

Some Exact

Results in

Four - dimensional

Non - perturbative

String Theory


BASED ON :

- D. Robles-Llana , M. Roček ,
F. Saueressig , U. Theis , S-V.


① MOTIVATION

String theory
effective actions
generically receive
quantum corrections
from both α' and g_s

worldsheet
CFT



g_s
string
loops



Non-perturbatively

worldsheet

spacetime

+

In string compactifications to $D = 4$

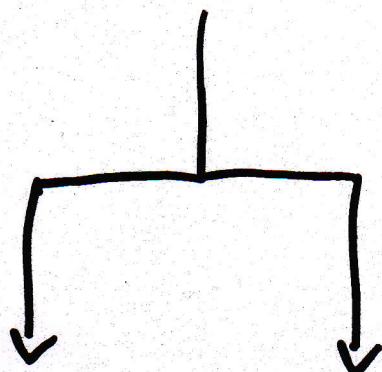
with $N > 2$, these quantum corrections contribute to higher derivative terms.

For $N = 2$ and $N = 1$

they contribute also to the Low Energy

Example ($N=2$)

IIA/CY

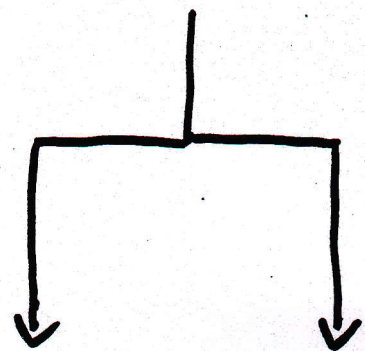


VM + HM

(d')

(g_s)

II B/ $\tilde{C}Y$



VM + HM

$(-)$

(d', g_s)

Vector Multiplet moduli

space : Special Kähler

geometry $F(X) =$

The hypermultiplet
moduli space has a
complicated geometrical
structure

$$S = \int d^4x \sqrt{g} \left[R - \frac{1}{2} g_{ij} \partial_\mu \phi^i \partial^\mu \phi^j + \dots \right]$$

Target space is

Quaternion-Kähler

(Bagger & Witten)

In non-perturbative
string theory

$$g_{ij}(\phi) = \overset{\text{class.}}{g_{ij}(\phi)} + \overset{\text{pert}}{g_{ij}(\phi)} + \overset{\text{inst.}}{g_{ij}(\phi)}$$

Q

Can we compute
the instanton contributions

IIA : Even (Euclidean) branes
wrapping odd cycles

II B : Odd branes
wrapping even
cycles

e.g. D1 over γ_2 .

(Becker, Becker,
Strominger)

In this talk, I will
determine

- . WORLDSHEET INST.
- . D(-1) INSTANTONS
- . D1

② IDEA

Use the $SL(2, \mathbb{Z})$

symmetry of type IIB

$S: F1 \leftrightarrow D1$

worldsheet
instantons



D1

instanton

(Witten)



Known from MIRROR
Symmetry (Candelas &

D(-1) instantons :

Use work by Green
and Gutperle, ...

about R^4 terms

in $D=10$

D-1 cannot probe the

CY_3 , so we can

compactify to $D=4$.

Q How to implement

this idea in $D=4$?

III SOME GEOMETRY

Dimension of HM

$$\text{moduli space} = 4(h_{1,1} + 1)$$

Quaternion-Kähler (QK

metric $(i=1, \dots, 4h_{1,1} + 4)$

$$ds^2 = g_{ij}(\phi) d\phi^i d\phi^j$$

Inherits $SL(2, \mathbb{R})$

symmetry (Isometry)

(Böhm, Günther,

Using the conformal calculus in $N=2$ SUGRA one can show there is a 1-to-1 correspondence between QK and conformal Hyperkähler geometry

HKC

$g_{AB}(\phi, \phi^{\circ})$

$4(m+2)$



compensator
for $D \times SU(2)_R$



Moreover, g_{AB} is
determined by a
SINGLE function

$$g_{AB} = D_A \zeta_B \chi$$

- de Wit, Kleijn, V.
- de Wit, Roček, V.
- Swann ; Galicki
- Bergshoeff, Cucu, T. de Wit
- T. Gheerardyn, S.V., Van

Any isometry of the
QK is lifted to an
invariance of $\chi(\phi, \phi')$

$$\delta_G \chi = 0$$

For $G = SL(2, \mathbb{Z})$, this
implies that χ is

MODULAR INVARIANT

\Rightarrow Key to the solution
of the problem!

IV

RESULTS

At tree-level in α'
and g_s , we determined

$$\chi_T^{\text{class}} = 4 r^0 (\text{Im } \tau)^2 V(z - \bar{z})$$

r^0 : compensator

τ : dilaton-axion complex

$$V(z - \bar{z}) = \frac{i}{48} d_{abc} (z - \bar{z})^a (z - \bar{z})^b (z - \bar{z})^c$$

\Rightarrow $SL(2, \mathbb{R})$ invariant

Perturbative corrections
in d' and g_s :

$$\chi^{\text{pert}} = \underbrace{4 \pi^2 (\text{Im } \tau)^{1/2}}_{\text{MODULAR INV.}} \chi_{\text{EULER}}$$

$$\times \underbrace{\left(\frac{1}{4} S(3) (\text{Im } \tau)^{3/2} + \frac{1}{2} S(2) (\text{Im } \tau)^{2/2} \right)}_{\text{Not modular inv!}}$$

$$S: \text{Im } \tau \rightarrow \frac{\text{Im } \tau}{|\tau|^2}$$

There is a **UNIQUE** way to complete this into a modular invariant consistent with susy :

$$\chi = 2\pi \chi_{\text{Euler}} \tau^0 (\text{Im } \tau)^{1/2} *$$
$$* E_{3/2}(\tau, \bar{\tau})$$

The Eisenstein series can be expanded and includes a sum over all $(D-1)$ instantons & anti-instantons

With $\tau = \tau_1 + i\tau_2$

$$4\pi E_{3/2} = 2 S(3) \tau_2^{3/2}$$

$$+ \frac{2\pi^2}{3} (\tau_2)^{-1/2}$$

} pert.
loops

$$+ 4\pi^{3/2} \sum_{m,n \geq 1} \left(\frac{m}{n^3}\right)^{1/2} \left(e^{\frac{2\pi i m \tau}{D-1}} + \frac{\text{c.c.}}{(D-1)} \right)$$

$$\times \left[1 + \sum_{k=1}^{\infty} \frac{\Gamma(k-1/2)}{\Gamma(k+1/2)} \frac{1}{(4\pi m n \tau_2)^k} \right]$$



Fluctuations around
the instantons

The real stuff:
applying the c-map
on the worldsheet
instanton corrections,
we get

$$\chi = -2\tau^0 (\tau_2)^2 \sum_{d_a} n_{d_a} \left[\text{Li}_3(e^{i w}) \right. \\ \left. + w_2 \text{Li}_2(e^{i w}) + \text{c.c.} \right]$$

$$w = w_1 + i w_2 = 2\pi d_a z^a$$

$$\text{Li}_n(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^n}$$

This function is not $SL(2, \mathbb{Z})$ invariant because $D1$ -instantons are not included.

One can apply the
METHOD OF IMAGES
to make it modular
invariant

$$\tilde{f}(z, \bar{z}) = \sum f(g\tau, g\bar{\tau})$$

The result of this method gives

$$\chi^{\text{inst}} = -2^0 (\tau_2)^{1/2} \sum_{d_a} n_{d_a} \times$$

$$\times \sum_{(m,n)} \frac{(\tau_2)^{3/2}}{|m\tau + n|^3} (1 + |m\tau + n| w_2)$$

$$\times e^{-S_{m,n}}$$

SUM OVER

"BOUND STATES"

OF MIXED

INSTANT

ACTION

The instanton action
gives the right answer
for D1-instantons when

$$n=0$$

$$S = \frac{|m|}{g_s} + i d_a A^a$$

↑
RR scalars

Modular invariance:

$$T: S_{m,n} \rightarrow S_{m,n+m}$$

$$S: S \rightarrow S$$