AdS Spacetimes in M-Theory

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Outline of the talk

1. Motivations and overview
2. Supersymmetry conditions for wrapped branes
3. The AdS limit and supersymmetry conditions
4. New explicit solutions
5. Conclusions

1. Motivations and overview

The main objectives of this project are to provide an exhaustive map of the supersymmetric AdS landscape of M-theory, and to construct explicit new solutions. We extract from the Killing spinor equation of d=11 supergravity, all the general local geometrical properties of various classes of AdS spacetimes, of different dimensionalities and preserving various amounts of supersymmetry, that it encodes. We then use this information to construct explicit new examples.

In keeping with the general philosophy of AdS/CFT, one would expect that all AdS spacetimes in M-theory could be obtained as a decoupling limit of some brane configuration. Therefore we first focus on deriving all the properties of branes wrapping calibrated cycles in special holonomy manifolds that are implied by the Killing spinor equation. We then take the AdS limits of these wrapped brane spacetimes, and their supersymmetry conditions, to obtain the AdS supersymmetry conditions.

In one case, we have been able to use the supersymmetry conditions so derived to construct eight new doubly countably infinite families of AdS_3 solutions of M-theory and type IIB.

1. Motivations and overview

In more detail, our procedure to derive first the wrapped brane, and then the AdS supersymmetry conditions, is as follows.

- First we consider probe branes wrapping supersymmetric cycles in a background which is the product of flat space with a special holonomy manifold. We deduce the Killing spinors these configurations preserve, and the associated algebraic structures. We demand that the Killing spinors of the supergravity description define the same algebraic structures, after backreaction has been turned on. We also require that as for the probe brane Killing spinors, they are simultaneous eigenspinors of five independent projection operators. This gives the fermionic part of our Killing spinor ansatz for the wrapped brane configurations.

- For the bosonic part, we assume that the metric contains a warped Minkowski factor of the appropriate dimensionality (representing the unwrapped brane worldvolume directions), that the Minkowski isometries are isometries of the full metric, and that the flux respects the Minkowski symmetries. Then supersymmetry implies that the metric takes the form

$$ds^2 = L^{-1}ds^2(\mathbb{R}^{1,p}) + ds^2(\mathcal{M}_{10-p-q}) + C^2 ds^2(\mathbb{R}^q).$$

In hep-th/0605146, we also assumed (for the cases with wrapped three-cycles) that the electric flux vanished.

1. Motivations and overview

- To take the AdS limit, we observe that written in Poincare coordinates, every AdS spacetime may be viewed as a special case of a Minkowski spacetime in one dimension lower. Thus we demand that $L = \lambda e^{2mr}$, and we must out pick the AdS radial direction from the transverse space to the Minkowski factor. Generically it will lie partly in the deformed special holonomy manifold and partly in the overall transverse space, so we may write

$$\begin{aligned} \lambda^{-1/2} dr &= \cos\theta \hat{u} + \sin\theta \hat{v}, \\ \hat{u} \in \mathcal{M}_{10-p-q}&, \quad \hat{v} \in \mathbb{R}^q. \end{aligned}$$

  The AdS metric is then given by

$$ds^2 = \lambda^{-1}[e^{-2mr} ds^2(\mathbb{R}^{1,p}) + dr^2] + ds^2(\mathcal{N}_{9-p}).$$

  We demand that the AdS isometries are isometries of the full metric, and that the flux has no components along the AdS radial direction. Inserting this limit into the brane supersymmetry conditions, we derive the AdS supersymmetry conditions.

1. Motivations and overview

There are many possibilities for supersymmetric cycles on which we can wrap fivebranes. Arranged according to the dimensionality of the unwrapped Minkowski worldvolume directions/ AdS limits, these are as follows.

$$\mathbb{R}^{1,3}, \quad AdS_5$$

Cycle	***G***	***N***
Kähler-2	*SU(3)*	1
Kähler-2	*SU(2)*	2

$$\mathbb{R}^{1,2}, \quad AdS_4$$

Cycle	***G***	***N***
Associative	*G_2*	1
SLAG-3	*SU(3)*	2

In these cases, one could also include space-filling membranes.

1. Motivations and overview

$$\mathbb{R}^{1,1}, \quad AdS_3$$

Cycle	***G***	***N***
Cayley	*Spin(7)*	(1,0)
Kähler-4	*SU(4)*	(2,0)
SLAG-4	*SU(4)*	(1,1)
Quaternionic Kähler	*Sp(2)*	(3,0)
Complex Lagrangian	*Sp(2)*	(2,1)
Kähler-4	*SU(2)xSU(2)*	(4,0)
Kähler-2 x Kähler-2	*SU(2)xSU(2)*	(2,2)
Co-associative	*G_2*	(2,0)
Kähler-4	*SU(3)*	(4,0)

In all cases where the special holonomy manifold is eight dimensional, we can include membranes intersecting the fivebranes in a string, and extended in the overall transverse direction.

1. Motivations and overview

$$\mathbb{R}, \quad AdS_2$$

Cycle	***G***	***N***
SLAG-5	*SU(5)*	2
Kähler-2 x SLAG-3	*SU(2)xSU(3)*	4

In these cases, we can also include membranes wrapping Kähler two-cycles.

II. Supersymmetry conditions for wrapped branes

As a specific example to illustrate the derivation of the brane supersymmetry conditions, consider probe M5s wrapping a co-associative four cycle in $\mathbb{R}^{1,3} \times \mathcal{M}_{G_2}$. We introduce a frame

$$ds^2 = 2e^+e^- + (e^1)^2 + ... + (e^9)^2,$$

where $e^{\pm}$ span the unwrapped worldvolume directions, and the overall transverse directions are $e^{8,9}$. In the abscence of the probe brane, the spacetime admits four Killing spinors, given by the projections

$$\Gamma^{1234}\epsilon^i = \Gamma^{3456}\epsilon^i = \Gamma^{1357}\epsilon^i = -\epsilon^i.$$

The probe brane breaks half of these supersymmetries; we may choose the brane projection to be

$$\Gamma^{+-1234}\epsilon^i = -\epsilon^i.$$

The probe brane configuration thus admits two Killing spinors, which together define a $(G_2 \ltimes \mathbb{R}^7) \times \mathbb{R}^2$ structure in eleven dimensions. From the spinor bilinears, we can construct the G_2 invariant forms $\Phi, \ \Upsilon$. We demand the existence of the same algebraic structure in the supergravity description of the system.

II. Supersymmetry conditions for wrapped branes

Once we turn on backreaction, $\mathcal{M}_{G_2}$ is deformed away from G_2 holonomy, but still admits a G_2 structure. The deviation from special holonomy is encoded by the intrinsic torsion. Supersymmetry implies that the metric is given by

$$ds^2 = L^{-1}ds^2(\mathbb{R}^{1,1}) + ds^2(\mathcal{M}_{G_2}) + L^2 ds^2(\mathbb{R}^2).$$

The supersymmetry conditions take the form of algebraic constraints on the intrinsic torsion, and algebraic relationships between the torsion and the flux. The conditions on the intrinsic torsion may be expressed as

$$\begin{aligned} \mathrm{Vol}[\mathbb{R}^2] \wedge \mathrm{d}\Phi &= 0, \\ \mathrm{d}\left(L^{-1}\mathrm{Vol}[\mathcal{M}_{G_2}]\right) &= 0, \\ \Phi \wedge \mathrm{d}\Phi &= 0. \end{aligned}$$

Given a solution of these conditions, the flux is completely determined by the torsion, according to

$$F = \star \mathrm{d}\mathrm{Vol}[M5],$$

where

$$\mathrm{Vol}[M5] = L^{-1}\mathrm{Vol}[\mathbb{R}^{1,1}] \wedge \Upsilon$$

II. Supersymmetry conditions for wrapped branes

For all the other cases, the torsion conditions are qualitatively similar. Given our assumptions, the flux for M5s wrapping cycles in manifolds of dimension d<8 is given by

$$F \quad = \quad \star \mathrm{dVol}[M5],$$

with Vol[M5] the wedge product of the volume form on the warped Minkowski space with the calibration form of the cycle. For M5s wrapping four-cycles in eight-manifolds of special holonomy, and allowing for membrane charges, the metric takes the form

$$ds^2 = L^{-1} ds^2(\mathbb{R}^{1,1}) + ds^2(\mathcal{M}_G) + C^2 (dx^9)^2.$$

The membranes extend along the Minkowski and 9 directions. In these cases, the flux is given by

$$F = \mathrm{dVol}[M2] + \star \mathrm{dVol}[M5] + \frac{1}{2} L^{\alpha} \mathcal{L}_{e^9} \left(L^{-\alpha} \Xi \right) + F_{\text{unfixed}}.$$

III. The AdS limit and supersymmetry conditions

To take the AdS limit of the wrapped brane metrics and supersymmetry conditions, we must pick an AdS radial direction out of the space transverse to the Minkowski factor, by performing an appropriate frame rotation. Generically, the AdS radial direction will be a linear combination of a radial direction transverse to the cycle in the backreacted special holonomy manifold, and the radial direction on the overall transverse space. We must also impose the AdS isometries and symmetries on the metric and the flux. Employing this procedure for the co-associative example, the metric becomes

$$ds^2 = \lambda^{-1}\Big[ds^2(AdS_3) + \frac{\lambda^3}{4}\Big(\frac{d\rho^2}{1-\lambda^3\rho^2} + \rho^2 d\theta^2\Big)\Big] + ds^2(\mathcal{M}_{SU(3)}).$$

Since we have picked out a preferred direction (that containing the AdS radial direction) in the G_2 structure manifold, the structure group of the spacetime reduces to *SU(3)*. The wrapped brane torsion conditions reduce in the AdS limit to

$$\begin{aligned} \mathrm{d}\Big(\frac{1}{\lambda^{3/2}\rho} J\wedge\hat{\rho} - \mathrm{Im}\Omega\Big) &= 0, \\ \mathrm{d}\Big(\frac{1}{2\lambda} J\wedge J + \lambda^{1/2}\rho\mathrm{Re}\Omega\wedge\hat{\rho}\Big) &= 0, \end{aligned}$$

while the flux is given by

$$F = -\mathrm{d}\theta\wedge\mathrm{d}(\lambda^{-1/2}\sqrt{1-\lambda^3\rho^2}J).$$

III. The AdS limit and supersymmetry conditions

For the remaining cycles in manifolds with d<8, the supersymmetry conditions are qualitatively similar. For cycles in eight- or ten-manifolds, we have so far only worked out the AdS conditions for Kähler four-cycles in *SU(4)* manifolds (remainder in progress). Some general comments may be made:

- Generically, supersymmetry conditions imply that some, but not all, field equations/ Bianchi identities are identically satisfied. For M5s wrapping cycles in manifolds with d<8, the wrapped brane supersymmetry conditions imply all field equations, but not the Bianchi identity. However the Bianchi identity is solved by the AdS limit. With d≥8, one must in contrast impose the four-form field equations and Bianchi identity in addition to the wrapped brane supersymmetry conditions. In the AdS limit, the four-form field equation is satisfied, but the Bianchi identity must still be imposed.

- In every case we have analysed, the isometries of the AdS limits (which are encoded in the wrapped brane geometries) match the R-symmetries expected for the CFT duals.

- Where general classifications of AdS spacetimes in M-theory have been performed independently (N=1, AdS_3, N=1, AdS_4, N=1,2, AdS_5) the conditions we derive via our procedure match those already in the literature. This is strongly suggestive evidence that we are getting the most general AdS supersymmetry conditions in each class.

IV. New explicit solutions

For the case of M5 branes wrapping Kähler four-cycles in Calabi-Yau four-folds, we have found many new infinite families of warped AdS_3 solutions, dual to two dimensional N=(2,0) CFTs. The construction has many features in common with, and was directly inspired by, the construction of the $Y^{p,q}$, which arise from the near-horizon geometry of M5s wrapping Kähler two-cycles in Calabi-Yau three-folds. The new solutions are all S^2 bundles over six-dimensional base spaces B_6 , which are products of Kähler-Einstein manifolds. The new compact regular solutions involve one of the following choices for B_6:

$$S^2 \times S^2 \times H^2,\ KE_4^+ \times H^2;$$
$$H^2 \times H^2 \times S^2,\ KE_4^- \times S^2;$$
$$S^2 \times S^2 \times T^2,\ KE_4^+ \times T^2.$$

Of particular interest are those with a T^2 factor, as these may be reduced and dualised to IIB. There are eight possibilities for a positive scalar curvature Kähler-Einstein four-manifold: $S^2 \times S^2,\ CP^2,\ dP_k,\ 3 \leq k \leq 8.$

For $B_6 = KE_4^+ \times T^2$, upon reduction and T-duality to IIB, the metric becomes

$$ds^2 = l^2 y[ds^2(AdS_3) + ds^2(\mathcal{M}_7)],$$

IV. New explicit solutions

where

$$ds^2(\mathcal{M}_7) = \frac{3}{8y} ds^2_{KE_4} + \frac{9dy^2}{4q(y)} + \frac{q(y) D\psi^2}{16y^2(y^2 - 2y + a)}$$
$$+ \frac{y^2 - 2y + a}{4y^2} Dz^2 ,$$

$$D\psi = d\psi + P, \; Dz = dz - g(y) D\psi,$$

and *a* is constant. Only the five-form flux is non-vanishing, and the dilaton is constant. Topologically M_7 is a U(1) bundle over a six-manifold which is itself a S^2 bundle over a positive scalar curvature Kähler-Einstein four-manifold. Global regularity conditions imply the quantisation of *a* and the period of the coordinate z. Furthermore, flux quantisation implies the quantisation of the AdS length *l*, and we may compute the central charges of the field theory duals according to

$$c = \frac{3l}{2G_{(3)}}$$

to be

$$c = \frac{9p\, q^2(p + mq)}{3p^2 + 3mpq + m^2q^2} \frac{Mq}{m^2h^2} n^2$$

Here *m* and *M* are integers which depend on the topology of M_7.

V. Conclusions

- We are systematically charting the supersymmetric AdS landscape of M-theory. We do this by first identifying the brane configurations which can give rise to AdS spacetimes of different dimensionalities and preserving different amounts of supersymmetry. Having derived the supersymmetry conditions for the wrapped branes, we employ a simple limiting procedure to obtain the supersymmetry conditions for the AdS spaces they contain in their near-horizon geometry.

- We have used these supersymmetry conditions in the case of M5 branes wrapping Kähler four-cycles in Calabi-Yau four-folds to obtain many new families of AdS_3 solutions, dual to two dimensional N=(2,0) CFTs. In particular, we have found eight doubly countably infinite families which may be reduced and dualised to give new regular AdS_3 compactifications of IIB, and have computed the central charge of the dual field theories. It would be very interesting to construct the field theory duals.

- It is to be hoped that the results of the classification will facilitate the discovery of more explicit new solutions in the future. The results for AdS_2 and AdS_3 will certainly be of relevance to the study of supersymmetric black holes and black rings in M-theory.