Thermodynamics at the BPS bound for Black Holes in AdS

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General knowledge

- String theory delivers General Relativity as an effective theory at low energies....but in general is too complicated to study but....

- AdS/CFT provide a duality between String Theory and CFT: we understand better a CFT !

- Therefore we can use CFT to study the quantum structure of space-time!!!

Motivations

- Supersymmetry brings powerful simplifications
- We can study SUGRA configurations using its dual susy configurations (LLM, Wilson loops, susy BH, etc)
- General Relativity show a clear thermodynamics properties (BH physics) $\longrightarrow$ ensembles in dual CFT

We study BPS Thermodynamics in Ads/CFT

Field theory Statistical Mechanics

- Def. of the partition function depending on the ensemble type as function of charges (Energy, angular momentum, etc) and potentials (temperature, angular velocity, etc)

$$Z_{(\beta,\gamma)} = \sum_{\nu} e^{(-\beta E_\nu + \beta \Omega J_\nu)}$$

- ...many thermodynamical relations and functions like (Gibbs free energy)

$$-\ln Z_{(\beta,\Omega)} = \beta E - \beta \Omega J - S$$

But if susy is present BPS condition: $E \pm J = 0$

- Example: Two dimensional systems.

$$E^{\pm} = \tfrac{1}{2}(E_\nu \pm J_\nu)\,, \quad \beta_\pm = \beta(1 \mp \Omega)$$

$$Z_{(\beta_\pm)} = \sum_\nu e^{-(\beta_+ E_+ + \beta_- E_-)}\,.$$

susy: $\beta_- \to \infty$ while $\beta_+ \to w$

$$X_{(w)} = \sum_{bps} e^{-wJ}$$

- The susy partition function can be defined as a multi-scaling limit where

$$\beta \to \infty \quad , \quad \Omega \to (1 - w/\beta)$$

SUGRA Statistical Mechanics

- Black Hole show Thermodynamic properties... like

$$I = \beta E - \beta\Omega J - \beta\Phi Q - S$$

where (E, J, Q) are charges and (β, ω, Φ) are potentials

- ...But in the BPS case we do not write "susy potentials"

Charges like (E,Q,J) but potential ($\beta =?, \Omega = 1, \Phi = 1$)

??????????????

SUGRA Statistical Mechanics

- ...so if:

  $E \to E_{bps} + O(\beta^{-2})\,, \quad J \to J_{bps} + O(\beta^{-2})\,, \quad Q \to Q_{bps} + O(\beta^{-2})$

  and

  $\beta \to \infty\,, \quad \Omega \to \Omega_{bps} - \dfrac{w}{\beta} + O(\beta^{-2})\,, \quad \Phi \to \Phi_{bps} - \dfrac{\phi}{\beta} + O(\beta^{-2})$

- then,

  $I = \beta\left(E_{bps} - \Omega_{bps} J_{bps} - \Phi_{bps} Q_{bps}\right) + w J_{bps} + \phi Q_{bps} - S_{bps} + O(\beta^{-1})$

$$\boxed{I_{bps} = w J_{bps} + \phi Q_{bps} - S_{bps}}$$

SUGRA Statistical Mechanics

- In SUGRA, solitons come in terms of different parameters

$$\beta(a,b,c,...) \quad , \quad \Omega(a,b,c,...) \quad , \quad \dots$$

choosing a to parameterize the off-bps condition such that $a = 0$ is BPS,

$$\beta = \sum_{n=-\infty}^{n=\infty} \beta_{(n)} a^{-n} = \dots + \beta_{(-1)}(b,c,..)a^{-1} + \beta_{(0)}(b,c,..) + \dots$$

Therefore the above equation for the supersymmetric Euclidean action is by no means a trivial relation!!!!

SUGRA Statistical Mechanics

- Consider SUSY BH in minimal gauge SUGRA in 5D.

  Here BH are characterized by the Energy, Two angular momenta and a single electric charge (E, J_1, J_2, Q). All these functions comes in terms of four parameters (m, a, b, q) like e.g.

$$\beta = \frac{2\pi r_+ \left[(r_+^2 + a^2)(r_+^2 + b^2) + abq\right]}{r_+^4[(1 + g^2(2r_+^2 + a^2 + b^2)] - (ab + q)^2}$$

  where r_+ is the position of outer horizon.

- In the BPS regime

$$E - J_1 - J_2 - \sqrt{3}\, Q = 0$$

- ...it has a smooth and well defined BPS limit...

$$J^1_{bps} = \frac{\pi(a+b)(2a+b+ab)}{4(1-a)^2(1-b)}, \quad J^2_{bps} = \frac{\pi(a+b)(a+2b+ab)}{4(1-a)(1-b)^2},$$

$$Q_{bps} = \frac{\sqrt{3}\pi(a+b)}{4(1-a)(1-b)}, \quad S_{bps} = \frac{\pi^2(a+b)r_0}{2(1-a)(1-b)},$$

$$w_1 = \frac{\pi(1-a)(a+2ab+b^2+2b)}{r_{bps}(3r^2_{bps}+1+a^2+b^2)}, \quad w_2 = \frac{\pi(1-b)(b+2ab+a^2+2a)}{r_{bps}(3r^2_{bps}+1+a^2+b^2)},$$

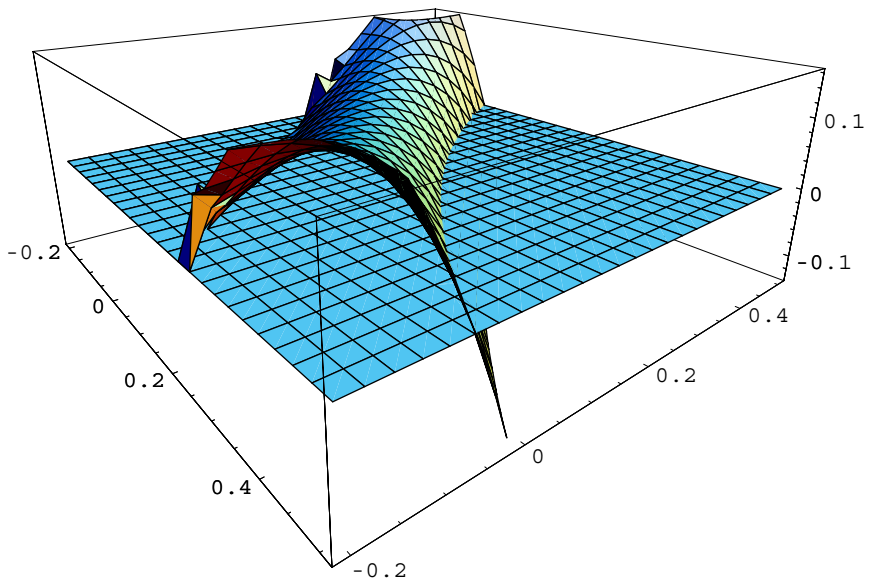
$$\phi = \frac{\pi\sqrt{3}(a+b)(1-ab)}{r_{bps}(3r^2_{bps}+1+a^2+b^2)}.$$

- With Euclidean action given by

$$I_{bps} = \phi Q_{bps} + w_1 J^1_{bps} + w_2 J^2_{bps} - S_{bps}$$

SUGRA Statistical Mechanics

- We can plot I_{bps} as a function of (a, b) to get



- Therefore study phase transitions at zero temperature!!!!

Concluding remarks

- We have defined new objects in SUGRA by studding dual CFT
- This is another check on ADS/CFT
- We found for BH, a rich phase diagram with phase transitions
- We can apply the above framework to any soliton, not only BH so again GR as statistical mechanics of space-time...
- Too many things to study !!!!!!