Towards the String Dual of N=1 SQCD-like Theories

based on hep-th/0602027 (RC, C. Núñez, A. Paredes)

Roberto Casero
CPhT - Ecole Polytechnique and CNRS

Centre de physique théorique
ÉCOLE POLYTECHNIQUE

CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

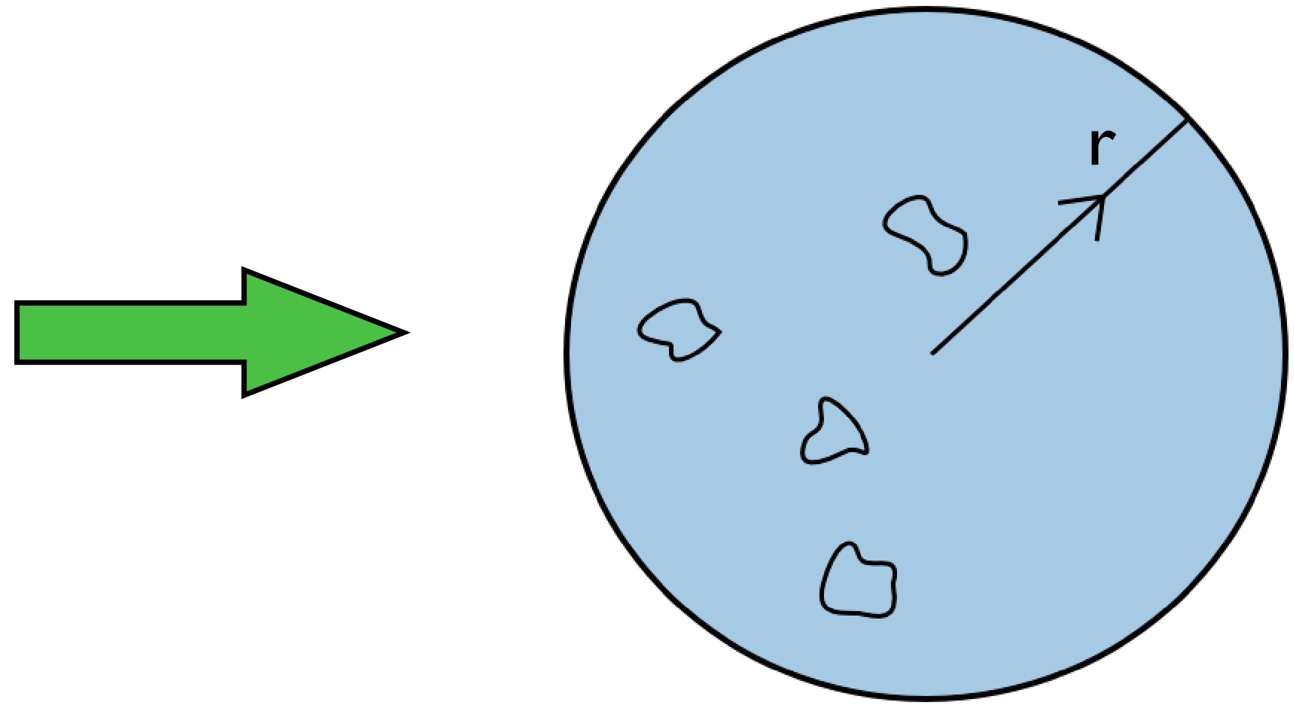
MARIE CURIE ACTIONS

RTN Workshop
Napoli - October 13th 2006

Motivations

The string/gauge correspondence proposes that

Color
field theory



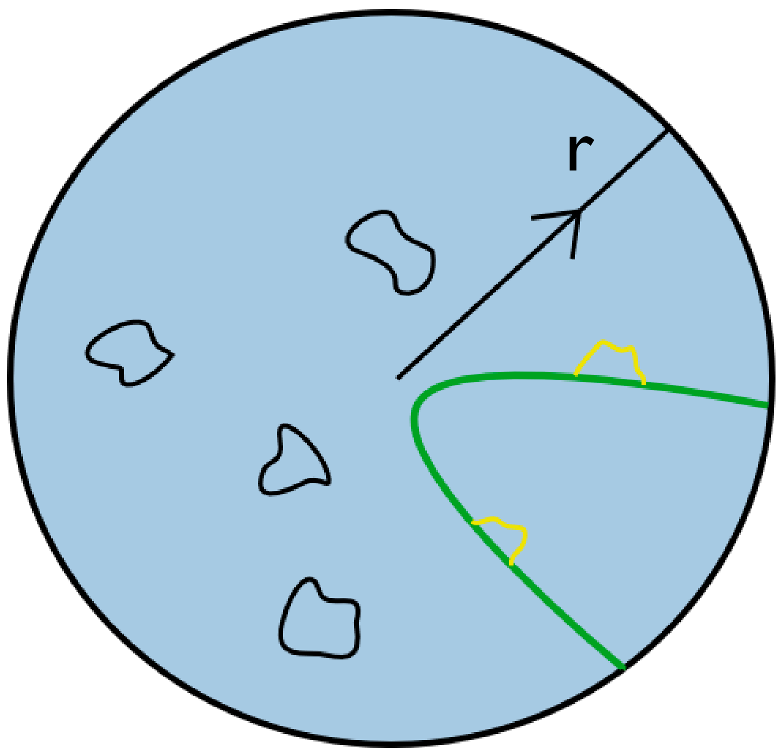
and might help us understand strong coupling effects:

- confinement
- gaugino condensation
- mass gap

Motivations

How do we include flavors in
the string/gauge correspondence?

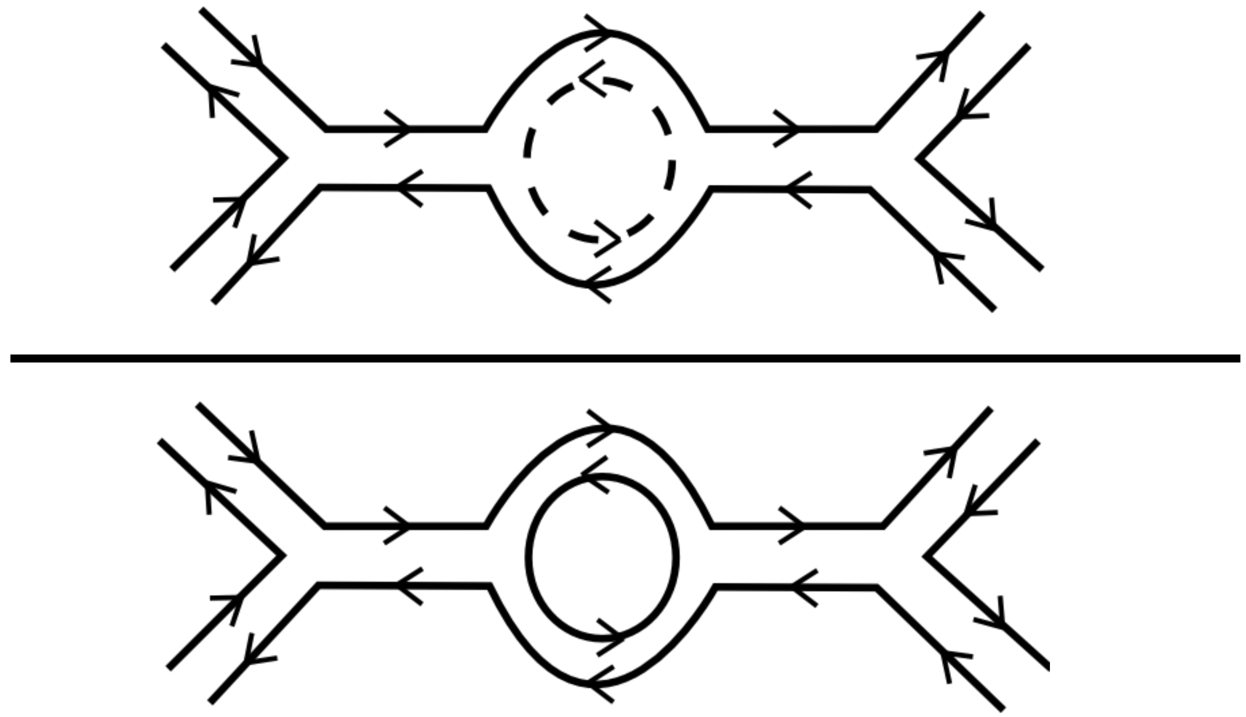
Field theory
with few flavors



We need to insert probe flavor branes

Karch Katz 2002

Motivations



$$\sim \frac{N_f}{N}$$

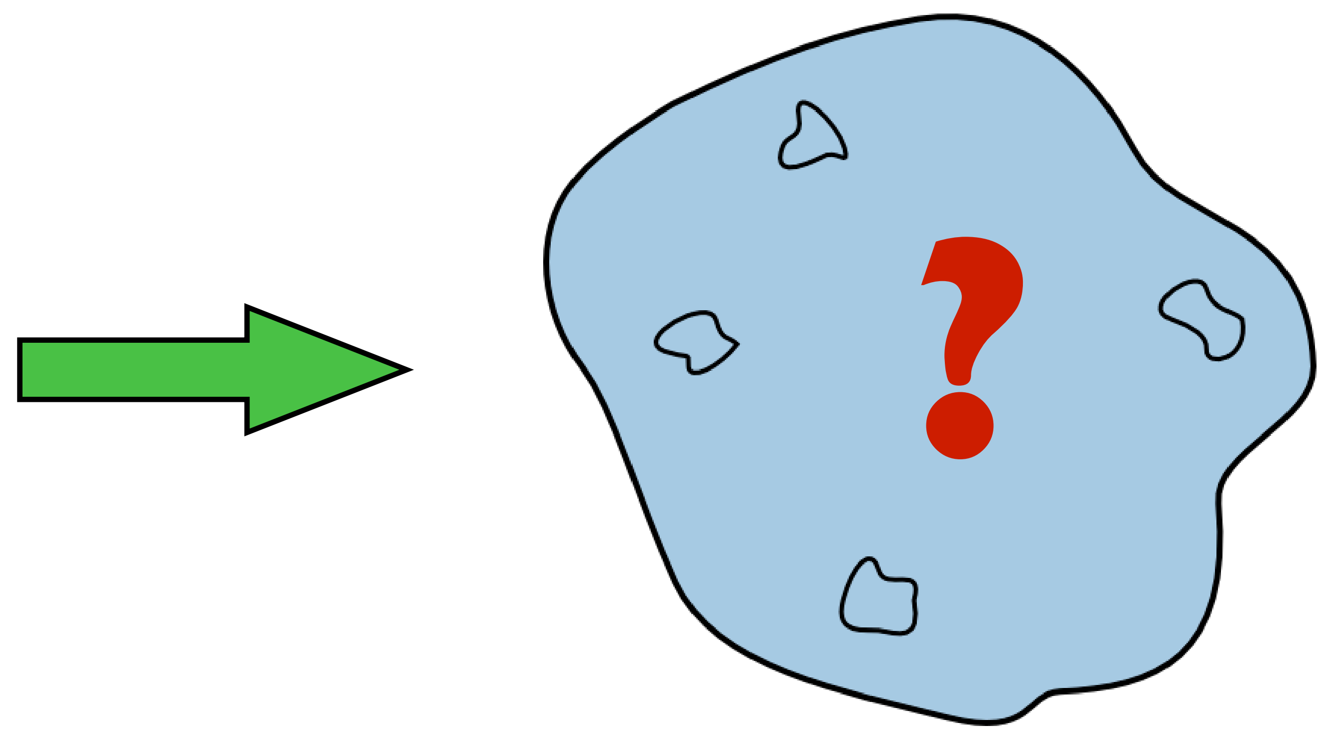
Real-world QCD has $N_f \sim N_c$ flavors:
we want to go beyond the probe approximation

Otherwise we would miss many
interesting phenomena, like:
baryons, screening, ...

Motivations

Real-world QCD has $N_f \sim N_c$ flavors:
we want to go beyond the probe approximation

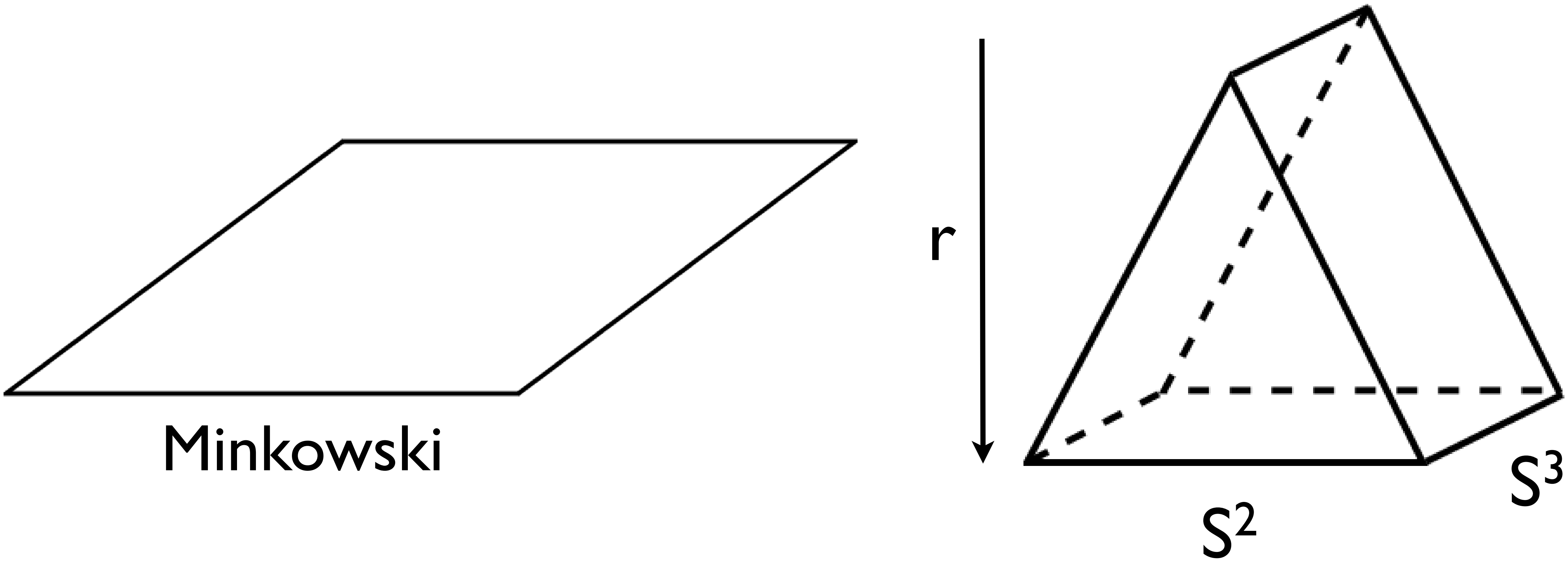
Field theory
with many flavors



What does the dual background look like?
What are its characteristics?

Adding many flavors to MN

Maldacena-Nunez background: $g_{\mu\nu}$, ϕ, F_3



Background symmetries: SU(2) x SU(2) x U(1)

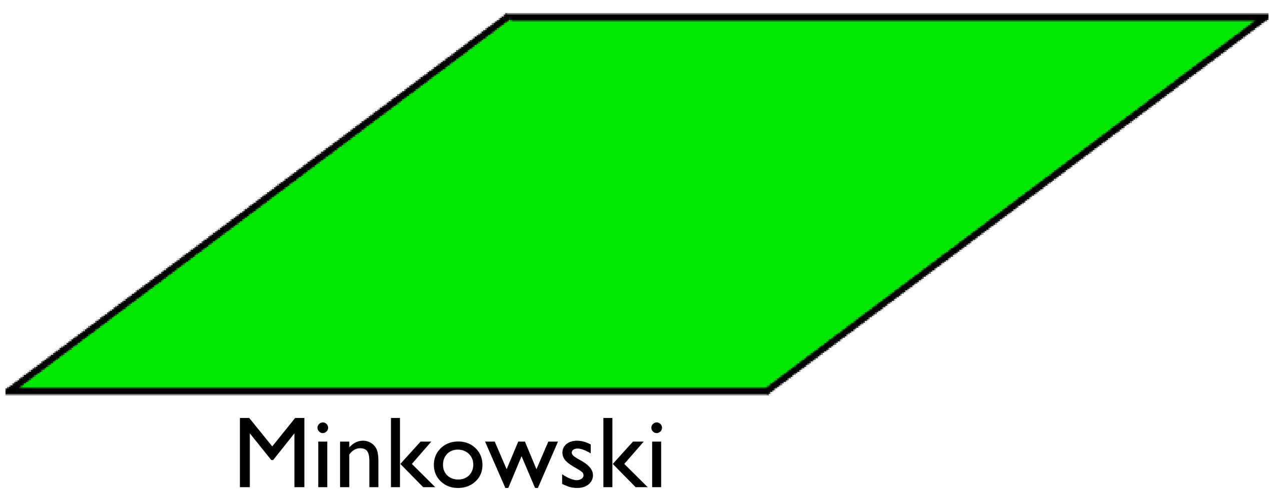
The spectrum also contains KK modes
and $M_{KK} \sim \Lambda_{QCD}$!

Adding many flavors to MN

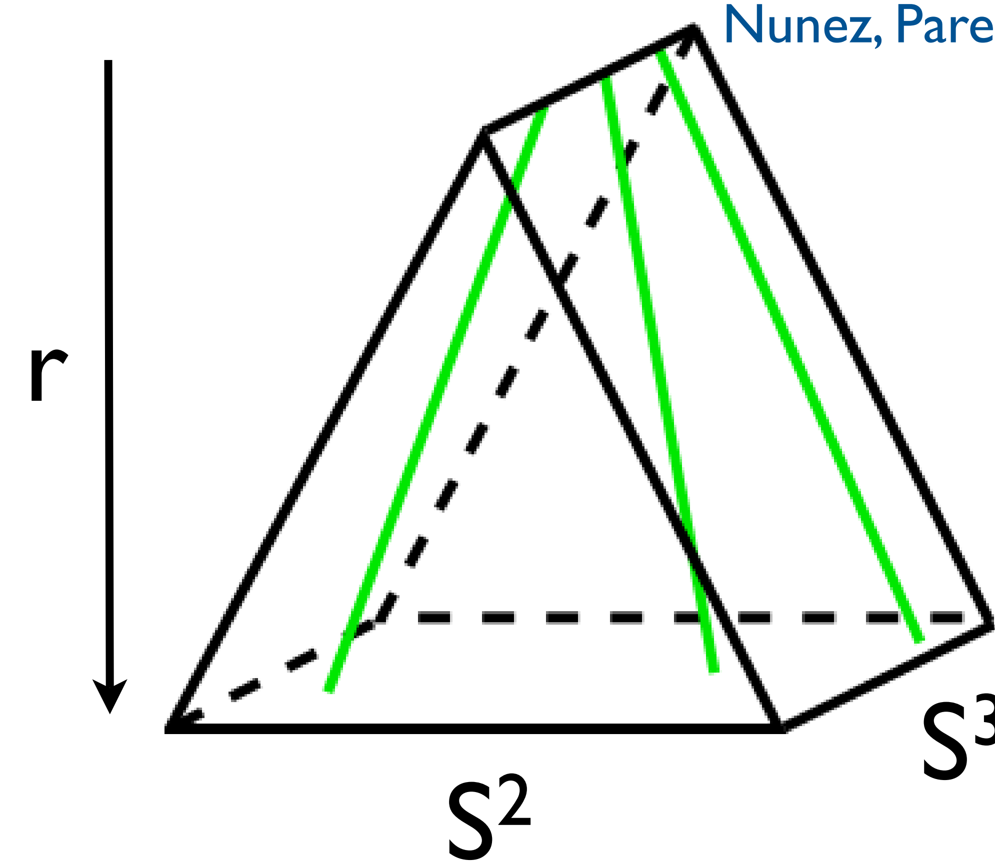
MN background: $g_{\mu\nu}$, ϕ, F_3

+

	$x_{1,3}$				ρ	θ	φ	$\tilde{\theta}$	$\tilde{\varphi}$	ψ
N_f D5	—	—	—	—	—	·	·	·	·	○



Nunez, Paredes, Ramallo
0311201



Background symmetries: SU(2) x SU(2) x U(1)

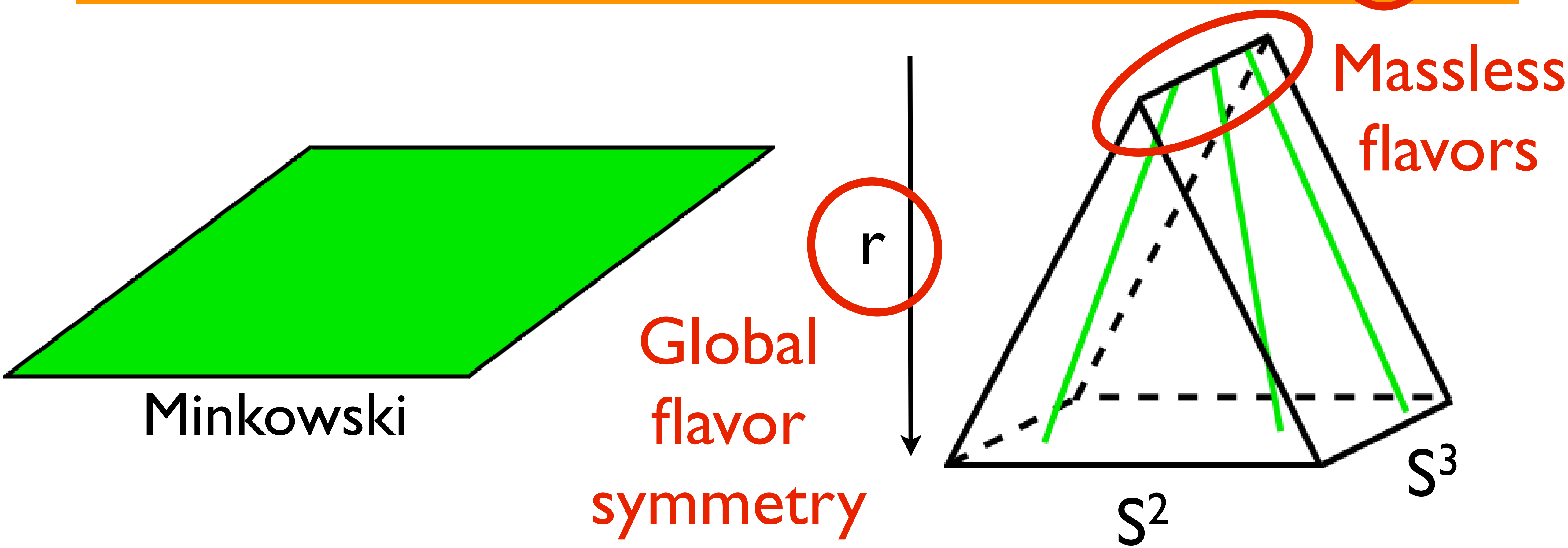
Adding many flavors to MN

MN background: $g_{\mu\nu}$, ϕ, F_3

$U(1)_R$ preserved

+

	$x_{1,3}$				ρ	θ	φ	$\tilde{\theta}$	$\tilde{\varphi}$	ψ
N_f D5	—	—	—	—	—	·	·	·	·	○



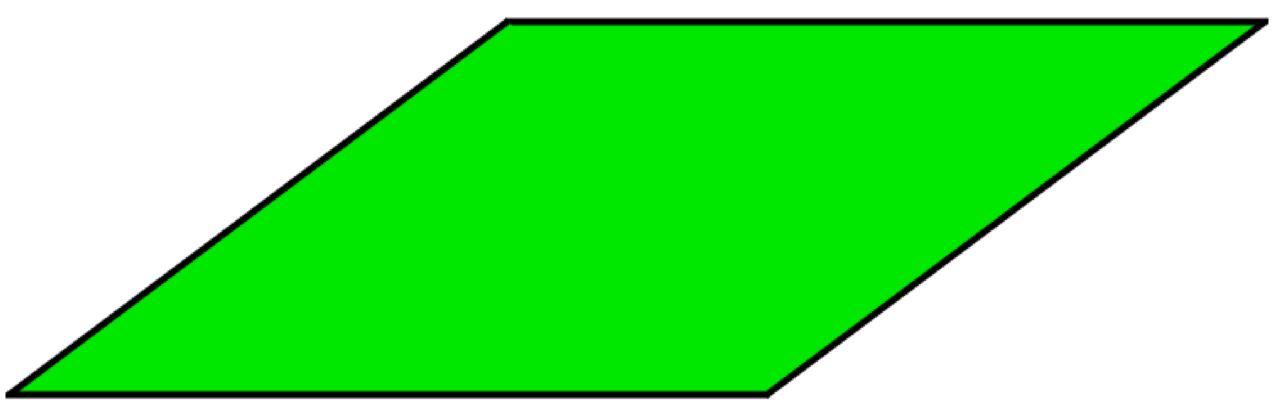
Background symmetries: ~~SU(2) x SU(2)~~ x U(1)

Adding many flavors to MN

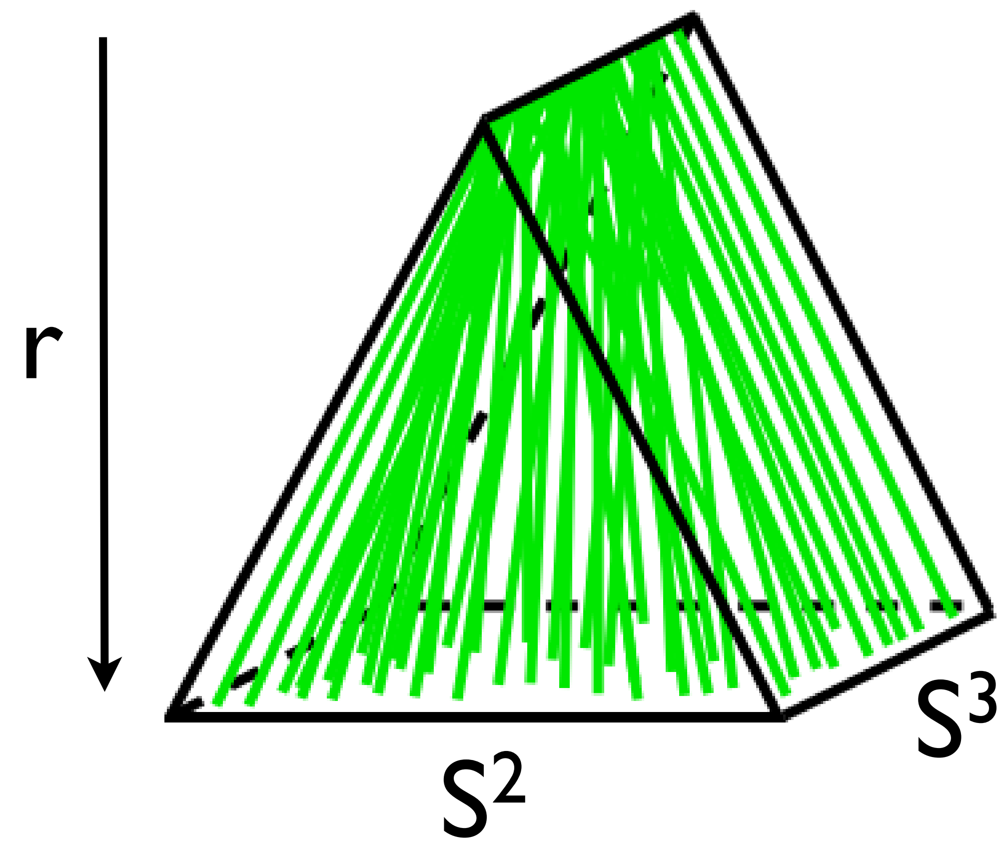
MN background: $g_{\mu\nu}$, ϕ, F_3

+

	$x_{1,3}$				ρ	θ	φ	$\tilde{\theta}$	$\tilde{\varphi}$	ψ
N_f D5	$-$	$-$	$-$	$-$	$-$	$\sim$	$\sim$	$\sim$	$\sim$	$\bigcirc$



Minkowski



Background symmetries: ~~SU(2) x SU(2)~~ x U(1)

Adding many flavors to MN

MN background: $g_{\mu\nu}$, ϕ, F_3

+

	$x_{1,3}$				ρ	θ	φ	$\tilde{\theta}$	$\tilde{\varphi}$	ψ
N_f D5	$-$	$-$	$-$	$-$	$-$	$\sim$	$\sim$	$\sim$	$\sim$	$\bigcirc$

since there are many D5 flavor branes
their backreaction on the background
cannot be neglected

closed strings

$$S = \frac{1}{2\kappa^2_{(10)}} \int d^{10}x \sqrt{-g} \left(R - \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{12} e^{\phi} F_3^2 \right) +$$

open strings

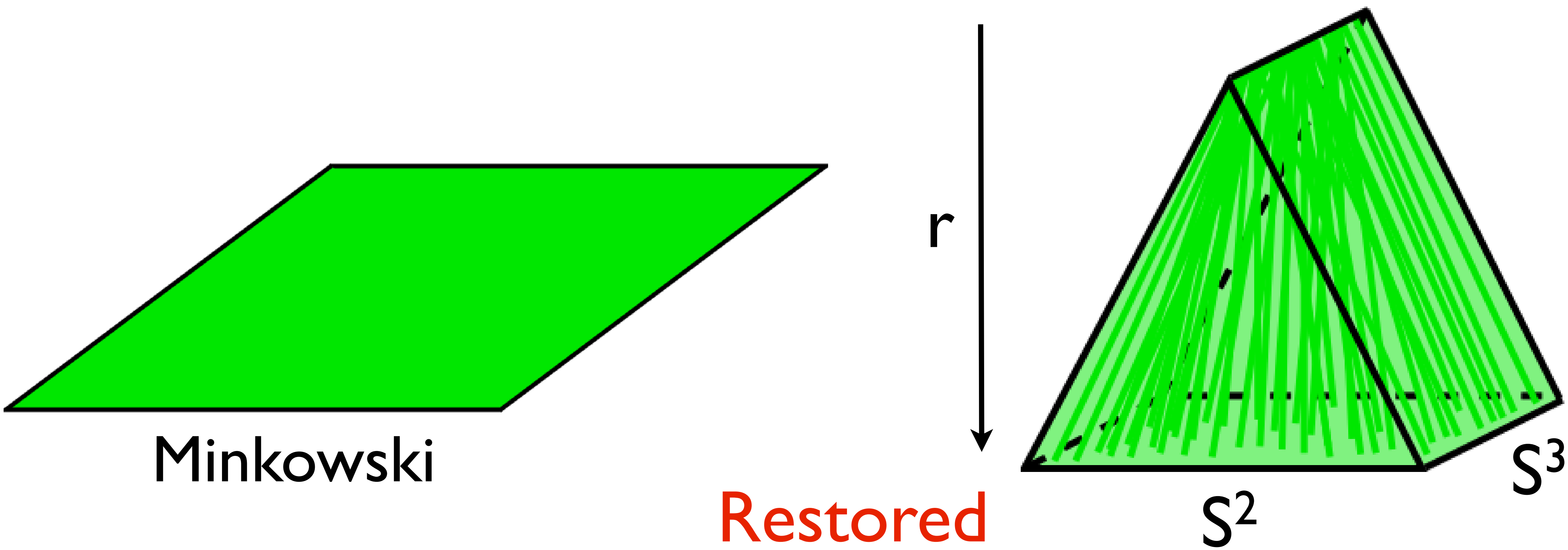
$$+ T_5 \sum^{N_f} \left(- \int_{\mathcal{M}_6} d^6 x e^{\frac{\phi}{2}} \sqrt{-\hat{g}_{(6)}} + \int_{\mathcal{M}_6} P[C_6] \right)$$

Adding many flavors to MN

MN background: $g_{\mu\nu}$, ϕ, F_3

+

	$x_{1,3}$				ρ	θ	φ	$\tilde{\theta}$	$\tilde{\varphi}$	ψ
N_f D5	—	—	—	—	—	$\sim$	$\sim$	$\sim$	$\sim$	$\bigcirc$



Restored

Background symmetries: SU(2) x SU(2) x U(1)

Adding many flavors to MN

MN background: $g_{\mu\nu}$, ϕ, F_3

+

	$x_{1,3}$				ρ	θ	φ	$\tilde{\theta}$	$\tilde{\varphi}$	ψ
N_f D5	—	—	—	—	—	$\sim$	$\sim$	$\sim$	$\sim$	$\bigcirc$

The smearing of the D5 flavor branes
restores all the original symmetries
of the background

$$S = \frac{1}{2\kappa_{(10)}^2} \int d^{10}x \sqrt{-g} \left(R - \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{12} e^{\phi} F_3^2 \right) +$$

$$-\frac{T_5 N_f}{(4\pi)^2} \left(\int d^{10}x \sin\theta \sin\tilde{\theta} e^{\frac{\phi}{2}} \sqrt{-\hat{g}_{(6)}} - \int Vol(Y_4) \wedge C_{(6)} \right)$$

smearing

Adding many flavors to MN

$$S = \frac{1}{2\kappa_{(10)}^2}\int d^{10}x\sqrt{-g}\left(R - \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{12}e^{\phi}F_3^2\right) +$$

$$-\frac{T_5 N_f}{(4\pi)^2}\left(\int d^{10}x \sin\theta\sin\tilde{\theta}e^{\frac{\phi}{2}}\sqrt{-\hat{g}_{(6)}} - \int Vol(Y_4)\wedge C_{(6)}\right)$$

$$\frac{1}{2\kappa_{(10)}}\int_{S^3} F_{(3)} = N_c T_5$$

$$dF_3 = \frac{N_f}{4}\sin\theta\sin\tilde{\theta}d\theta\wedge d\varphi\wedge d\tilde{\theta}\wedge d\tilde{\varphi}$$

We solve the first-order BPS equations numerically

All solutions are singular at the origin

$\mathcal{N}$=1 SQCD + quartic + ...

The MN background is dual to *N*=1 SYM plus KK modes

$$\mathcal{L} = \mathcal{L}_{\mathcal{N}=1\,\mathrm{SYM}} + \text{kinetic terms} + \mu\Phi_{KK}^2 + \kappa\,\tilde{Q}\Phi_{KK}Q + \dots$$

we add massless quarks

If we integrate out the massive KK modes,
N=1 SQCD receives a quartic superpotential contribution

$$\mathcal{W} \sim \frac{\kappa^2}{\mu}\,\tilde{Q}Q\tilde{Q}Q + \dots$$

- The flavor group is explicitly broken $SU(N_f)\times SU(N_f) \to SU(N_f)_{\mathrm{diag}}$
- $\mu \longrightarrow \infty$ $\Leftrightarrow$ "pure" *N*=1 SQCD $\longrightarrow$ non-critical strings

Gauge theory features

β function

For large ρ we have $\frac{4\pi^2}{g^2_{sqcd}} \sim N_c(1-\frac{x}{2})\rho + \ldots.$ $\log\frac{\mu}{\Lambda} \sim -\frac{1}{3}\log a \sim -\frac{1}{3}\log b \sim -\frac{2}{3}\rho$

and therefore

$$\beta = \frac{dg_{sqcd}}{d\log\frac{\mu}{\Lambda}} = -\frac{3g^3_{sqcd}}{32\pi^2}(2N_c - N_f) + \ldots$$

U(1)$_R$ breaking

The U(1)$_R$ symmetry is associated to shifts of the ψ angle

$$\psi \rightarrow \psi + 2\varepsilon$$

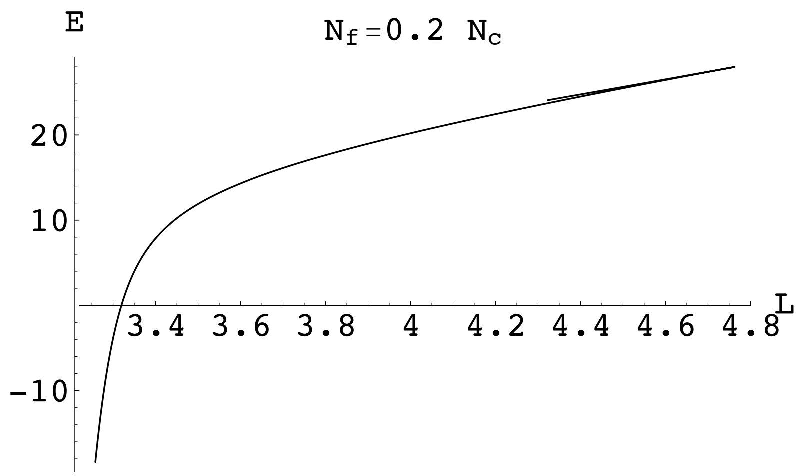
This changes $\theta_{sqcd} \sim \theta_{sqcd} + 2\pi n$ unless $\varepsilon = \frac{2\pi n}{2N_c - N_f}$ $n = 1, 2, \ldots, 2N_c - N_f$

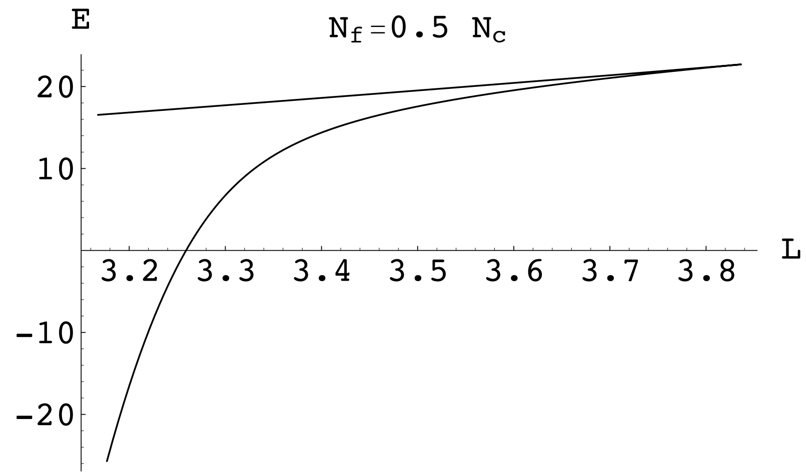
This matches the anomaly of the U(1)$_R$ in *N*=*1* SQCD ⟶ $U(1)_R \rightarrow \mathbb{Z}_{2N_c - N_f} \rightarrow \mathbb{Z}_2$

Gauge theory features

Wilson loop and screening

The Wilson loop can be evaluated as the energy of a string joining two very massive quarks, as a function of its length





This is what one expects from a confining theory with fundamental degrees of freedom: screening

Gauge theory features

Seiberg duality

Very schematically Seiberg duality states the identity of the IR dynamics of two different theories with

$$SU(N_c) + N_f \text{ flavors} \leftrightarrow SU(N_f - N_c) + N_f \text{ flavors}$$

In our setup the number of colors is

$$N_c T_5 = \frac{1}{2\kappa_{(10)}} \int_{\tilde{S}^3} F_3$$

and the number of flavors is fixed by

$$dF_3 \sim N_f \dots$$

The internal space in our geometry is a fibration of a 3-sphere over a 2-sphere

There is an ambiguity in the identification of the 3-sphere

Seiberg duality

With one choice for the three-sphere in the geometry we have

$$N_c T_5 = \frac{1}{2\kappa_{(10)}} \int_{\tilde{S}^3} F_3 \qquad\qquad dF_3 \sim N_f \ldots$$

with the other, instead

$$(N_f - N_c)\, T_5 = \frac{1}{2\kappa_{(10)}} \int_{S^3} F_3 \qquad\qquad dF_3 \sim N_f \ldots$$

which is the way colors and flavors transform under Seiberg duality.

Moreover the two ways of writing the geometry coincide in the IR region: we interpret this as identity of the IR dynamics

Seiberg duality is encoded very naturally in this dual description of *N=1* SYM + flavors

Summary

- Backreaction of flavor branes by adding an open string sector to the gravity action
- Gravity + branes dual of N=1 SQCD + quartic
- Many field theory features match
  - Wilson loop and screening
  - $U(1)_R$ anomaly and β function
  - Seiberg duality
  - $x<1$ and $x>1$