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Pure Spinor Formalism in Non-Critical Superstrings and Applications

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Outline

- Pure Spinor Formalism (Brief Review)
- Pure Spinor Formalism in Lower Dimensions (D=2,4,6) and Non-Critical Superstrings
- Results and Perspectives

Brief Review of Pure Spinor String Theory

We start from the fields $x^m, \theta^\alpha, p_\alpha$ (θ^α are Majorana-Weyl spinors in $d=(9,1)$) and the free field action

$$S = \int d^2 z \Big(\partial x^m \bar{\partial} x_m + p_\alpha \bar{\partial}\theta^\alpha + \hat{p}_\alpha \partial \hat{\theta}^\alpha \Big)$$

i) The total conformal charge is $c_T = (10)_x + (-32)_{p,\theta}$

ii) Inserting $p_\alpha = p^*_\alpha \equiv \frac{1}{2}\partial x_m \gamma^m_{\alpha\beta}\theta^\beta + \frac{1}{8}(\gamma^m_{\alpha\beta}\theta^\beta)(\theta\gamma_m\partial\theta)$ in S,

$$S|_{p=p^*} = S_{Green-Schwarz}$$

iii) So, $d_\alpha \equiv p_\alpha - p^*_\alpha \approx 0$ must be identified with the fundamental constraint.

$$Q = \oint dz \lambda^\alpha d_\alpha$$

iv) The nilpotentcy

$$\{Q, Q\} = \oint \lambda^\alpha(z) \oint \lambda^\beta(w) d_\alpha(z) d_\beta(w) = \oint \lambda^\alpha \gamma^m_{\alpha\beta} \lambda^\beta \Pi_m = 0$$

$$\lambda^\alpha \gamma^m_{\alpha\beta} \lambda^\beta = 0$$

Pure Spinor Constraints

There is a gauge symmetry due to these constraints

$$\delta_{\Lambda^m} w_\alpha = \Lambda_m (\gamma^m \lambda)_\alpha$$

$$Q d_\alpha = \Pi^m (\gamma_m \lambda)_\alpha, \quad Q w_\alpha = d_\alpha, \quad Q^2 w_\alpha = \delta_{\Pi^m} w_\alpha$$

Conformal Algebra

$$T = -\frac{1}{2}\Pi^m \Pi_m - d_\alpha \partial\theta^\alpha + \frac{1}{10} : N_{mn} N^{mn} : -\frac{1}{8} : JJ : +\partial J$$

$$J = w_\alpha \lambda^\alpha$$

$$j_{BRST} = \lambda^\alpha d_\alpha$$

where $N_{mn} = \frac{1}{2} w \gamma^{mn} \lambda$

$$TJ = -8\frac{1}{(z-w)^3} + \frac{J}{(z-w)^2} + \frac{\partial J}{(z-w)} \qquad JJ = -4\frac{1}{(z-w)^2}$$

$$TN^{mn} = \frac{N^{mn}}{(z-w)^2} + \frac{\partial N^{mn}}{(z-w)} \qquad N^{mn} N^{pq} = \frac{-3\eta^{m[p}\eta^{q]n}}{(z-w)^2} + \frac{\eta^{m[p} N^{q]n}}{(z-w)}$$

N=1 SYM D=10 OPEN STRING MASSLESS SPECTRUM

$$\mathcal{U}^{(1)} = \lambda^\alpha A_\alpha(x,\theta) \qquad \{Q, \mathcal{U}^{(1)}\} = 0$$

$$\gamma^{\alpha\beta}_{[5]} D_{(\alpha} A_{\beta)} = 0$$

$$\gamma^{\alpha\beta}_{[5]} = (\gamma_{[m_1} \dots \gamma_{m_5]})^{\alpha\beta}$$

SUGRA IIA/B D =10 CLOSED STRING MASSLESS SPECTRUM

$$\mathcal{U}^{(1,1)} = \lambda^\alpha_L \lambda^{\hat\beta}_R \hat A_{\alpha\hat\beta}(x,\theta,\hat\theta) \qquad Q \to Q_L, \quad Q_R$$

$$\gamma^{\alpha\beta}_{[5]} D_{(\alpha} A_{\beta)\hat\beta} = 0\,, \qquad \gamma^{\hat\alpha\hat\beta}_{[5]} \hat D_{(\hat\alpha} A_{\alpha\hat\beta)} = 0$$

vi) Amplitudes

Vertex operators: $\int dz d\bar{z} \mathcal{V}^{(0,0)}_{z\bar{z}}$ $\oint dz \mathcal{V}^{(1,0)}_{z}$, $\oint d\bar{z} \mathcal{V}^{(0,1)}_{\bar{z}}$ and $\mathcal{U}^{(1,1)}$

$$\langle\langle \quad \mathcal{U}^{(1,1)}_1 \mathcal{U}^{(1,1)}_2 \mathcal{U}^{(1,1)}_3 \prod_{j=1}^{n} \int dz_j d\bar{z}_j \mathcal{V}^{(0,0)}_j \quad \rangle\rangle$$

After all OPE's are performed, we need a measure for zero modes

$$\langle \mathcal{M} \rangle = \int d^{16}\theta_0 d^{10}x_0 \mathcal{D}\lambda_0 \mu(\theta_0, \lambda_0) \mathcal{M}(x_0, \theta_0, \lambda_0)$$

Top Form in the Pure Spinor Space

$$\mathcal{D}\lambda_0 = d\lambda^{\alpha_1} \wedge \ldots \wedge d\lambda^{\alpha_{11}} \epsilon_{\alpha_1 \ldots \alpha_{16}} (\gamma^m \gamma^n \gamma^p \gamma_{mnp})^{[\alpha_{12} \ldots \alpha_{16}](\beta_1 \ldots \beta_3)} \frac{\partial}{\partial \lambda^{\beta_1}} \cdots \frac{\partial}{\partial \lambda^{\beta_3}}$$

$$\mu(\lambda_0, \theta_0) = \prod_{i=1}^{11} (C^i_\alpha \theta^\alpha) \delta(C^i_\alpha \lambda^\alpha)$$

Motivations

- The supergravity approximation is no longer valid
- We would like to study AdS + RR backgrounds
- We need to keep supersymmetry and Poincare symmetry manifest at all stages of computation
- We need a gauge fixed action and (possibly) a conformal field theory model
- Study of physical states spectrum
- Computation of Amplitudes manifestly supersymmetric.

Pure Spinor String Theory in Lower Dimensions

In the same way as in 10 dimensions, we can construct lower dimensional models which have

- Conformal Field Theory
- Vanishing central charge
- D=2,4,6 manifest Lorentz and susy manifest
- BRST charge(s) + Pure Spinor constraints
- Non-trivial spectrum and non-trivial amplitudes (definition of zero mode measure)
- Off-shell Physical States

Pure Spinor Constraints (and gauge invariances) in Lower Dimensions

D=2 (rather degenerate example, susy N)

$$\bar{\lambda}\lambda = 0 \qquad \delta w = \Lambda\bar{\lambda}\,, \qquad \delta\bar{w} = \Lambda\lambda\,,$$

D=4 (susy N=1)

$$\bar{\lambda}^{\dot{\alpha}}\lambda^{\alpha} = 0 \qquad \delta w_{\alpha} = \Lambda_{\alpha,\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}}\,, \qquad \delta\bar{w}_{\dot{\alpha}} = \Lambda_{\alpha,\dot{\alpha}}\lambda^{\alpha}\,,$$

$$\alpha, \dot{\alpha} = 1, 2$$

D=6 (susy N=2)

$$\bar{\lambda}^{A}_{I}\lambda^{B}_{J}\epsilon^{IJ} = 0 \qquad\qquad \delta w^{I}_{A} = \Lambda^{AB}_{IJ}\lambda^{J}_{B}$$

$$I, J = 1, 2\,, \qquad A, B = 1, \ldots, 4$$

Mapping the RNS Non-Critical Strings to Pure Spinors

- We start from the linear dilaton LD (this is due to the simplicity and to the fact that some only in LD framework part of the spectrum is known): R x U(1)
- We map the variables of RNS supestrings (together with the ghost fields) after the fermionization and bosonization of superghosts (and after the GSO projection) to Green-Schwarz-like variables (superspace coordinates) + pure spinor variables
- In order to have manifest susy we need to duplicate the superspace introducing more superspace coordinates
- We add the pure spinors in order to establish the correct balance between the original coordinates and the duplication of the superspaces coords.
- Then, we write the ``geometrical" BRST charge and we recover the pure spinor constraints in d=2,4,6 dimensions.

Example (D=2)

$$Q_+ = \oint dz e^{-\frac{\phi}{2}+\frac{i}{2}H-ix}, \qquad Q_+^2 = 0 \qquad\qquad q_{\dot{+}} = e^{-\frac{\phi}{2}+\frac{i}{2}H+ix},$$

They are mutually local w.r.t. each other, the first is BRST invariant and the second not.

Conformal Field Theory

To map correctly the theory, we need to map the generators of the conformal transformations and the action. For that we found that we need to modify the original action and energy momentum tensor as follows

$$S = S_0 + S_1 \qquad S_1 = \int d^2 z r^{(2)} \log \Omega(\lambda)$$

where we define it from the top form

$$\Omega = \Omega(\lambda) d\lambda \wedge \ldots \wedge d\lambda$$

$$T = w_\alpha \partial \lambda^\alpha - \boxed{2 r^{(2)} \log \Omega(\lambda)}$$

The definition of the pure spinor space requires some care: 1) we need to be sure that there are no anomalies, 2) the space is a singular space and need to be regulated or patched, 3) the action is a non-linear beta-gamma system and the Fock space is defined by the Cech cohomology.

AdS backgrounds

As an example, we construct the sigma model for
AdS in D=4 with N=2 supersymmetry.

The basic supergroup is

$$\frac{OSp(2|4)}{SO(1,3) \times SO(2)}$$

$$g^{-1}dg = L_\mu P^\mu + L_{\mu\nu} J^{\mu\nu} + L_{IJ} J^{IJ} + L^I_\alpha Q^\alpha_I$$

The action is decomposed into

$$S = S_{GS} + S_d + S_{ghost}$$

$$S_{GS} = \int_\Sigma d^2 z \eta_{\mu\nu} L^\mu \bar{L}^\nu + \int_{\mathcal{M}} d^3 y L^\mu L^\alpha_I (\gamma_5 \gamma_\mu)_{\alpha\beta} L^\beta_J \epsilon^{IJ}$$

$$S_d = \int_\Sigma d^2 z (\delta^{ij} + i\epsilon^{ij}) d_{\alpha i} \bar{L}^\alpha_j + (\delta^{ij} - i\epsilon^{ij}) \bar{d}_{\alpha i} L^\alpha_j + q_{RR} d_{\alpha i} \gamma^{5\alpha\beta} d_{\beta j} \delta^{ij}$$

Coupling with RR field strenghts

Results and Future Perspectives

- Construction of Pure Spinor Sigma model for Non-Critical and Lower dimensional models
- Correspondence one-to-one between the spectrum of RNS non-critical superstrings and the pure spinor formulation in d=2,4
- Mapping between RNS approach and PS framework
- AdS backgrounds, Integrability, Conformal Invariance,......
- AdS/CFT, D-branes for non-critical models and coupling with RR fields.