pp-Wave Matrix Models from Point-Like Gravitons

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Motivation

1. The fuzzy 5-sphere vacuum in BMN

2. Type IIB pp-wave matrix models

Results

Starting with the action for coincident gravitons:

1. Physical matrix model admitting a fuzzy 5-sphere as a supersymmetry preserving solution

2. Type IIB pp-wave matrix model with the right fuzzy 3-sphere solutions

Outline

1. The action for coincident gravitons

2. pp-wave matrix models from point-like gravitons

THE ACTION FOR COINCIDENT GRAVITONS

- Motivation: Microscopical description of giant gravitons
- Construction
- Check: Giant gravitons in $AdS_m \times S^n$

The construction

D0-branes are gravitons in M-theory, moving along the 11th direction:

M:		M0		M2		M5
IIA:	wave	D0	F1	D2	D4	NS5

$\Rightarrow$ Uplift Myers action for N D0-branes

Matrix Theory calculation:

- Matrix string theory: (Non-Abelian) Type IIA strings with non-zero light-cone momentum
- Sen-Seiberg limit + Static gauge $\rightarrow$ (Non-Abelian) massless particles with spatial momentum (IIA gravitational waves) (Janssen & Y.L. '02)

Dielectric couplings?: Matrix string theory in a weakly curved background (Schiappa '00; Brecher, Janssen & Y.L. '01):

$$S = S_{\text{flat}} + S_{\text{linear}}$$

$$S_{\text{linear}} \supset \text{dielectric couplings}$$

$$S_{\text{linear}} = \int \text{STr}\{\frac{1}{2}h_{AB}T^{AB} + C^{(3)}_{ABC}J^{ABC} + C^{(6)}_{ABCDEF}M^{ABCDEF}\}$$

(Kabat & Taylor '98)

Sen-Seiberg limit:

$$S^{(\text{linear})}_{ND0} = \int \text{STr}\{\frac{1}{2}h_{ab}I^{ab}_{h} + \phi I_{\phi} + B^{(2)}_{ab}I^{ab}_{s} + C^{(1)}_{a}I^{a}_{0} + C^{(3)}_{abc}I^{abc}_{2} + \ldots\}$$

(Taylor & Van Raamsdonk '99)

with

$$C^{(1)}_{a}I^{a}_{0} = P[C^{(1)}]\,; \qquad C^{(3)}_{abc}I^{abc}_{2} = iP[i_X i_X C^{(3)}]$$

$\rightarrow$ Precise agreement with the linear expansion of Myers action

The action for M-theory gravitons

$$S = T_0 \int d\tau \mathrm{STr}\{-k^{-1}\sqrt{E_{00} + E_{0i}(Q^{-1} - \mathbb{1})^i_k E^{kj} E_{j0}}\sqrt{\mathrm{det}Q}$$

$$-k^{-2} k_i \partial X^i + i(i_X i_X)C^{(3)} + \frac{1}{2}(i_X i_X)^2 i_k C^{(6)} + \ldots\}$$

k^μ: Killing vector pointing along the direction of propagation (isometric $\leftrightarrow$ momentum eigenstate ($\leftrightarrow$ superstar solutions smeared in the ϕ-direction (Myers & Tafjord '01)))

$$E = \mathcal{G} + k^{-1}(i_k C^{(3)})\,, \qquad Q^i_j = \delta^i_j + ik[X^i, X^k]E_{kj}$$

In the Abelian limit: Legendre transformation :

$$S[\gamma] = -\frac{T_0}{2} \int d\tau \sqrt{|\gamma|}\gamma^{-1}\partial X^\mu \partial X^\nu g_{\mu\nu}$$

Check: Giant gravitons in $AdS_m \times S^n$

- S^2 giant gravitons:
  - Fuzzy 2-sphere solutions
  - $P_\phi \neq 0$, $C^{(3)}_{\phi ij} \neq 0$
  - Exact agreement with GST in the large N limit

- S^3 giant gravitons:

  $AdS_5 \times S^5$

  IIB background $\Rightarrow$ Action for Type IIB gravitons

  T-duality direction isometric $\leftrightarrow l^\mu$

$$E = \mathcal{G} - k^{-1} l^{-1} e^{\phi} (i_k i_l C^{(4)})$$

  - Fuzzy 3-sphere solutions: S^1 bundles over fuzzy 2-spheres
  - $P_\phi \neq 0$, $C^{(4)}_{\phi\chi ij} \neq 0$
  - Exact agreement with GST in the large N limit

- S^1 giant gravitons: Fuzzy cylinders

- S^5 giant gravitons: S^1 bundles over fuzzy CP^2

PP-WAVE MATRIX MODELS FROM POINT-LIKE GRAVITONS

The BMN matrix model gives the dynamics of DLCQ M-theory in the sector with momentum $2p^+ = -p_- = N/R$

It contains a dielectric coupling to the 3-form potential which supports a fuzzy 2-sphere vacuum solution

There should be as well a fuzzy 5-sphere vacuum which is however not supported by a dielectric coupling (BMN: non-perturbative origin)

x^- is a null direction, but BMN can be derived from the action for M-theory gravitons taking into account that before Penrose limit $p_\phi = -p_- \frac{\mu L^2}{3}$, so that $p_- = -N/R \Leftrightarrow p_\phi = N$ and $R = \mu L^2/3$

The BMN matrix model with coupling to the 6-form potential

1. Take in the BMN background

$$ds^2 = -4dx^+dx^- - [(\frac{\mu}{3})^2(x_1^2 + x_2^2 + x_3^2) + (\frac{\mu}{6})^2 y^2](dx^+)^2$$
$$+dx_1^2 + dx_2^2 + dx_3^2 + dy^2 + y^2 d\Omega_5^2$$

$$C^{(3)}_{+ij} = \frac{\mu}{3}\epsilon_{ijk}x^k \quad i,j = 1,2,3 \qquad C^{(6)}_{+\gamma_1\ldots\gamma_5} = -\frac{\mu}{6}y^6\sqrt{g_\gamma}$$

$d\Omega_5^2 = (d\chi - A)^2 + ds^2_{CP^2}$ and $k^\mu = \delta^\mu_\chi$

2. Use that CP^2 can be defined as the submanifold of $\mathbb{R}^8$ determined by the constraints:

$$(z^a)^2 = 1\,, \qquad d^{abc}z^b z^c = \frac{1}{\sqrt{3}}z^a$$

In these coordinates

$$ds^2_{CP^2} = \sum_{a=1}^{8} (dz^a)^2 \qquad C^{(6)}_{+\chi abcd} = \frac{\mu}{3} y^6 f_{[abe} f_{cd]f} z^e z^f$$

and $C^{(6)}$ couples in the CS action through $(i_Z i_Z)^2 i_k C^{(6)}$

Using the action for M-theory gravitons in this background

$$\begin{aligned} H &= -\int dx^+ \mathrm{STr}\Big\{ \frac{1}{4R}(\dot{X}^2 + \dot{y}^2 + y^2 \dot{Z}^2) - \frac{1}{4R}(\frac{\mu^2}{9} X^2 + \frac{\mu^2}{36} y^2) + \\ &\quad + \frac{1}{2} R [X,X]^2 - \frac{1}{16} R\, y^{10} [Z,Z]^4 + i \frac{\mu}{3} \epsilon_{ijk} X^k X^j X^i - \\ &\quad + \frac{\mu}{6} y^6 f_{[abe} f_{cd]f} Z^d Z^c Z^b Z^a Z^e Z^f \Big\} \end{aligned}$$

The fuzzy 5-sphere solution

Make non-commutative the CP^2 base manifold

CP^2 base manifold $\rightarrow$ Fuzzy CP^2
Fuzzy S^5=S^1 bundle over a fuzzy CP^2

The constraints

$$(z^a)^2 = 1\,, \qquad d^{abc} z^b z^c = \frac{1}{\sqrt{3}} z^a$$

can be realized at the level of matrices taking

$$Z^a = \frac{1}{\sqrt{C_N}} T^a \qquad \text{with} \qquad T^a,\ a = 1, \ldots 8,\ \text{generators of } SU(3)$$

in the irreps $(n, 0)$ or $(0, n)$

Taking the ansatz $r = X^i = 0$, y, Z^a time independent: 5-sphere vacuum solution with radius

$$y = \left(\frac{\mu(n^2+3n)}{3R}\right)^{1/4} \sim \left(-\frac{2}{3}\mu p_-\right)^{1/4} \qquad \text{for large } N$$

$\rightarrow$ Exactly as the 5-sphere giant graviton of GST

Compare to the S^n_{fuzzy}, $n > 2$, constructed as $SO(n+1)$-covariant matrix realizations of the condition $\sum_{i=1}^{n+1}(X^i)^2 = \mathbb{1}$:

- Complicated technically (Ramgoolam'01,'02)
- The construction with a fuzzy S^5 scales like $(-p_\phi)^{1/5}$ (Nastase'04)

Type II pp-wave matrix models

Type IIA : Matrix string theory in the pp-wave background of Sugiyama & Yoshida'02 and Das, Michelson & Shapere'03 from the action for Type IIA gravitons

Type IIB

1. Take in the maximally supersymmetric IIB pp-wave background

$$ds^2 = -2dx^+dx^- - \mu^2(r^2+y^2)(dx^+)^2 + dr^2 + r^2 d\Omega_3^2$$
$$+dy^2 + y^2 d\tilde{\Omega}_3^2$$
$$C^{(4)}_{+\alpha_1\alpha_2\alpha_3} = -\mu r^4 \sqrt{g_\alpha}\,, \qquad C^{(4)}_{+\gamma_1\gamma_2\gamma_3} = -\mu y^4 \sqrt{g_\gamma}$$

$d\Omega_3^2 = (d\chi - A)^2 + d\Omega_2^2$ and $k^\mu = \delta^\mu_\phi$, $l^\mu = \delta^\mu_\xi$ with $\xi = (\chi + \tilde{\chi})/2$

2. Use Cartesian coordinates to parametrize the S^2

Proposal for the Type IIB matrix model:

$$S = \int dx^+ \mathrm{STr}\Big\{ \frac{1}{2R}\Big(\dot{r}^2 + \dot{y}^2 + \frac{r^2}{4}\dot{X}^2 + \frac{y^2}{4}\dot{Z}^2\Big) - \frac{\mu^2}{2R}(r^2+y^2) + \\ + \frac{1}{256}R(r^2+y^2)\Big(r^4[X,X]^2 + y^4[Z,Z]^2 + 2r^2y^2[X,Z]^2\Big) + \\ -i\frac{\mu}{8}r^4\epsilon_{ijk}X^iX^jX^k - i\frac{\mu}{8}y^4\epsilon_{abc}Z^aZ^bZ^c\Big\}$$

Check: It contains fuzzy 3-sphere vacua with the right continuum limit. These fuzzy S^3 are defined as S^1 bundles over fuzzy S^2.

Candidate to the holographic description of strings in the pp-wave background. Compare to Sheikh-Jabbari's *Tiny graviton matrix theory*

CONCLUSIONS

1. pp-wave matrix model containing the fuzzy 5-sphere giant graviton as a supersymmetry preserving solution

   - BMN plus a quadrupolar coupling to the 6-form potential
   - More "non-perturbative" $\leftrightarrow$ The direction of propagation of the gravitons and the geometrical S^1 have been interchanged
   - BMN as a theory of coincident gravitons

2. Same idea in Type IIB: pp-wave matrix model containing fuzzy 3-sphere giant graviton solutions

   Compare to Sheikh-Jabbari's *Tiny graviton matrix theory*

Sheikh-Jabbari'04:

Regularize the light-cone 3-brane action

3-sphere vacua carrying N units of light-cone momentum in terms of N expanding gravitons each carrying one unit of light-cone momentum $\rightarrow$ Tiny gravitons

- $U(N)$ gauge theory with 6 non-Abelian scalars plus 2 Abelian vs 8 non-Abelian scalars plus $\mathcal{L}_5$
- Simpler realization of the fuzzy 3-sphere + right continuum limit