(1)$

- The 1st class of anomaly-free theories is formulated in $N = 4$ supertwistor space

  Gauging a symmetry of the string theory generated by 1 bosonic and 4 fermionic currents gives a theory with the spectrum of $N = 4$ Einstein SUGRA coupled to $N = 4$ SYM with arbitrary G

  Gauging a single bosonic current gives a theory with the spectrum of $N = 8$ Einstein SUGRA, `provided the number of` $N = 4$ `vector multiplets is 6`

- In the YM sector, the string theory is identical to that of Berkovits, so that it gives the same tree level YM amplitudes

- Both gauged theories have the MHV 3-graviton interaction (with 2 positive helicity gravitons and 1 negative helicity one) of Einstein gravity

  The other interactions are still being computed, and the results should determine the form of the interacting theories...

- There are different interacting theories with the spectrum of $N = 4$ Einstein SUGRA (coupled to $N = 4$ SYM):

  – the standard non-chiral Einstein SUGRA

  – an Einstein SUGRA with `chiral interactions`

- For the theory with $\dim G = 6$ and the spectrum of $N = 8$ SUGRA, the interactions could be those of:
  – the standard $N = 8$ SUGRA
  – Siegel's `chiral` $N = 8$ SUGRA
- The 2nd class of string theories is obtained by gauging different numbers of bosonic and fermionic symmetries so that anomalies are cancelled against ghost contributions for strings in twistor spaces with 3 complex bosonic dimensions and any number N of complex fermionic dimensions, corresponding to theories in 4-dimensional spacetime with N supersymmetries

- Analysing the spectrum of states arising from ghost-independent vertex operators, one finds:

  For $N = 0$, a theory with the bosonic spectrum of SD gravity together with SD YM and a scalar

  For $N < 4$, supersymmetric versions of this $N = 0$ SD theory

  For $N = 4$, a (second) theory with the spectrum of $N = 4$ Einstein SUGRA coupled to $N = 4$ SYM with arbitrary G

- Consistent non-linear interactions are possible classically for the $N = 0$ theory
  The field equations are given by a scalar-dependent modification of the equations for SD gravity coupled to SD YM, and the (noncovariant) action is presumably of the Plebanski type

- This $N = 0$ theory may be closely related to the interacting theory of SD gravity coupled to SD YM arising from the (Ooguri-Vafa) $N = 2$ world-sheet supersymmetric string...

  The theories with $N \leq 4$ are supersymmetric extensions of the $N = 0$ theory, and could be consistent at the interacting level if the $N = 0$ theory is

  However there are issues about the quantum consistency of such interactions, and Siegel has argued that only the $N = 4$ supersymmetric case is fully consistent...

- The determination of the precise form of the interactions must await a detailed investigation of the scattering amplitudes in the new twistor string models...

Gauged Berkovits Models

- Consider the Berkovits twistor string with target a supermanifold $M = \mathcal{T}_{D|N}$, with D bosonic dimensions and N fermionic ones; the flat twistor spaces are $\mathbb{C}^{D|N}$, $\mathbb{R}^{D|N} \times \mathbb{R}^{D|N}$ or $\mathbb{C}^{D|N} \times \mathbb{C}^{D|N}$

  The case of physical interest is $D = 4$, and $N = 4$ is special in twistor theory, but can consider the general case

- The fibration $\mathcal{T} \rightarrow \mathbb{C}^2 - 0$ respectively $P\mathcal{T} \rightarrow \mathbb{CP}^1$ in the Ricci-flat case implies the existence of the 1-form

$$k = I_{\alpha\beta} Z^\alpha dZ^\beta$$

In the supersymmetric case

$$PT_{[D-1|N]} \to \mathbb{CP}^{1|0}$$

or

$$PT_{[D-1|N]} \to \mathbb{CP}^{1|N}$$

A local basis of 1-forms on $\mathbb{CP}^{1|0}$ or $\mathbb{CP}^{1|N}$ pull back to 1-forms k and k^i, $i = 1 \ldots k^i$ on $\mathcal{T}_{D|N}$ and we can gauge the corresponding symmetries

- Given 1-forms $k^i = k^i_I(Z)dZ^I$, $\tilde{k}^i = \tilde{k}^i_I(\tilde{Z})d\tilde{Z}^I$ of scaling weights $h_i, \tilde{h}_i$ there are currents

$$K^i = k^i_I \partial Z^I, \qquad \tilde{K}^i = \tilde{k}^i_I \tilde{\partial} \tilde{Z}^I$$

which are conserved Kač-Moody currents

We assume that the $k^i_I, \tilde{k}^i_I$ are horizontal so that the currents can be gauged:

$$Z^I k^i_I = 0 \qquad \tilde{Z}^I \tilde{k}^i_I = 0$$

NB For Euclidean Σs ($\tilde{\sigma} = \sigma^*$ and $\tilde{Z} = Z^*$), $K^i = \tilde{K}^i$; otherwise the $\tilde{K}^i$ and K^i are independent currents satisfying $K^i = \tilde{K}^i$ on $\partial\Sigma$

- The gauged action is

$$S' = \int_\Sigma d^2\sigma \left(Y_I \tilde{\partial} Z^I + \tilde{Y}_J \partial \tilde{Z}^J - \tilde{A} J - A \tilde{J} - B_i \tilde{K}^i - \tilde{B}_i K^i \right) + S_C$$

and this is invariant under the gauge transformations

- Gauge fixing: choose conformal gauge and set

$$A = \tilde{A} = B_i = \tilde{B}_i = 0 \qquad \forall i$$

Then in addition to the ghosts (u, v) and $(\tilde{u}, \tilde{v})$ of the Berkovits string, one has the ghost system (r^i, s_i) and its conjugate system $(\tilde{r}^i, \tilde{s}_i)$

The open string boundary conditions then include the conditions

$$c = \tilde{c} \quad b = \tilde{b} \quad v = \tilde{v} \quad u = \tilde{u} \quad r^i = \tilde{r}^i, \quad s_i = \tilde{s}_i$$

on the ghosts

- The left-moving BRST operator is

$$Q = \oint d\sigma \left(cT + vJ + s_i K^i + cu\partial v + cb\partial c + cr^i \partial s_i - \sum_i v h_i s_i r^i \right)$$

and calculation gives

$$Q^2 \sim C \oint c\partial^3 c + \kappa \oint v\partial v$$

The Virasoro central charge is

$$C = D - N + c_C - 28 - 2(d - n)$$

Here $D - N$ comes from the YZ system, c_C is from the extra CFT, $-28 = -26 - 2$ is from the bc and uv systems and $-2(d-n)$ is from the (r^i, s_i) systems with d fermionic and n bosonic ghosts

The Kač-Moody central charge is

$$\kappa = D - N - \sum_i \epsilon_i (h_i)^2$$

where $\epsilon_i = 1$ for bosonic symmetries (with α_i bosonic) and $\epsilon_i = -1$ for fermionic symmetries (with α_i fermionic)

(There is a similar story for the right-movers)

Quantum consistency requires $C = \tilde{C} = 0$ and $\kappa = \tilde{\kappa} = 0$ so

$$\begin{aligned} D - N + c_C - 28 - 2(d-n) &= 0 \\ D - N - \sum_i \epsilon_i (h_i)^2 &= 0 \end{aligned}$$

- There are many formal solutions to these equations and thus many anomaly-free twistor string models

- The n-point tree-level scattering amplitudes for the Berkovits string are calculated from open string correlation functions with vertex operators V_f, V_g, V_ϕ inserted on $\partial\Sigma$:

$$\sum_d \left\langle cV_1(\sigma_1)cV_2(\sigma_2)cV_3(\sigma_3) \int d\sigma_4 V_4(\sigma_4) \ldots \int d\sigma_n V_n(\sigma_n) R \right\rangle_d$$

where V_i are any of the vertex operators V_f, V_g, V_ϕ and $\langle \ldots \rangle_d$ is the correlation function on a disc of degree d, corresponding to a gauge instanton on the disc with a topologically non-trivial configuration for the gauge field A characterised by the integer d

- For e. g. the new theories corresponding to $N = 8$ or $N = 4$ SUGRA coupled to SYM, the main difference is the additional ghost sector

  The extra terms in the BRST operator give extra constraints and extra gauge invariances for the f^I, g_I

  e. g. in the $N = 8$ theory, there is an extra anti-commuting ghost s of conformal weight zero, which has one zero mode on the disc, so that one insertion of the s zero-mode is needed to obtain a non-zero amplitude

- For the $N = 4$ theories there is in addition one zero-mode for each of the four commuting ghosts s^a, and the integral over these can be handled by choosing appropriate pictures for the vertex operators V_i

- Upon integrating out the zero-modes of the new ghosts, the tree-level correlation functions for the new $N = 4$ and $N = 8$ theories have the same form as for the Berkovits string when written in terms of f^I, g_I, ϕ_r but these are subject to further constraints and have further gauge invariances

  These can be used to write f^I, g_I in terms of unconstrained wave-functions $h, \tilde{h}$ for the $N = 4$ theory, or $h, \tilde{h}, f^a, g_a$ for the $N = 8$ theory

- The $h, \tilde{h}$ resp. the $h, \tilde{h}, f^a, g_a$ are wave-functions for SUGRA and matter systems whose field equations are of 2nd order in space-time derivatives for bosons (1st order for fermions), not those for CSUGRA with 4th order equations for bosons

- When written in terms of $h, \tilde{h}$ or $h, \tilde{h}, f^a, g_a$, the scattering amplitudes of the new twistor strings then give interactions for Einstein gravitons and matter...

- e. g. at degree zero we find

$$\langle V_g V_g V_g \rangle = 0 \qquad \langle V_f V_g V_g \rangle = 0$$

and

$$\langle V_{f_1} V_{f_2} V_{g_3} \rangle = \int_{\mathbb{RP}^{3|4}} f_1^I f_2^J \partial_{[I} g_{3J]} \Omega_s$$

In terms of functions $h, \tilde{h}$ which are momentum eigenstates:

$$\langle V_{f_1} V_{f_2} V_{g_3} \rangle = \delta^4(P_1 + P_2 + P_3) \left(\frac{p_{1A} p_2^A}{p_{3B} p_1^B p_{2C} p_3^C} \right)^2$$

- Thus the new $N = 4$ and $N = 8$ twistor string theories each have at least one non-trivial interaction, and this gives precisely the helicity $(+ + -)$ 3-graviton interaction of Einstein gravity

- Under scaling the infinity twistor $I^{IJ} \to RI^{IJ}$, $\epsilon^{AB} \to R\epsilon^{AB}$, so that if f^I, g_I are kept fixed, then $h \to R^{-1}h$ and $\tilde{h} \to R\tilde{h}$

  Then the amplitude scales as R^{-1}, so that R^{-1} sets the strength of the gravitational coupling

- Computing the remaining 3-point functions will determine whether the interactions in spacetime are of the chiral or the nonchiral type...

Chiral Gauge and Gravity Actions

- Consider Einstein gravity, formulated in terms of a vierbein ${e^a}_\mu$ and spin-connection ${\omega_\mu}^{bc}$

  The torsion and curvature 2-forms are given by

$$T^a = de^a + {\omega^a}_b \wedge e^b \qquad {R^a}_b(\omega) = d{\omega^a}_b + {\omega^a}_c \wedge {w^c}_b$$

  In a second order formalism, one imposes the constraint

$$T^a = 0$$

This determines the spin-connection in terms of the vierbein:

$$\omega_{\mu ab} = \Omega_{\mu ab}(e)$$

Here $\Omega_{\mu ab}(e)$ is the usual expression for the Lorentz connection in terms of the vierbein:

$$\Omega_\mu{}^{ab}(e) \equiv e^{\nu a}\partial_{[\mu}e_{\nu]}{}^b - e^{\nu b}\partial_{[\mu}e_{\nu]}{}^a - e^{\rho a}e^{\sigma b}\partial_{[\rho}e_{\sigma]c}e_\mu{}^c$$

- The Einstein-Hilbert action is

$$\frac{1}{4\kappa^2}\int e^a \wedge e^b \wedge R^{cd}(\omega)\varepsilon_{abcd}$$

The same action can be used in the first order formalism in which the torsion is unconstrained and the vierbein ${e_\mu}^a$ and the connection ${\omega_\mu}^{ab}$ are treated as independent variables

The field equation obtained by varying ω implies that the Lorentz connection is the Levi-Civita connection

The vierbein field equation then gives the Einstein equation

- In e. g. split signature the spin group factorises as

$$Spin(2,2) = SU(1,1) \times SU(1,1)$$

The spin-connection decomposes into the SD piece $\omega^{(+)ab}$ and the ASD piece $\omega^{(-)ab}$
These are the independent gauge fields for the two factors of the spin group

- The curvature 2-form can also be split into SD and ASD pieces

$$R_{bc}^{(\pm)} \equiv \frac{1}{2}\left(R_{bc} \pm \frac{1}{2}\varepsilon_{bc}{}^{de} R_{de}\right)$$

It is easily seen that $R^{(+)ab}$ depends only on $\omega^{(+)}$ while $R^{(-)ab}$ depends only on $\omega^{(-)}$

- An equivalent form of the Einstein-Hilbert action is given using $R^{(+)}$ instead of R:

$$\frac{1}{2\kappa^2}\int e^a \wedge e^b \wedge R^{(+)}_{ab}(\omega)$$

This gives the chiral action plus the topological term

$$\frac{1}{2\kappa^2}\int e^a \wedge e^b \wedge R_{ab}(\omega)$$

Using the above this can be written as

$$\frac{1}{2\kappa^2}\int d(T^a \wedge e_a)$$

which vanishes in the second order formalism in which one sets $T^a = 0$ and in the first order formalism is a total derivative that does not contribute to the field equations or Feynman diagrams

- As $R^{(+)}$ depends only on $\omega^{(+)}$, the action is independent of $\omega^{(-)}$ and depends only on the vierbein and the SD spin-connection

  Moreover, the first order action is polynomial in these variables

- It is remarkable that one only needs the SD part of the spin-connection in order to formulate gravity

  The torsion constructed from $e, \omega^{(+)}$ is

$$\tilde{T}^a = de^a + \omega^{(+)a}{}_b \wedge e^b$$

  If one imposes the constraint $\tilde{T}^a = 0$, one obtains

$$\omega^{(+)ab} = \Omega^{(+)ab}(e) \qquad \Omega^{(-)ab}(e) = 0$$

  This implies $R^{(-)ab}(e) = 0$, where $R^{(-)ab}(e)$ is the ASD part of the curvature of the connection $\Omega(e)$

- Then the Riemann curvature constructed from the vierbein is SD and hence Ricci-flat, so that the torsion constraint imposes the field equations of SD gravity as well as solving for the spin-connection in terms of the vierbein

- The chiral action we wrote for gravity is

$$+\frac{1}{2}\int e^a \wedge e^b \wedge \left(d\omega_{ab} + \kappa^2 \omega_{ac} \wedge \omega^c{}_b\right)$$

where $\omega \equiv \omega^{(+)}$ and we have rescaled the connection by the gravitational coupling κ^2

Varying independently with respect to $\omega^a{}_b$ and e^a_μ, gives the usual expressions for the connection in terms of the vierbein, and the

Einstein equation

$$e^a \wedge \left(d\omega_{ab} + \kappa^2 \omega_{ac} \wedge \omega^c{}_b\right) = 0$$

- Now taking the limit $\kappa \to 0$ yields a weak-coupling limit of gravity with action

$$\frac{1}{2}\int e^a \wedge e^b \wedge d\omega_{ab}$$

which can be rewritten as

$$-\int e^a \wedge e^b \wedge \omega_{ac} \wedge \Omega^{(+)c}{}_b$$

where $\Omega^{(+)} = \Omega^{(+)}(e)$ is the SD part of the connection

- This is an action for two independent fields, the vierbein ${e_\mu}^a$ and the SD connection ω^{ac}

  The latter now plays the role of a Lagrange multiplier field

  Note that the self-duality of ω^{ac} implies that only the SD part $\Omega^{(+)}$ of $\Omega(e)$ occurs in the action

- The field equation from varying the Lagrange multiplier field ω^{ac} sets the SD part of $\Omega(e)$ to zero,

$$\Omega^{(+)a}{}_b(e) = 0$$

This implies that the SD part of the curvature constructed from the Levi-Civita connection $\Omega(e)$ vanishes

$$R_{\mu\nu}{}^{ab}(\Omega^{(+)}) = 0$$

so that the vierbein gives a metric with ASD Riemann curvature

The field equation for the vierbein gives

$$e_b \wedge d\omega^{ab} = 0$$

This can be seen to be a version of the Einstein equation linearised around the ASD background spacetime described by the tetrad e^a, where the linearised graviton field is the SD connection ω^{ac}

- The fact that ω^{ab} and $\Omega^{(-)}(e)$ are respectively SD and ASD means that they describe particles of opposite helicity:

  e describes a particle of helicity $+2$ and ω describes a particle of helicity -2

  The linearized spectrum is the same for the chiral action and the Einstein-Hilbert action, but the interactions differ as the chiral gravity action has no $--+$ vertex

- This formulation of gravity extends to one of $N=8$ SUGRA in which the Einstein term is written in the chiral form above and the vector field kinetic terms take the Chalmers-Siegel form

  In the weak coupling limit, it gives Siegel's form of $N=8$ SUGRA