Constituents, Fundamental forces and Symmetries of the Universe
2nd RTN Workshop

A COMMENT ON
BLACK HOLES, TOPOLOGICAL STRINGS AND QUANTUM DISTRIBUTION FUNCTIONS

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INTRODUCTION

$$\langle\Psi|\hat{A}(\hat{p},\hat{q})|\Psi\rangle = \int dq\, dp\, A(p,q)\, F_{|\Psi\rangle}(p,q)$$

$$e^{\frac{i}{\hbar}(\xi q - \eta p)} \longrightarrow f(\xi,\eta)\, e^{\frac{i}{\hbar}(\xi\hat{q}-\eta\hat{p})}$$

$$A^{f}(p,q) = \frac{1}{2\pi}\int d\xi\, d\eta\, \frac{\tilde{A}(\xi,\eta)}{f(\xi,\eta)}\, e^{\frac{i}{\hbar}(\xi q - \eta p)}$$

where $\hat{A} = \frac{1}{2\pi}\int d\xi\, d\eta\, \tilde{A}(\xi,\eta)\, e^{\frac{i}{\hbar}(\xi\hat{q}-\eta\hat{p})}$

$$F^{f}_{|\Psi\rangle}(p,q) = \frac{1}{4\pi^2}\int d\xi\, d\eta\, \langle\Psi| f(\xi,\eta)\, e^{\frac{i}{\hbar}(\xi\hat{q}-\eta\hat{p})}|\Psi\rangle\, e^{-\frac{i}{\hbar}(\xi q - \eta p)}$$

$H^3(CY_3,\mathbb{R})$ phase space

$Z_{top} \longrightarrow |\Psi_{top}\rangle$

$\updownarrow \longrightarrow$ OSV conjecture

$$Z_{BH}(p,\phi) = |Z_{top}|^2$$

BHW entropy

CONTENT

- "Attractor" Black Holes
- Topological Strings and the Quantization of H^3
- Black Hole Quantum Distribution Function

$N=2$ OR "ATTRACTOR" BLACK HOLES

4d static, spherically symmetric dyonic BHs in type IIB compactifications on a CY_3 M

$$ds^2 = e^{2U(\tau)} dt^2 - e^{-2U(\tau)} \left(\frac{c^4}{\sinh^4 c\tau} d\tau^2 + \frac{c^2}{\sinh^2 c\tau} d\Omega_2^2 \right)$$

Charges p^I, q_J $\qquad I = 0, 1, \dots h_{21}$; $\; i = 1, \dots h_{21}$

$U(\tau)$, $z^i(\tau)$

$-\infty$ (horizon) $\longrightarrow \tau \longrightarrow$ 0

$$\mathcal{L} = \left(\frac{dU}{d\tau}\right)^2 + G_{i\bar{j}} \frac{dz^i}{d\tau} \frac{d\bar{z}^{\bar{j}}}{d\tau} + e^{2U} V_{BH}(z, \bar{z}; p, q)$$

$$\left(\frac{dU}{d\tau}\right)^2 + G_{i\bar{j}} \frac{dz^i}{d\tau} \frac{d\bar{z}^{\bar{j}}}{d\tau} - e^{2U} V_{BH}(z, \bar{z}; p, q) = c^2$$

$$V_{BH} = \left| Z_{p,q}(z, \bar{z}) \right|^2 + \left| \left(\partial_i + \tfrac{1}{2} \partial_i K\right) Z_{p,q} \right|^2$$

$$Z_{p,q}(z, \bar{z}) = e^{-\frac{K}{2}} \left(X^I q_I - p^I F_I \right)$$

$$K = -\log\left[i \left(\bar{X}^I F_I - X^J \bar{F}_J \right) \right]$$

$$F_J = \frac{\partial F_0}{\partial X^J}$$

- Extremal BH $c \to 0 \Rightarrow \begin{cases} \left.\dfrac{\partial V_{BH}(z,\bar{z};p,q)}{\partial z^i}\right|_h = 0 \Rightarrow z^i\big|_h = z^i_{p} \\[2ex] S_{BH} = \dfrac{A}{4} = \pi V_{BH}\big|_h = S_{BH}(p,q \end{cases}$

- BPS BH $\Rightarrow \begin{cases} \dfrac{dU}{dw} = e^{U-w}\,|Z_{p,q}| \\[2ex] \dfrac{dz^i}{dw} = 2\,e^{U-w}\,G^{i\bar{j}}\,\partial_{\bar{j}}\,|Z_{p,q}| \end{cases}$ Attractor flow =

$$\Rightarrow \gamma_{p,q} = \mathrm{Re}(\lambda^{-1}\Omega) \in H^{3,0} \oplus H^{0,3}$$

where $\gamma_{p,q} = p^I \alpha_I + q_I \beta^I \in H^3(M,\mathbb{Z})$

– Microscopic description: $\boxed{\text{D3 on } C_{p,q} = q_I A_I - p^J B^J \in H_3(M,}$

$\updownarrow$ mirror

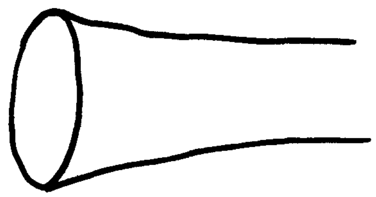
$\boxed{\text{M-theory branes}} \xleftarrow{\text{lift}} \boxed{\text{Type IIA branes}}$

$\Big\downarrow$ $\xrightarrow{\text{Maldacena limit}}$ AdS_3/CFT_2 conjecture

Macroscopic description

Connection with the quantization of $H^3(M,\mathbb{R})$

1º



$M\times S^2\times S^1 \leftarrow \tau$

$$\Psi_{p,q}\left(\frac{X}{\lambda},\frac{\bar{X}}{\lambda}\right)$$

BPS attractor flow eq. as constraints $\Rightarrow \left[\frac{\hat{X}}{\lambda},\frac{\hat{\bar{X}}}{\lambda}\right]=2i\hbar(\mathrm{Im}\,\tau)^{-1}$

$$\hat{\gamma}_{\hat{p},\hat{q}}=\mathrm{Re}\left(\lambda^{-1}\hat{\Omega}\right) \Rightarrow$$

$$\Rightarrow \left[\hat{q}_{X_I},\hat{p}_X^{\,J}\right]=i\hbar\,\delta_I^J \Rightarrow$$

$$\Rightarrow |\Psi_{p,q}\rangle \in \mathcal{H}_{H^3(M,\mathbb{R})}$$

$$|\Psi_{0,0}\rangle \underset{OVV}{=} |\Psi_{top}\rangle$$

2º

$$S_{BHW} \text{ in terms of } Z_{top}$$

$$+$$

$$Z_{top} \longrightarrow |\Psi_{top}\rangle$$

TOPOLOGICAL STRINGS AND THE QUANTIZATION OF H^3

$$S[G] = 2\int_{M\times[0,1]} G\wedge d'G \underset{\substack{\uparrow \\ G = \gamma + w\wedge dt'}}{=} \int \left[2\gamma\wedge(-\dot{\gamma} + dw) + 2w\wedge d\gamma\right]\wedge dt'$$

$$\delta\mathcal{L} = 4\, d'G\wedge\delta G + d'\Theta[\delta G] \Rightarrow \omega(\delta_1 G, \delta_2 G) = \delta_1\Theta_2 - \delta_2\Theta_1 = -4\,\delta_1 G\wedge$$

$$\left.\begin{aligned} \pi_\gamma &= -2\gamma \\ \pi_w &= 0 \end{aligned}\right\} \Longrightarrow \left\{\begin{aligned} &\mathcal{H}_{sd}(\gamma, w, \pi_\gamma, \pi_w) = -4\gamma\wedge dw - d(\gamma\wedge w) \\ &\Phi^{(1)}_\gamma \equiv \pi_\gamma + 2\gamma = 0 \\ &\Phi^{(1)}_w \equiv \pi_w = 0 \\ &\Phi^{(2)}_w \equiv 4\, d\gamma = 0 \end{aligned}\right.$$

$$\Omega(1,2) = \int_M \omega(\delta_1 G, \delta_2 G) = -4\int_M \delta_1\gamma\wedge\delta_2\gamma \Rightarrow$$

$\Rightarrow H^3(M,\mathbb{R})$ is the physical phase space

$$\gamma = \frac{1}{2}\left[\lambda^{-1}\Omega + x^i\left(\frac{\partial}{\partial t^i} + \partial_i B\right)\Omega + cc\right] \Rightarrow$$

$$\Rightarrow \left\{\begin{aligned} &[\hat{\lambda}^{-1}, \hat{\bar{\lambda}}^{-1}] = -\hbar\, e^{+B} \\ &[\hat{x}^i, \hat{\bar{x}}^{\bar{j}}] = +\hbar\, e^{B} G^{i\bar{j}} \end{aligned}\right.$$

$$\begin{cases} \hat{\lambda}^{-1} \, | \lambda^{-1}, x^i \rangle = \lambda^{-1} \, | \lambda^{-1}, x^i \rangle \\ \hat{x}^i \, | \lambda^{-1}, x^i \rangle = x^i \, | \lambda^{-1}, x^i \rangle \end{cases}$$

$$\gamma = p^I q_I + q_I \beta^I \Rightarrow [q_I, p^J] = i\hbar \delta_I{}^J$$

$\left| \lambda^{-1}_{p,q;\Xi,\bar{\Xi}} , X^i_{p,q;\Xi,\bar{\Xi}} \right\rangle_{\Xi,\bar{\Xi}}$ are squeezed states

- centered around (p, q)
- with width given by $\mathrm{Im}\, \tau_{IJ}(\Xi)$
- with squeezing parameter given by $\mathrm{Re}\, \tau_{IJ}(\Xi)$

If $|q\rangle = e^{q\hat{a}^{+}} |0\rangle$ instead of $|q\rangle = e^{q\hat{a}^{+} - \bar{q}\hat{a}} |0\rangle \Rightarrow$

$$\Rightarrow \Psi_{top}(\lambda^{-1}, x ; t, \bar{t}) = \langle \Psi_{top} | \lambda^{-1}, x \rangle_{t,\bar{t}}$$

$$\hat{\Phi}^{(2)}_{w} \, | \Psi_{top} \rangle = \widehat{d\gamma} \, | \Psi_{top} \rangle = 0$$

BLACK HOLE QUANTUM DISTRIBUTION FUNCTION

Modified attractor equations:

$$\begin{cases} p^I = \mathrm{Re}\left[\lambda^{-1} X\right] \\ q_I = \mathrm{Re}\left[\dfrac{\partial F_{mod}(\lambda^{-1}X, \bar\lambda^{-1}\bar X, 2\Im S)}{\partial(X\lambda^{-1})} - \dfrac{\partial F_{mod}(\lambda^{-1}X, \bar\lambda^{-1}\bar X, 2\Im S)}{\partial(\bar\lambda^{-1}\bar X)} \right. \end{cases}$$

$$S_{BHW}(p,q) = F_{top}(\lambda; t, \bar t) + \bar F_{top}(\bar\lambda; t, \bar t) + \frac{\pi}{2} q_I \phi^I \Big|_{\substack{\text{consmet} \\ \text{attract} \\ \text{values}}}$$

where $\dfrac{X^I}{\lambda} = p^I + \dfrac{i\phi^I}{2}$

$$\text{BPS BH}(p,q) \longrightarrow \left| \lambda^{-1}_{p,\tilde q; X^{quan}_{p,q}}, X^i_{p,\tilde q; X^{quan}_{p,q}} = 0 \right\rangle_{X^{quan}_{p,q}, \bar X^{q}_{p,}}$$

with $$\frac{X_{p,\tilde q}}{\lambda_{p,\tilde q}} = \frac{X^{quan}_{p,q}}{\lambda^{quan}_{p,q}}$$

$$\frac{\left\langle \lambda^{-1}_{p,\tilde q; X^{quan}_{p,q}}, 0 \,\middle|\, \lambda^{-1}_{p,\tilde q; X^{quan}_{p,q}}, 0 \right\rangle_{X^{quan}_{p,q}, \bar X^{quan}_{p,q}}}{\langle 0,0 | 0,0 \rangle} = e^{-S_{BH}(p,\tilde q)}$$

$$\hbar = \frac{4}{\pi}$$

$$e^{-S_{BH}(p,\tilde{q})} = e^{-i\frac{\pi}{2}F_0\left(\frac{X^{quan}_{p,q}}{\lambda^{quan}_{p,q}}\right)+i\frac{\pi}{2}\bar{F}_0\left(\frac{\bar{X}^{quan}_{p,q}}{\bar{\lambda}^{quan}_{p,q}}\right)-\frac{\pi}{2}\tilde{q}_I\phi^{I\,quan}_{p,q}}$$

$$\text{If } |q\rangle = e^{q\hat{a}^+ - \bar{q}\hat{a}}|0\rangle \Rightarrow$$

$$\Rightarrow e^{S_{BHW}(p,q)} = \underbrace{\left|\langle\Psi_{top}|\lambda^{-1}_{p,\tilde{q};X^{quan}_{p,q}},0\rangle_{X^{quan}_{p,q},\bar{X}^{quan}_{p,q}}\right|^2}_{F^{BH}_{|\Psi_{top}\rangle}(p,\tilde{q};X^{quan}_{p,q},\bar{X}^{quan}_{p,q})} e^{\frac{\pi}{2}(q-\tilde{q})\phi^{q\ldots}_{p,\ldots}}$$

$$F^{BH}_{|\Psi_{top}\rangle} = \int d\mu_{X''}\,d\mu_{\lambda^{-1}''}\langle\Psi_{top}|\exp\left[-\frac{1}{\hbar}e^{-K}\hat{\lambda}^{-1}\bar{\lambda}^{-1}{}'' - \frac{1}{\hbar}e^{-K}G_{i\bar{j}}\hat{X}^i\bar{X}^{\bar{j}}{}'' + \right.$$

$$\left. + \frac{1}{\hbar}e^{-K}\lambda^{-1}{}''\hat{\bar{\lambda}}^{-1} + \frac{1}{\hbar}e^{-K}G_{i\bar{j}}X^{i}{}''\hat{\bar{X}}^{\bar{j}}\right]|\Psi_{top}\rangle\,\cdot$$

$$\cdot\exp\left[\frac{1}{\hbar}e^{-K}\lambda^{-1}_{p,\tilde{q};X_{p,\tilde{q}}}\bar{\lambda}^{-1}{}'' + \frac{1}{\hbar}e^{-K}G_{i\bar{j}}X^i_{p,\tilde{q};X_{p,\tilde{q}}}\bar{X}^{\bar{j}}{}'' - \right.$$

$$\left. - \frac{1}{\hbar}e^{-K}\lambda^{-1}{}''\bar{\lambda}^{-1}_{p,\tilde{q};X_{p,\tilde{q}}} - \frac{1}{\hbar}e^{-K}G_{i\bar{j}}X^{i}{}''\bar{X}^{\bar{j}}_{p,\tilde{q};X_{p,\tilde{q}}}\right] =$$

= mixed Husimi-antiHusimi distribution function

$$S_{BHW}(p,q) = F_{top}(\lambda,t,\bar{t}) + \bar{F}_{top}(\bar{\lambda},\bar{t},t) + \frac{\pi}{2} q_I \phi^I \Big|_{\substack{\text{connected} \\ \text{attractor} \\ \text{values}}}$$

$$Z_{BH}(p,\phi) = \sum_q \Omega(p,q)\, e^{\frac{\pi}{2} q_I \phi^I} \underset{\substack{OSV \\ \frac{X}{\lambda} = p + i\frac{\phi}{2}}}{=} \left| e^{F_{top}} \right|^2 =$$

$$= F^{BH}_{|\Psi_{top}\rangle}\left(p, q_\phi; X = p + \frac{i\phi}{2}, \bar{X} = p - \frac{i\phi}{2}\right) e^{\pi q_\phi (\mathrm{Im}\,\tau)^{-1}(q_\phi - \mathrm{Re}\,\tau\, p)}$$

where $q_\phi = -\mathrm{Im}\,\tau \frac{\phi}{2} + \mathrm{Re}\,\tau\, p$

$$\sum_q \longrightarrow \int dq \quad \Rightarrow$$

$$\Rightarrow F^{BH}_{|\Psi_{top}\rangle}(p, q_\phi; X, \bar{X}) = \int dq'\, \Omega(p,q')\, e^{\pi (q' - q_\phi)(\mathrm{Im}\,\tau)^{-1}(q_\phi - \mathrm{Re}\,\tau\, p)}$$

$$F^{BH}_{|\Psi_{top}\rangle}(p, q_\phi; X, \bar{X}) = \int dq' \int dp'\, F^{W}_{|\Psi_{top}\rangle}(p', q') \,\cdot$$

$$\cdot \exp\Big[-\pi \big((q_\phi - q') - \mathrm{Re}\,\tau\,(p - p') \big) (\mathrm{Im}\,\tau)^{-1} \big((q_\phi - q') - \mathrm{Re}\,\tau\,(p - p') \big) -$$

$$- \pi (p - p')\, \mathrm{Im}\,\tau\, (p - p') \Big]$$

$$\tau \longrightarrow i\infty$$

CONCLUSIONS AND DISCUSSION

- BHW entropy and OSV partition function can be interpreted in terms of a mixed Husimi-antiHusimi quantum distribution function associated with $|\Psi_{top}\rangle$
- OSV degeneracies can be related to the Wigner function at $\tau \to i\infty$

antiHusimi (χ^{-1} sector)

$\big\downarrow$ coarse-graining

Wigner $\quad e^{\frac{i}{\hbar}(\xi q - \eta p)} \xrightarrow{\text{Weyl map}} e^{\frac{i}{\hbar}(\xi \hat{q} - \eta \hat{p})}$

$\big\downarrow$ coarse-graining $\quad \int dq\, F^{W}_{|\psi\rangle}(p,q) = |\psi(p)|^2$ (but non-definite-positive)

Husimi (χ-sector) (definite-positive)