Three-Charge Black Holes on a Circle

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(with **Harmark, Kristjansson, Roenne**)

0503020 (**review**), **0407094** (JHEP), **0403103** (JHEP),

0309230 (NPB), **0309116** (CQG),, **0204047** (JHEP)
(with **T. Harmark**)

0407050 (JHEP) (with **H. Elvang and T. Harmark**)

0509011 (JHEP) (with **V. Niarchos and T. Harmark**)

Introduction/Motivation

- 3-charge black holes (in 5D) play prominent role in ST:

  • BH entropy can be explained by microscopic counting of degeneracy of states (D-brane configurations)

  Strominger,Vafa/Callan,Maldacena/
  Horowitz,Strominger/Horowitz,Maldacena,Strominger

  • fertile ground for further exploration of microscopic origin of entropy (also: 2-charge case)

  Mathur et al

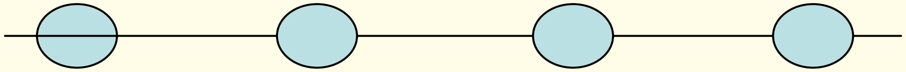
  • AdS_3/CFT_2 correspondence

- what happens to entropy of these 3-charge systems when we compactify one of the directions, i.e. asymptotic space is $\mathcal{M}^4 \times S^1$

extremal

→ infinite array on covering space of circle $S = 2\pi\sqrt{N_1 N_4 N_0}$.

non-extremal



Main results

- can compute corrections to the entropy in small mass (large radius) limit

  - result for near-extremal limit

$$S = 2\pi\sqrt{N_1 N_4 N_0}\left(1 + \sqrt{\frac{\epsilon}{8}} + \frac{\epsilon}{16} + \mathcal{O}(\epsilon^{3/2})\right)$$

  - also compute: partial near-extremal limit (one finite charge)

- show that: first correction to finite entropy is in agreement with microscopic entropy formula

$$S = 2\pi\sqrt{N_1 N_4}\left[\sqrt{N_0} + \sqrt{\bar{N}_0}\right]$$

need to take into account that number of D0 and anti-D0 branes shift as consequence of interactions across circle

- also obtain:
  - entire phase (numerically) of non- and near-extremal three-charge black holes localized on circle
  - rich phase structure that includes new phase of three-charge black holes that are non-uniformly distributed on circle

Method: Map GR to String/M-theory

static and neutral Kaluza-Klein black holes

solutions of pure gravity with event horizon asymptoting to d-dimensional Minkowski-space times a circle

boost/U-duality → non-extremal branes on a transverse circle (II/M) (Hassan,Sen)

↓ near-extremal limit

near-extremal branes

- was applied to get:

Harmark,NO

Bostock,Ross/Aharony,Marsano,Minwalla,Wiseman/

near-extremal p-branes on a transverse circle (p = 9 - d) ⟷ gauge/gravity correspondence ⟷ thermal physics of (p +2)-dim. SYM on S^1; (2,0) LST

- here: focus on KK BHs in five dimensions (d =4) Harmark,Kristjansson,NO,Roenne

↓

non- and near-extremal three charge branes on transverse circle

- D0-D4-F1 charges (related by T-duality to D1-D5-P)
- reducing on w.v. gives 5D Sugra soln

Kaluza-Klein Black Holes

- Kaluza-Klein black holes are interesting in their own right

  - richer phase structure when compact directions present

  - gravitational phase transitions between different solutions with event horizons (sometimes: topology change)

  - end-point of Gregory-Laflamme instability (new phases)

  - (non)-uniqueness theorems in higher dimensional gravity
    Cf. situation in asymptotically flat spaces: black rings etc.
  - possible objects in universe/accelerators

- phase structure of Kaluza-Klein black holes related to objects and phenomena in string theory/gauge theory

→ many of these aspects have direct relevance for the physics of 3-charge black holes on circle

in particular: much is know about phase structure of 5D KK BHs

Mass and tension for KK BHs

consider static solutions of vacuum Einstein equations with event horizon

- coordinates for $\mathcal{M}^d \times S^1$ $(d \geq 4)$

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega_{d-2}^2 + dz^2 \qquad z \sim z + L$$

asymptotics $$g_{tt} \simeq -1 + \frac{c_t}{r^{d-3}} \ , \quad g_{zz} \simeq 1 + \frac{c_z}{r^{d-3}}$$

→ 2 (gauge-invariant) asymptotic quantities

Harmark,NO/Kol,Sorkin,Piran/
Traschen,Fox/Towsend,Zamaklar

$$M = \frac{\Omega_{d-2} L}{16\pi G_N}\left[(d-2)c_t - c_z\right] \ , \quad \mathcal{T} = \frac{\Omega_{d-2}}{16\pi G_N}\left[c_t - (d-2)c_z\right]$$

mass → M; tension → $\mathcal{T}$

1st law of thermo $$\delta M = T\delta S + \mathcal{T}\delta L$$

Smarr formula $$TS = \frac{d-2}{d-1}M - \frac{\mathcal{T}L}{d-1}$$

using Komar integral
(time-translation sym.)

(μ,n) phase diagram for KK BH's

dimensionless physical quantities

$$\mu = \frac{16\pi G_{\rm N}}{L^{d-2}} M \ , \quad n = \frac{\mathcal{T} L}{M}$$

rescaled mass

relative tension

bounds $\mu \geq 0 \ , \quad 0 \leq n \leq d-2$

Traschen/Shiromizu,Ida,Tomizawa

positive tension

gravitational force on test particle at infinity attractive

plot phase structure in terms of (μ,n)-phase diagram

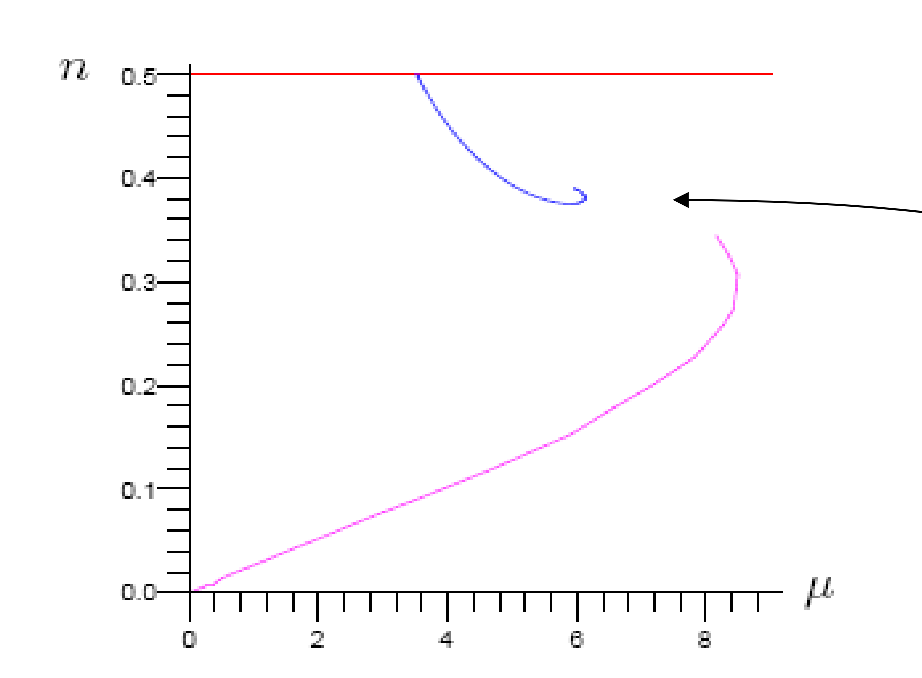
1st law of thermo $\delta\mu = \mathfrak{t}\,\delta\mathfrak{s}$

generalized Smarr $\mathfrak{t}\mathfrak{s} = \frac{d-2-n}{d-1}\mu$

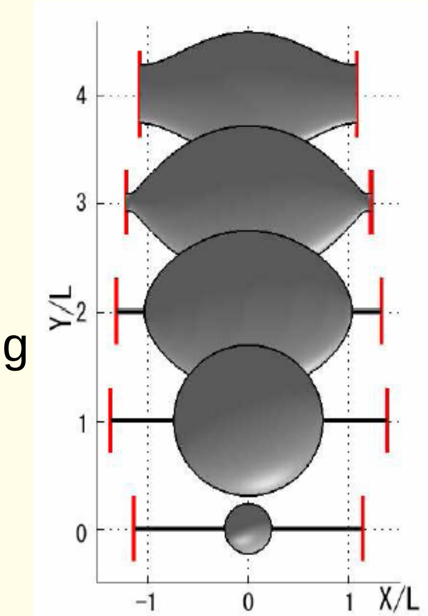
knowledge of curve $n(\mu)$ in phase diagram

can derive entire thermodynamics

black hole/string region $0 \leq n \leq \frac{1}{d-2}$

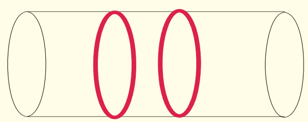


black hole/black string transition



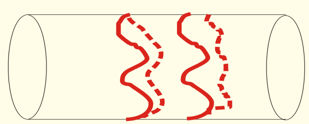
Koduh,Wiseman

•uniform black string (US)



Schwarzschild-Tangherlini(d) $\times$ S^1

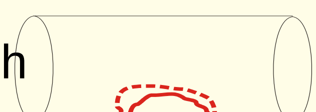
•non-uniform black string (NUS)



emanates from uniform at Gregory-Laflamme point

Gregory,Laflamme/Gubser/Wiseman/Sorkin
motivated in part by: Horowitz,Maeda

•localized black hole branch (LBH)



Schwarzschild (d +1) + $\mathcal{O}$ (μ)

Harmark,NO/Harmar/Kol,Gorbonos/
Sorkin,Kol,Piran/Koduh,Wiseman

Black holes and strings on cylinders

- uniform black string phase

$$ds^2 = -\left(1 - \frac{r_0}{r}\right) dt^2 + \left(1 - \frac{r_0}{r}\right)^{-1} dr^2 + r^2 d\Omega_2^2 + dz^2$$

- non-uniform black string phase

Wiseman, Sorkin

Kleihaus,Kunz,Radu

d <13 : branch numerically known

$$n(\mu) = \frac{1}{2} - 0.14(\mu - \mu_{\mathrm{GL}}) + \mathcal{O}((\mu - \mu_{\mathrm{GL}})^2)\ , \quad 0 \leq \mu - \mu_{\mathrm{GL}} \ll 1$$

- localized black hole phase

Kudoh,Wiseman

d=4,5: branch numerically known

Harmark/Kol,Gorbonos

all d : (analytic) first correction to the metric known

recently: also second correction

Karasik et al/Chu,Goldberger,Rothstein

$$n(\mu) = \frac{\mu}{6^2} - \frac{\mu^2}{6^4}$$

The seeding solution

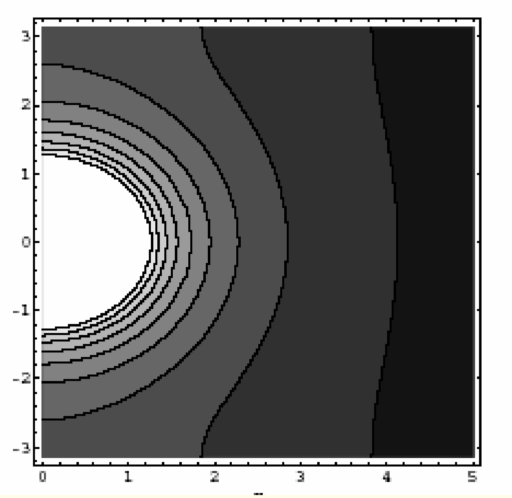
- solutions with $0 < n < \frac{1}{2}$ ⟶ SO(3) symmetry Harmark,NO//Wiseman

$$ds^2_{4+1} = -f dt^2 + \frac{L^2}{(2\pi)^2}\left[\frac{A}{f}dR^2 + \frac{A}{K^2}dv^2 + KR^2 d\Omega_2^2\right] \ , \quad f = 1 - \frac{R_0}{R}$$

•involves two functions

•$A(R,v)$ determined in terms of $K(R,v)$

•(R,v) coordinates: interpolate between spherical - cylindrical



lines of constant R are equipotential surfaces
of extremal p-branes with transverse space
$R^{d-1} \times S^1$

(equivalently: charged particle on circle)

Generating non- and near-extremal 3-charge black holes

- lift 5D KK BH to 10D → act with three boosts + various U-dualities

  ⇨ non-extremal D0-D4-F1 system with circle in transverse space

  → take near-extremal limit that keeps non-trivial physics of circle

  ⇨ near-extremal D0-D4-F1 system with circle in transverse space

- map of physical quantities from neutral (μ, n) phase diagram to near-extremal 3-charge system

$$\epsilon = \frac{1}{2}\mu n, \qquad r = 2, \qquad r_a = 1 \ , \quad a = 1, 4, 0$$

energy above extremality

$$\hat{\mathfrak{t}} = \mathfrak{t}(\mathfrak{t}\mathfrak{s})^{3/2}, \quad \hat{\mathfrak{s}} = \mathfrak{s}(\mathfrak{t}\mathfrak{s})^{-3/2}$$

new feature (compared to 1-charge):

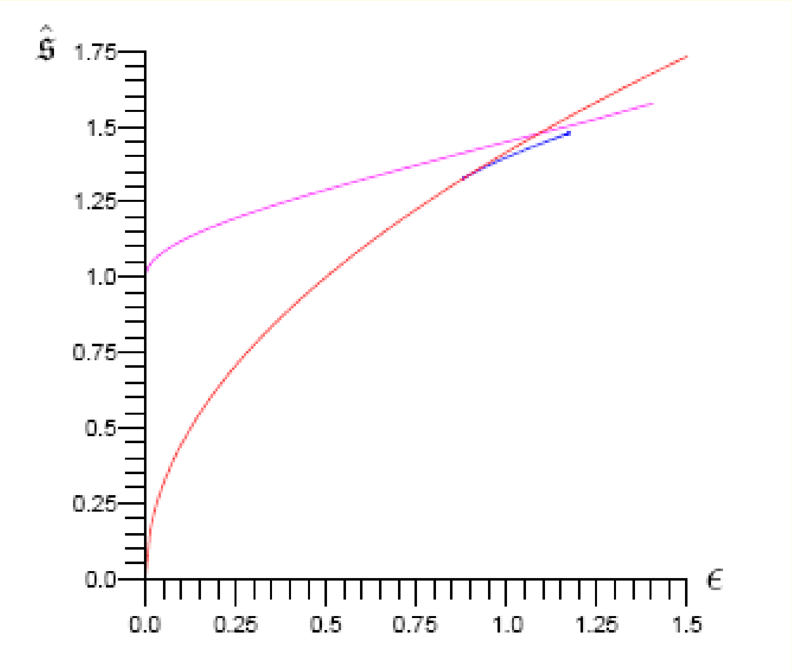
constant relative tension r in transverse circle (collapse of the phases on top of each other)

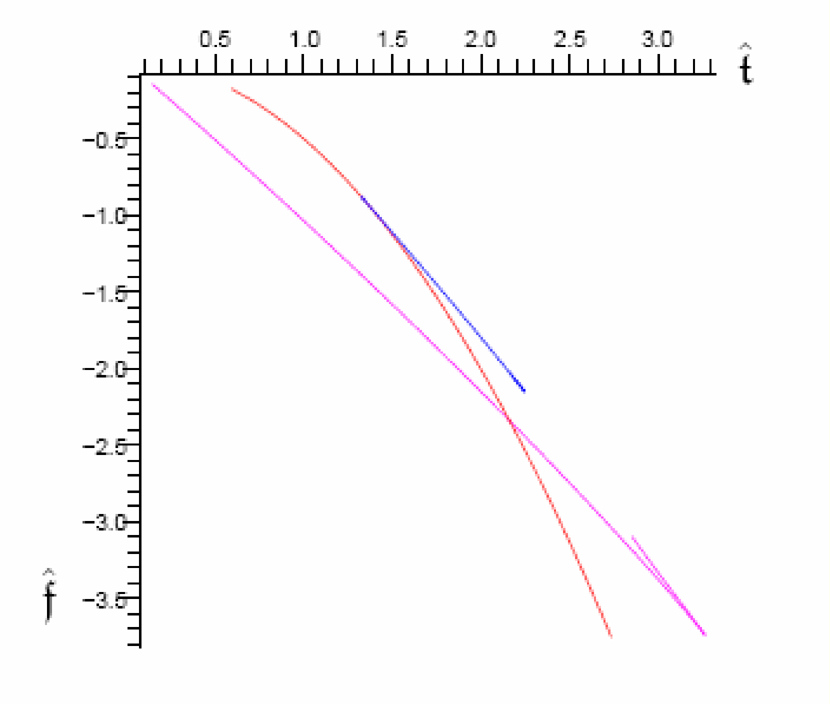
intimately related to the finite non-vanishing entropy in the extremal limit

Three phases of near-extremal 3-charge black holes

black hole/string phase map directly onto phases of near-extremal 3-charge black holes on a circle

- **uniform phase**: uniformly smeared over transverse circle
- **non-uniform phase**: non-uniformly distributed around circle
- **localized phase**: localized on transverse circle





Entropy for localized phase

- from analytic results for the corrected entropy of small localized KK BHs (in pure GR) we can thus get correction to the finite entropy for localized phase of near-extremal 3-charge BHs on circle

$$S = 2\pi\sqrt{N_1 N_4 N_0}\left(1 + \sqrt{\frac{\epsilon}{8}} + \frac{\epsilon}{16} + \mathcal{O}(\epsilon^{3/2})\right)$$

- can also consider other cases + limits

  • 2 charges large, 1 finite
  • 1 charge large, 2 finite
  • 2-charge BHs (1 charge =0)

Hagedorn behavior in near-extremal limit

$$\epsilon = \rho_0^2 \sinh^2 \alpha_1 + \frac{1}{2}\rho_0^2\left(1 + \frac{1}{16}\rho_0^2\right) + \mathcal{O}(\rho_0^6)$$

$$\hat{\mathfrak{s}} = \rho_0 \cosh \alpha_1 \left(1 + \frac{1}{16}\rho_0^2 + \frac{1}{512}\rho_0^4\right) + \mathcal{O}(\rho_0^7)$$

⇒ leading correction can be reproduced by microscopic derivation

Microscopic computation on circle

Consider D0-D4-F1 + go slightly off extremality by adding anti-D0

$$S = 2\pi\sqrt{N_1 N_4}\left(\sqrt{N_0} + \sqrt{N_{\bar{0}}}\right).$$

(entropy additive in dilute gas approximation)

write total mass of 3-charge system as

$$\bar{M} = Q_1 + Q_4 + \delta E + V_{\rm int}$$

(see: Costa,Perry/Emparan,Teo
for application to collinear BHs/ diholes)

→ energy stored in tension

→ find effective number of D0/anti-DO branes from

$$\delta E \simeq N_0' + N_{\bar{0}}' \ , \quad Q_0 \simeq N_0' - N_{\bar{0}}'$$

(shift due to interactions across circle)

⇒ microscopic entropy formula above reproduces first order corrected macroscopic entropy

higher corrections ?

Conclusion/Outlook

- generated new three-charge BH solutions when there is circle in transverse space
- found first corrections to the finite entropy of (extremal) localized 3-charge BHs
  + matched first correction with microscopic calculation
- new phase of non-uniformly distributed non-and near-extremal 3-charge BHs on circle

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- matching with microstate counting for localized phase to higher order ?
- other new 3-charge solutions (from black hole-Kaluza Klein bubble phase of neutral KK BHs)
- further examination of non-uniform phase of 3-charge BHs and the classical stability of the uniform phase (cf correlated stability conjecture)
  (for 1-charge case: map of instability of neutral uniform string to that of smeared branes
  Aharonoy et al/Harmark, Niarchos,NO
- relation to Mathur's proposal
  very recently: Chowdhury,Giusto,Mathur: use `brane fractionation' to propose microscopic picture of transition between black hole/string transition that reproduces many broad features of the phase diagram

Upper part of phase diagram

black holes/strings in region $0 \leq n \leq \frac{1}{d-2}$

what about region ? $\frac{1}{d-2} \leq n \leq d-2$

⇨ contains solutions with Kaluza-Klein bubbles Witten

↘ minimal S^2 surfaces in space-time (``bubble of nothing'')

static Kaluza-Klein bubble

$$ds^2 = -dt^2 + \left(1 - \frac{R^{d-3}}{r^{d-3}}\right) dz^2 + \frac{1}{1 - \frac{R^{d-3}}{r^{d-3}}} dr^2 + r^2 \Omega_{d-2}^2$$

to avoid conical singularity: z periodic with $L = \frac{4\pi R}{d-3}$

⇨ static KK bubble is static solution of pure gravity asymptoting to KK space

specific point in the phase diagram with $(\mu, n) = (\mu_b, d-2)$

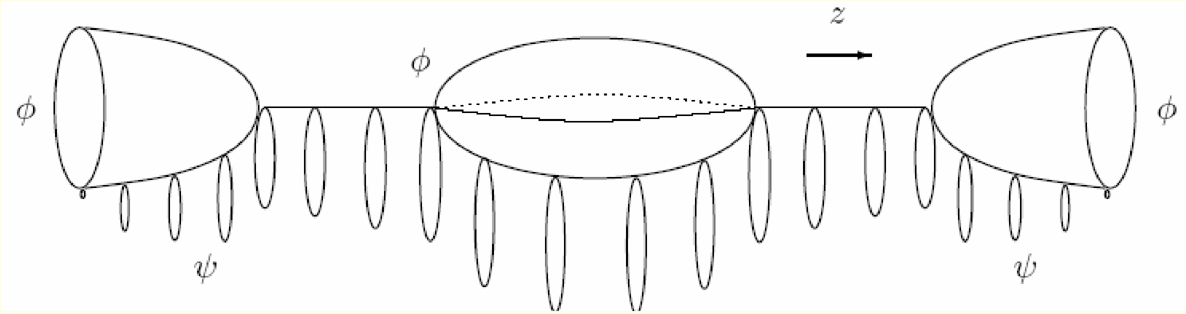
other instance of critical dimension: for $D \leq 10$ $\mu_b < \mu_{GL}$

Bubble-black hole sequences

Emparan,Reall/Elvang,Horowitz/Elvang,Harmark,NO

- found explicitly using generalized Weyl ansatz in five and six dimensions (with compact circle)
- exact, regular and static solutions of vacuum Einstein equations
- new topologies (black ring $S^2 \times S^1$, black tuboid $S^2 \times S^1 \times S^1$)
- infinite non-uniqueness

black hole – bubble – black hole- bubble – black hole

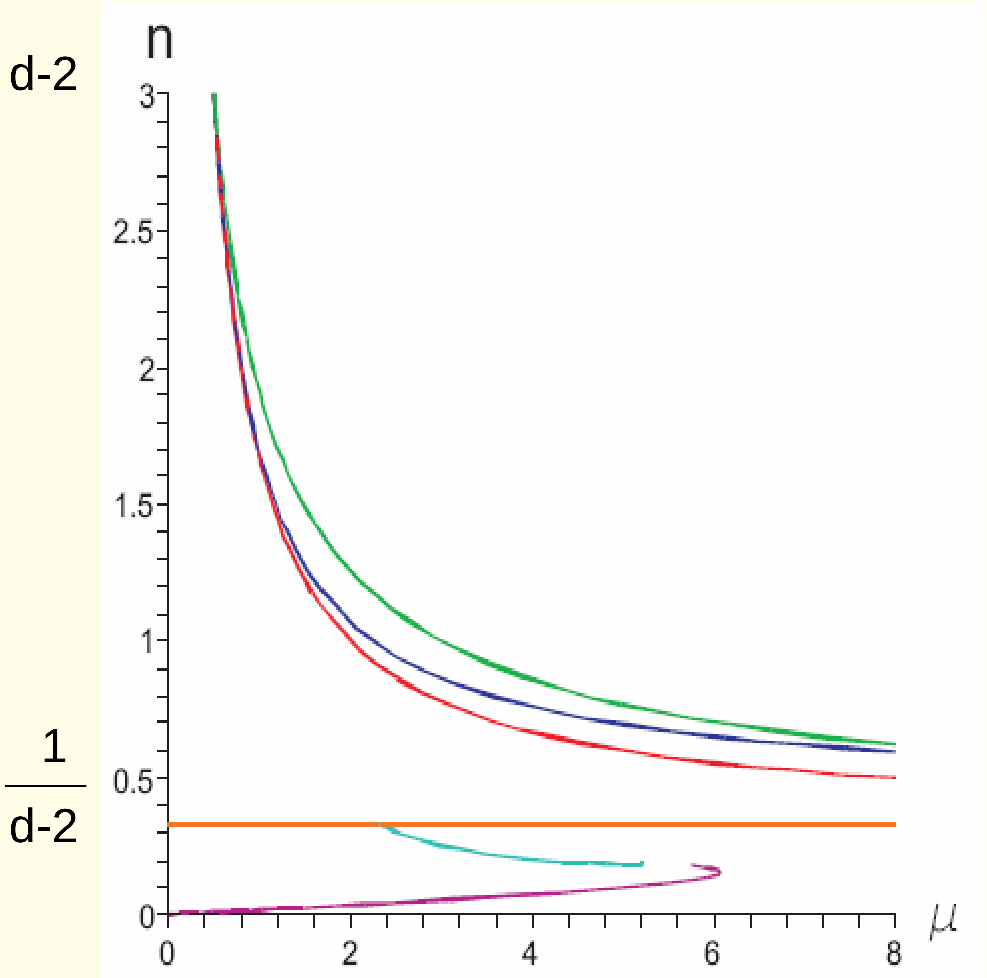


S^3 $S^1 \times I$ $S^2 \times S^1$ $S^1 \times I$ S^3

⇒ bubbles play double role:

- holding the black holes apart
- support S^1 's on horizon against gravitational collapse

Present knowledge of KK black holes



solutions with
Kaluza-Klein bubbles

BH – bub –BH –… - bub

Emparan,Reall/Horowitz,Elvang/
Elvang,Harmark,NO

solutions without KK bubbles
and local SO(d-1)symmetry

- black holes: S^{d-1}
- black strings: $S^{d-2} \times S^1$

phase diagram for six dimensions

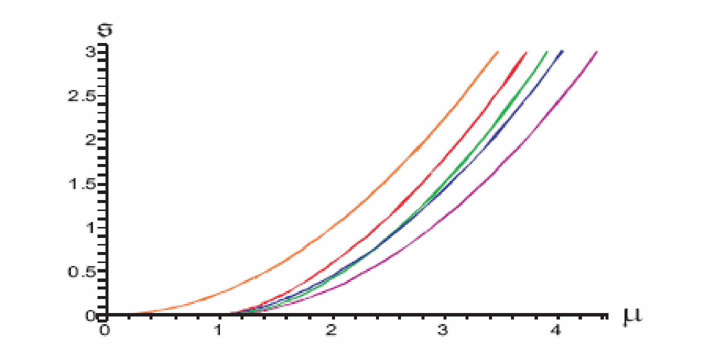
Some features of bubble-black hole sequences

- (p,q) bubble-black hole sequences have q parameters (size of each black hole) ⟶ large degree of non-uniqueness

- map between 5D and 6D case

- can associate temperature to each disconnected horizon component
  -special 1-parameter class of solutions exists with all temperatures equal (minimizes entropy)

- entropy of (1,1) solution always less than uniform string of same mass



- stability ?
  -static bubble is unstable (GPY negative mode): expands/collapses

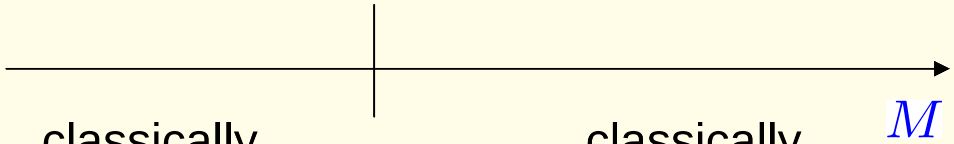
Brill,Horowitz/Corley,Jacobson/Sarbach,Lehner

Gregory-Laflamme instability

Gregory,Laflamme (1993)

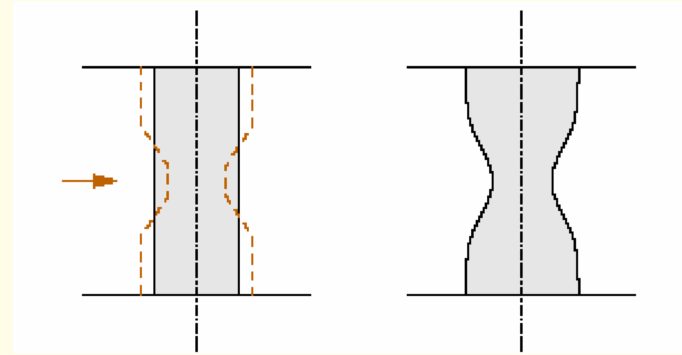
GL instablity of neutral black strings: perturbations oscillating in direction
In which string extends $\lambda \gtrsim r_{\mathsf{hor}}$

$M < M_{\mathsf{GL}}$



classically unstable

classically stable



$$\delta g_{\mu\nu} \sim e^{i\mu z} h_{\mu\nu} \;, \quad h_{\mu\nu} \propto e^{\Omega t}$$

$$\Delta_{\mathsf{Lich}} \; \delta g_{\mu\nu} = 0$$

threshold mode $(\Omega = 0)$

non-uniform static solution emerges

thermodynamic argument for instability: $S_{\mathsf{bh}} > S_{\mathsf{bs}}$ for mall masses

interpretation: black string decays to black holes

important observation:
classical Lorentzian threshold unstable mode
corresponds to Euclidean negative mode

$$\tilde{\Delta}_{\mathsf{Lich}} h_{\mu\nu} = -\mu^2 h_{\mu\nu}$$

From Neutral Strings To Smeared Dp-branes

solutions of pure gravity with event horizon asymptoting to d-dimensional Minkowski-space times a circle (d = 9- p)

for recent reviews: Harmark,NO/Kol

boost/
U-duality

Harmark,NO/Bostock,Ross/
Aharony,Marsano,Minwalla,Wiseman

static and neutral
Kaluza-Klein black holes

non- and near-extremal
Dp-branes
on a transverse circle

neutral black strings

smeared Dp-branes

GL point

critical mass/energy point

unstable mode at threshold:
new static phase of non-uniform
strings emerges

Gregory,Laflamme/Gubser/Wiseman

new phase of non- and near-extremal
branes non-uniformly distributed
on circle emerges

suggests that non-and near-extremal smeared branes exhibit classical instabilities